Preparation for CAS Problem Solver: Fraction Machine
USING **DEFINE, SOLVE** AND **SUBSTITUTION** ON TI-89 CAS:

PART A

1. What do you notice about the following pairs of fraction calculations?

$$\frac{3}{5} - \frac{3}{8} \qquad\qquad \frac{3}{5} \times \frac{3}{8}$$

$$\frac{4}{7} - \frac{4}{11} \qquad\qquad \frac{4}{7} \times \frac{4}{11}$$

$$\frac{3}{4} - \frac{3}{7} \qquad\qquad \frac{3}{4} \times \frac{3}{7}$$

$$\frac{5}{8} - \frac{5}{13} \qquad\qquad \frac{5}{8} \times \frac{5}{13}$$

$$\frac{6}{5} - \frac{6}{11} \qquad\qquad \frac{6}{5} \times \frac{6}{11}$$

2. Find some other pairs of fractions whose difference is equal to their product.

3. Suppose that a and b are positive integers. Form a fraction $\frac{a}{b}$. Use your CAS calculator to answer the following question: What is the fraction that when subtracted from $\frac{a}{b}$ or multiplied by $\frac{a}{b}$ produces the same answer?

Calculator Hint:

4. a) Take a pair of distinct positive integers. Form a fractional number bigger than 1 by dividing the larger by the smaller.

b) Form a new fraction whose numerator is the same as your first fraction and whose denominator is the difference of your two original integers.

c) You now have two fractions. Consider their sum and their product. What do you notice? Can you explain?

d) Use your CAS calculator to generalise your answer to part c).

Calculator steps for part d:
Clear a-z first

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┌F1┬┐┌F2┬┐┌F3┬┐┌F4┬┐┌F5─┐┌──F6┬──┐
│Tools│Algebra│Calc│Other│PrgmIO│Clean Up│
 1:Clear a-z…
 2:NewProb
 3:Restore custom default
```

```
■ NewProb                    Done
NewProb
TYPE OR USE ←→↑↓ + [ENTER] OR [ESC]
```

Define $f_1 = \dfrac{a}{b}$

Define $f_2 = \dfrac{a}{a-b}$

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┌F1┬┐┌F2┬┐┌F3┬┐┌F4┬┐┌F5─┐┌F6┬┐┌──┐
│Tools│Algebra│Calc│Other│PrgmIO│Clean Up│  │
■ NewProb                    Done
■ Define f1 =ᵃ/ᵦ             Done
■ Define f2 =ᵃ/ₐ₋ᵦ           Done
Define f2=a/(a-b)
MAIN      DEG AUTO    FUNC      3/30
```
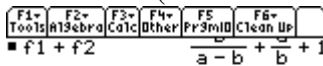
Find the general expressions for:

$f_1 + f_2$

comDenom(

```
┌F1┬┐┌F2┬┐┌F3┬┐┌F4┬┐┌F5─┐┌F6┬┐┌──┐
│Tools│Algebra│Calc│Other│PrgmIO│Clean Up│  │
■ f1 + f2          ᵇ/ₐ₋ᵦ + ᵃ/ᵦ + 1
■ comDenom( ᵇ/ₐ₋ᵦ + ᵃ/ᵦ + 1 )
                        a²
                     ─────────
                     a·b - b²
comDenom(b/(a-b)+a/b+1)
MAIN      DEG AUTO    FUNC      5/30
```

$f_1 \times f_2$

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┌F1┬┐┌F2┬┐┌F3┬┐┌F4┬┐┌F5─┐┌F6┬┐┌──┐
│Tools│Algebra│Calc│Other│PrgmIO│Clean Up│  │
comDenom( ᵃ/ₐ₋ᵦ  ᵇ/ᵦ  ... )
                        a²
                     ─────────
                     a·b - b²
■ f1·f2                 a²
                     ─────────
                     (a - b)·b
f1*f2
MAIN      DEG AUTO    FUNC      6/30
```

PART B

Example: The algebraic fraction is defined as $f_r = \dfrac{f + \dfrac{1}{f}}{1 - f^2}$.

1. Use Define function on your calculator to define f_r.

2. Use the substitution feature on your calculator to find the values of f_r for the following pairs of fractions f: $\dfrac{1}{2}, -\dfrac{1}{2}$; $\quad \dfrac{1}{4}, -\dfrac{1}{4}$; $\quad \dfrac{1}{5}, -\dfrac{1}{5}$; $\quad \dfrac{3}{7}, -\dfrac{3}{7}$.

Toois F1▾ F2▾ F3▾ F4▾ F5 PrgmIO F6▾

$$\blacksquare\ Define\ fr = \dfrac{f + \dfrac{1}{f}}{1 - f^2}\qquad \boxed{Done}$$

$\blacksquare\ fr\,|\,f = \{1/2\quad {}^-1/2\}$

$\{10/3\quad {}^-10/3\}$

fr|f={1/2, -1/2}

MAIN   DEG AUTO   FUNC   2/3

F1▾ F2▾ F3▾ F4▾ F5 F6▾
Tools Algebra Calc Other PrgmIO Clean Up

$\blacksquare\ fr\,|\,f = \{3/7\quad {}^-3/7\}$

$\left\{\dfrac{203}{60}\quad -\dfrac{203}{60}\right\}$

fr|f={3/7, -3/7}

MAIN   DEG AUTO   FUNC   1/30

3. Look closely at the results. What do you notice? Write down a statement which will summarise what happens.

4. Now find the values of f_r for the following pair of fractions: $f = \dfrac{a}{b}$ and $f = -\dfrac{a}{b}$.

F1▾ F2▾ F3▾ F4▾ F5 F6▾
Tools Algebra Calc Other PrgmIO Clean Up

$\blacksquare\ fr\,|\,f = \dfrac{a}{b}\qquad \dfrac{-\left(a^2 + b^2\right)\cdot b}{a\cdot\left(a^2 - b^2\right)}$

$\blacksquare\ fr\,|\,f = \dfrac{-a}{b}\qquad \dfrac{\left(a^2 + b^2\right)\cdot b}{a\cdot\left(a^2 - b^2\right)}$

fr|f=-a/b

Note: Domain of result may be larger

5. Copy the results from your calculator. What do you notice?