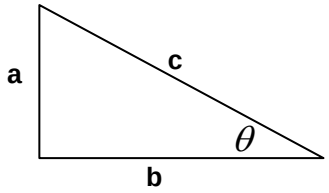
NUMB3RS Activity: Telling Time with the Sun
Episode: "Obsession"

Topic: Uses of Trigonometry **Grade Level**: 7 - 10
Objective: Be able to use values of tangent to compute angles in a right triangle.
Time: 10 - 15 minutes
Materials: Calculator, protractor, ruler

Introduction

Trigonometry is the study of triangle measurements and has been used by mathematicians for centuries to accurately determine distances without having to physically measure them. More importantly, it can help to calculate angles that would otherwise be very difficult to measure. In this activity, students estimate the angle of elevation using the tangent ratio. In a right triangle, the tangent of an angle is the ratio of the side opposite the angle to the side adjacent to the angle, excluding the hypotenuse.



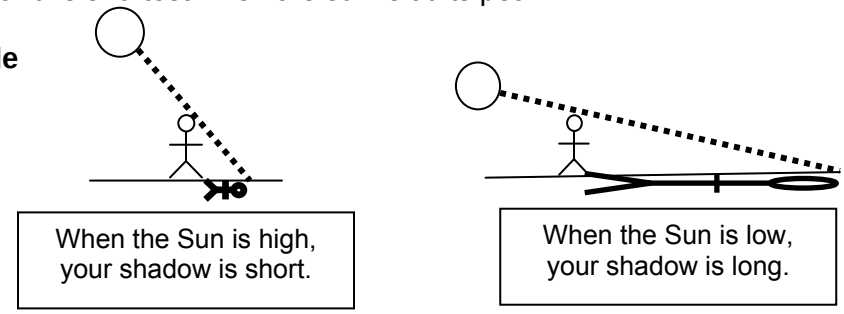
$$\tan(\theta) = \frac{a}{b}$$

Discuss with Students

***NUMB3RS* Example** In "**Obsession**," Charlie has a photograph that shows a basketball hoop and its shadow from the sun. He then uses trigonometry to determine the sun's angle of elevation. Knowing the sun's elevation and the time of day, Charlie could then figure out *where* the photograph was taken.

If you know where the photograph was taken but you don't know when, you might be able to figure out the angle of elevation of the sun if the photo contains enough information. As the sun moves across the sky, its angle of elevation changes—it first increases and then decreases before setting. Because the sun is moving, your shadow looks different at different times of the day. Your shadow is longer when the sun is low in the sky and is shortest when the sun is at its peak.

Example



Student Page Answers: **1**. *a = 3.5 cm, a = 7 cm, angle B = 64°, angle A = 26°, so a ÷ b = 0.50* **2**. *0.49* **3**. *they are equal (or very close)* **4**. **a**. *check drawing* **b.** *23/33 = 0.697* **c.** *≈ 35°* ***d.*** *34.9°* ***e.*** *less than 45°* ***f.*** *Answers will vary. For example, he could find the height of a suspect if he knows the shadow length and the angle of elevation. Note: Answers may change if the size of the figure changes.*

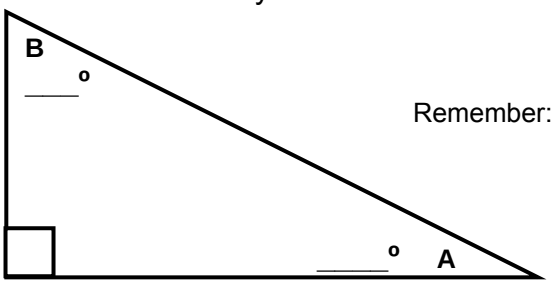
Patrick Flynn, Turner High School, Kansas City, KS

Name: _____ Date: _____

NUMB3RS Activity: Telling Time with the Sun

Explore the tangent of angles: Using a ruler and protractor, measure the labeled sides and angles of the right triangle to the nearest tenth of a centimeter and the nearest degree. Be sure to label your answers in the diagram.

Side a = _____cm



Remember: ∠A + ∠B = 90°

Side b = _____ cm

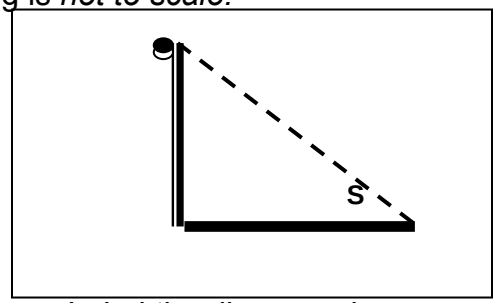
1. Give the value of the ratio $\frac{a}{b}$ (to the hundredths place): _____

2. Use your calculator to find the tangent (**TAN**) of ∠**A**: _____

3. What is the relationship between the tangent of ∠**A** and the ratio of sides a and b? _____

Use tangent to find angles: You can use tangent to find angles without measuring them. In the problem below, the drawing is *not to scale.*

4. Charlie has a photograph and is asked to determine what time of day the photograph was taken. If he can figure out the sun's angle of elevation, he can determine the time.



a. He knows that the streetlight in the picture is 23 feet tall, and from the size of bricks in the sidewalk he finds that the shadow is 33 feet long. Label the diagram above.

b. Angle **S** in the diagram is the angle of elevation of the sun. Use the ratios of the sides from part **a** to find the tangent of ∠**S**.

_____

c. To find the measure of ∠**S,** choose an angle between 0º and 90º, then find the tangent of that angle by pressing **TAN** on your calculator. Compare that measurement to the one you found for part **b.** What is your best estimate for the measure of ∠**S?** _____

Patrick Flynn, Turner High School, Kansas City, KS

Name: _____ Date: _____

d. You can also find the measure of $\angle$**S** by using the **TAN**$^{-1}$ function on your calculator. Verify that your answer to part **c** is correct by pressing **TAN**$^{-1}$ (23/33).

e. For what angles of elevation would your shadow be longer than your actual height? _____

f. Describe another situation where Charlie would use the tangent ratio to help solve a crime. _____
_____
_____
_____

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Activity: Make a Sundial

Introduction

In "Obsession," Charlie and Larry use two photographs of the same scene, taken at different times. By comparing the two photographs and using spherical geometry and trigonometry, they are able to determine the location of where the photos were taken. For an example closer to home, measure the length of the shadow of your school's flagpole (you must know its height) at two different times of day, on the same day (record the time to the nearest second). Then, you can use spherical geometry to compute the latitude and longitude of the flagpole. The formula needed is:

$$\sin\alpha = \sin\varphi\sin\delta + \cos\delta\cos\varphi(\tau - e_\delta - \lambda)$$

α = the sun's angle of elevation, φ = the geographic latitude, δ = the sun's declination (determined by the date), τ = the standard time as an angle, e_δ = the equation of time (determined completely by the date), and λ = the longitude difference from the central meridian of your time zone. To obtain the value of the rest of the necessary variables, here is a website, which gives the sun's declination and the equation of time, and standard time as an angle: **http://www.jgiesen.de/deceot/**

For the Student

Explain why you need to take latitude and longitude into consideration when you build a sundial. Think about the extreme examples of living on one of the poles or at the equator. How would this affect your construction?

Resources

http://www.rbduncan.com/sn4a2.doc.htm
An instructional guide to building a sundial.

http://www.qwerty.co.za/sundials/howto/steps.html
Another instructional guide to building a sundial.

http://www.astrosociety.org/education/publications/tnl/31/arch2.html
Learn how the ancient astronomers determined the elevation angle.

Related Topic

Most students study Euclidean Geometry which is based on a 2-dimensional plane. However, there are other types of geometries as well. In the show, Charlie uses spherical geometry, or geometry based on a sphere to determine the latitude and longitude of a photograph's subject based on the comparison of two photos. More advanced students can learn more about spherical geometry. A good starting point is the website:
http://www.math.uncc.edu/~droyster/math3181/notes/hyprgeom/node5.html

Patrick Flynn, Turner High School, Kansas City, KS