Deciding About Planting & Harvesting - An Application of the Definite Integral

NOTE: While working with this lesson, we recommend that you open a 2D-plot Window and then choose the option Window > Tile Vertically. Return to the Plot Window and choose the option Set >Plot Range. Choose a range of -50 to 400 with 9 intervals for *x* and -3 to 24 with 9 intervals for *y*. Now you will be able to follow the lesson and display your graphical results side by side.

Background

A farmer is planning to "rest" one of his fields by planting it in clover for hay. The farmer would like to get three cuttings of hay from the field. Because of the need to perform other duties around the farm, it is best if the last cutting can be made close to September 15th. The main question for the farmer is when to plant the field and when to plan for the first two cuttings.

Your task is to help the farmer plan. You do some research by contacting a local seed supplier and find that the hay requires, assuming normal weather conditions, approximately 600 hours of sunlight during the growing season to reach a maturity that would yield a proper level of nourishment for the larger farm animals (cattle and horses).

Your next step is to find a way of calculating the hours of sunlight during each day of the year. You make the assumption that this is a periodic phenomenon with a period of one year (= 365.242 days). Thus a graph for the number of hours of daylight *t* days after the beginning of the year should follow a curve of the form:

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year should follow a curve of the form:

$$\#1: \quad a + b \cdot \text{SIN}\left(\frac{2 \cdot \pi}{365.242} \cdot (t - \alpha)\right)$$

The assumption is that the number of hours of daylight will follow a sinusoidal curve. The constant $2\pi/365.242$ controls the length of the period of the curve.

Exercise

1) Substitute values for *a*, *b* and α in expression #1. Graph the results. Can you determine what aspects of the curve are controlled by each of these parameters.

As a result of your investigations in exercise #1, you may have determined that α is the offset or phase shift of the sine curve. Thinking about the course of the sun we can set the values for our parameters. After the Vernal Equinox on March 21 the days get longer until the Summer Solstice on June 21; then they get shorter returning to equal hours of sunlight on the Autumnal Equinox on September 21, and continue getting shorter with less daylight than darkness until the Winter Solstice on December 21 when the hours of daylight are a minimum. Finally, the days become longer until the cycle repeats itself. The phase shift for the cycle must be to the position of the Vernal Equinox, or March 21, the 80th day of the year. Thus, $\alpha = 80$.

$$\#2: \quad a + b \cdot \text{SIN}\left(\frac{2 \cdot \pi}{365.242} \cdot (t - 80)\right)$$

The values for the other parameters depend on one's location on the globe. The farmer in our example is at Latitude 39° 55' 49” and Longitude 77° 14' 54”. You can go to the web site located at the URL:
http://triton.srrb.noaa.gov/highlights/sunrise/sunrise.html,
which is run by NOAA, to find the following information for this location.

#3:

Date	Sunrise	Sunset	Hours
Mar 21	6:11	18:22	12:11
Jun 21	5:40	20:41	15:01
Sep 21	6:56	19:08	12:12
Dec 21	7:27	16:47	9:20

Exercise

2) The web site mentioned above will help you to find the Latitude and Longitude for your location. Using this information construct a table similar to the one in expression #3 for your location.

From the information in the table we can substitute into expression #2 for the hours of daylight in our area. Note that the hours of daylight for the Vernal and Autumnal Equinoxes differs by a minute. We will use their average as our baseline. We will also need to make a slight adjustment for the two solstices. This results from our model being an approximation to the physical phenomenum. All of the adjustments are in the second decimal place.

#4: $$12.192 + 2.84 \cdot \text{SIN}\left(\frac{2 \cdot \pi}{365.242} \cdot (t - 80)\right)$$

<u>Exercise</u>
3) Using the informatioin from your area, write the expression for the hours of daylight on day, *t*, that is appropriate for the area, i.e. the analog of expression #4 for your location.

Now we are ready to solve the farmer's problem.

Relating Total Hours of Daylight to the Integral

What is this assignment doing in a section on the Definite Integral? Stated simply, the integral is an extension of the idea of summing the values of a function over an interval. Let's take an example.

In the plot window restrict the range of the *t*-variable to 50 to 100 with 5 intervals and the *y*-variable to -3 to 15 with 6 intervals. This is done using the Set >Plot Range dialog box. If we want to calculate the total number of hours of daylight from day 60 (March 1) to day 80 (March 21), we can take a representative value for the hours of daylight function for each day and add those values. In expression #6 we have done just that. We divided the interval from 20 to 80 into subintervals of one day and chose the midpoint of the interval as the representative value. Simplify expression #6.

#5: $$\text{HDL}(t) := 12.192 + 2.84 \cdot \text{SIN}\left(\frac{2 \cdot \pi}{365.242} \cdot (t - 80)\right)$$

#6: $$\sum_{t=60}^{79} \text{HDL}(t + 0.5)$$

We can represent this sum graphically by plotting the following points and connecting them.

#7: $$\text{VECTOR}\left(\begin{bmatrix} i & 0 \\ i & \text{HDL}(i + 0.5) \\ i + 1 & \text{HDL}(i + 0.5) \\ i + 1 & 0 \end{bmatrix}, i, 60, 79\right)$$

Plot this expression and enlarge the plot about the tops of the rectangles. Then return plot settings to those given prior to expressions #5 and #6. When you looked at a close up of the tops of the rectangles, you saw that essentially we had as much area under the curve lying above of the rectangle as we had area of the rectangle lying above of the curve. In other words, the area under the curve should be the same as the sum of the areas of the rectangles. But the sum of the areas of the rectangles (remember each rectangle has width 1) is the value of expression #6, the total number of hours of daylight. Also the graph of expression #7 looks very much like an approximating sum for the definite integral. Thus, the value of the sum and the value of the definite integral should be approximately the same. Check this statement by simplifying the following expression. Compare your result with that of simplifying expression #6.

#8: $\int_{60}^{80} \mathrm{HDL}(t)\, dt$

The agreement is not perfect, but it is quite close.

Exercise

4) In the example we chose an interval where the graph of the function was rather straight. Choose an interval of about 20 days that contains one of the solstices in its interior and repeat the comparison of the number of hours of daylight over the interval and the value of the definite integral. Comment on the accuracy of the approximation and issues affecting the integral as an estimate.

Helping the Farmer Plan

We have seen that in order to calculate the number of days of daylight for a group of consecutive days from $t = a$ to $t = b$ can be done in two ways:

#9: $\sum_{t=a}^{b-1} \mathrm{HDL}(t + 0.5)$

#10: $\int_{a}^{b} \mathrm{HDL}(t)\, dt$

If the goal is to simply find the total hours of daylight between day a and day b, then expression #9 will give the best answer. The time to perform the calculation is not appreciably longer than using the approximation given by expression #10. However, the farmers question is of a different nature. We know b which is 258 (the value of t corresponds to September 15). What we want to do is find the value for a that will yield a value approximately 1800 = 3 *X* 600. In this situation the form of expression #10 makes sense. The answer will be the solution to expression #12. Simplify expression #11 and then find the soution to the farmers problem using expression #12.

#11: $\int_{a}^{258} \mathrm{HDL}(t)\, dt$

#12: $\mathrm{SOLVE}\left(\int_{a}^{258} \mathrm{HDL}(t)\, dt = 1800,\ a\right)$

What day of this year is this? It remains for you to complete the farmer's schedule by determining the dates when the first and second cuttings should be done.

Exercises

5) Find the dates for the first and second cuttings for the farmer. Remember that you need 600 hours of daylight between cuttings and also 600 hours from planting to the first cutting. You can do this either by working backwards from September 15 or forwards from the date found as a result of simplifying expression #12.

6) Work out a planting and harvesting schedule for clover for a farmer in your location.

7) (Project) Another phenomenum that is cyclic and related to planting is called Growing Degree Days. The GDD for any day is given by taking the average Maximum Temperature for the day and the Minimum Temperature for the day (using degrees Fahrenheit) and subtracting 50. Using a weather page (your local TV or radio station my have such a web page, NOAA is another source, or one of the channels that specialize in weather forcasts may have one) that archives the average highs and lows for each date, choose the 15th of each month and plot the average GDD for these dates. Fit a sine curve such as expression #1 to the data by approximating the maximum and minimum GDD values for the year. Now go to a local seed dealer and get a seed catalogue for corn or some other crop of interest. These catalogs list the total number of GDD from planting to harvest. Work out a planting schedule for the farmer to plant this crop in your area.

Carl Leinbach: leinbach@gettysburg.edu