EXPLORATIONS ™

Activity 8

Rotations in the Plane

Introduction

Rotations about a point constitute a third type of rigid transformation. In this activity, you will explore the invariant properties of rotations and the results of rotations in the coordinate plane.

This activity makes use of the following definitions:

Rotation — a transformation that turns a figure through a specified angle and direction about a fixed point

Pre-image — the original object that is to be transformed

Image — the new object created by applying the conditions of a transformation

Part I: Properties of a Rotation
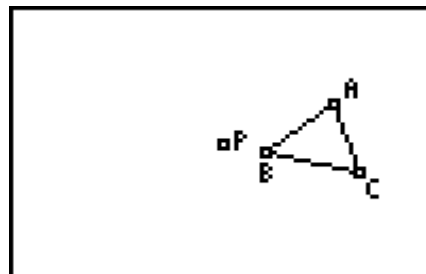
Construction

I. Construct a triangle to rotate and a point that will be the center of rotation.

Draw a small scalene triangle $\triangle ABC$ on the right side of the screen.

Draw point P near the center of the screen.

II. Construct a rotation angle tool.

⊙ Draw a small circle near the lower left corner of the screen.

🖉 Hide the defining radius point of the circle since it will not be used in the construction.

╱ Construct two radii of the circle to form a central angle.

∠° Measure the central angle.
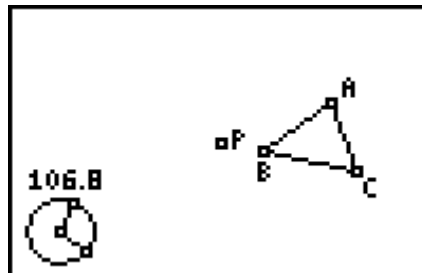
III. Rotate the triangle.

To use the **Rotation** tool you must do the following in this order:

• Select what is to be rotated.

• Select a point to be the center of rotation.

• Identify the angle of rotation by selecting three points that define an angle.

The rotation angle is selected last and is done in the same way that an angle would be named, side-vertex-side. The order in which the angle-defining points are selected controls the direction of the rotation, clockwise or counter-clockwise.

↻ 𝐀 Rotate Δ*ABC* around point *P* using the central angle of the circle as the angle of rotation. The new triangle formed is the image of the original triangle rotated about point *P* by an angle equal to the central angle. Label the corresponding vertices of Δ*ABC*, *A*1, *B*1, *C*1.
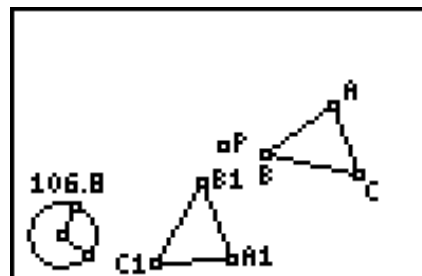
Note: This is a clockwise rotation. If the entire image does not appear on the screen, try making ΔABC smaller or moving it closer to point P.

Exploration

🖎 Observe the relationships between the pre-image and the image formed by the rotation by dragging the vertices and sides of Δ*ABC*, Δ*ABC*, point *P,* and the points on the circle.

⇄ The **Animate** tool automatically sets into motion a point on a defined path such as a segment or a circle. Use the **Animate** tool to animate the rotation angle by selecting one of the radius points on the circle. Confirm that the relationships you observed are still valid by dragging Δ*ABC* or point P as the animation takes place.

Note: To stop the animation, select the **Animate** tool, and then press ⎡2nd⎤ ⎡ENTER⎤.

Questions and Conjectures

1. Make a conjecture about how the **Rotation** tool determines the direction of the rotation. Explain how you would test your conjecture.

2. Make a list of the properties of the pre-image that are preserved in the image. Use various measurement tools (**Distance and Length**, **Angle**, and **Slope**) to verify your answers.

3. Can the **Translation** tool or the **Reflection** tool be used to create the same image that can be created using the **Rotation** tool? Explain your reasoning and be prepared to demonstrate.

Part II: Rotations in the Coordinate Plane
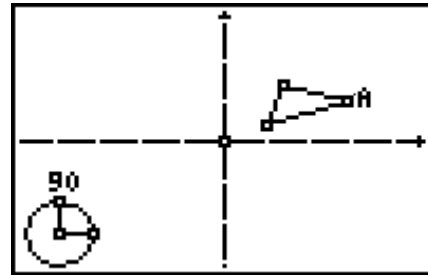
Construction

I.   Construct a triangle on a coordinate axis to rotate.

⌫   Clear the previous construction.

▨   Show the axes on the screen. Drag the origin to the center of the screen.

◿ 𝐀 Draw a small scalene triangle in the first quadrant. Label one vertex *A*.

⊙ ◠ Create a rotation angle tool by constructing a central angle of a small circle located near the lower left corner of the figure. This angle will be used to define the angle of rotation.

▱ Measure the central angle.

II. Rotate the triangle and identify coordinates of some verticies.

⋮ Rotate the triangle counter-clockwise around the origin using the central angle of the small circle as the rotation angle.

⊙ Construct a circle with the center at the origin and having *A* as its radius point. Notice that the image of *A* is also on the circle.
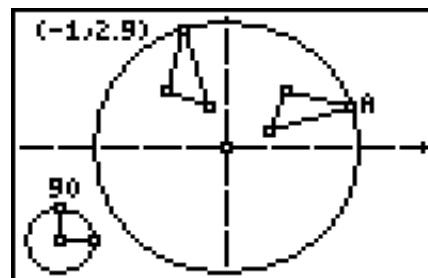
Note: *Coordinates of A are not shown.*

▦ Show the coordinates of *A* and its image.

Exploration

Form a 90° counter-clockwise rotation. Drag *A* to different locations and look for patterns in the coordinates of *A* and its image.

Repeat the previous step using counter-clockwise rotations of 180°, 270°, and 360°.

Note: The angle measurements of the central angle will always be less than or equal to 180°, even though the image rotates around the complete circle.

Questions and Conjectures

Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and those of its image when the triangle has been rotated counter-clockwise as defined in the exploration. Complete the following table based on your observations.

Rotation Angle	Central Angle	*A*	Image of *A*
90°		(x, y)	
180°		(x, y)	
270°		(x, y)	
360°		(x, y)	

Teacher Notes

Activity 8

Rotations in the Plane

Additional Information

It is recommended that you complete Activity 6, Translations in the Plane, and Activity 7, Reflections in the Plane, prior to doing this activity.

A great way to summarize this activity would be to develop a working definition for rotation as a whole class activity.

Part I: Properties of a Rotation

Answers to Questions and Conjectures

1. Make a conjecture about how the **Rotation** tool determines the direction of the rotation. Explain how you would test your conjecture.

    The order used to select the three defining points of the angle Rotation tool determines the direction of the rotation. Use the **Rotation** tool twice, defining the rotation angle counter-clockwise or clockwise each time.

2. Make a list of the properties of the pre-image that are preserved in the image. Use various measurement tools (**Distance and Length**, **Angle**, and **Slope**) to verify your answers.

    A rotation preserves the following properties from the pre-image to the image: length, area, angle measure, perimeter, and orientation. Unlike a translation, the slopes of corresponding sides of the pre-image and image are not always equal.

3. Can the **Translation** tool or the **Reflection** tool be used to create the same image that can be created using the **Rotation** tool? Explain and be prepared to demonstrate.

    In general, neither the **Translation** tool nor the **Reflection** tool can produce the same images as the **Rotation** tool. Special cases such as rotating an equilateral triangle 120° can be duplicated using a translation.

Part II: Rotations in the Coordinate Plane

Answers to Questions and Conjectures

Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and those of its image when the triangle has been rotated counter-clockwise as defined in the exploration. Complete the following table based on your observations.

Rotation Angle	Central Angle	A	Image of A
90°	90°	(x, y)	$(-y, x)$
180°	180°	(x, y)	$(-x, -y)$
270°	90°	(x, y)	$(y, -x)$
360°	0°	(x, y)	(x, y)