Spreading Doom

About the Lesson

Students explore a geometric sequence that models the spread of the 2004 Mydoom virus. After finding a rule for the sequence, they apply it recursively to extend it and graph the resulting data as a scatter plot. Then they derive, evaluate, and graph an exponential function to model the data. The activity concludes by discussing the meaning of the constants *a* and *b* in the exponential function $f(x) = ab^x$. As a result, students will:

- Display the terms of a geometric sequence on a spreadsheet using the formula for the *n*th term.
- Display the terms of a geometric sequence on a spreadsheet using recursion.
- Graph the terms of a geometric sequence as a scatterplot.
- Graph an exponential function of the form $f(x) = ab^x$ where *a* and *b* are non-zero real numbers and $b \neq 1$.

Vocabulary

- geometric sequence
- exponential function

Teacher Preparation and Notes

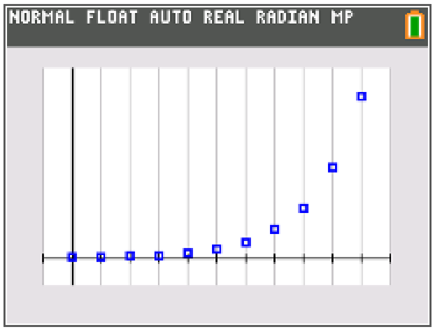
- This activity is appropriate for an Algebra 1 classroom. Students should have been exposed to arithmetic and geometric sequences and work competently with exponents and scientific notation.
- The student worksheet provides detailed instructions for the completion of the activity and a place for students to record their work. You may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

*** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

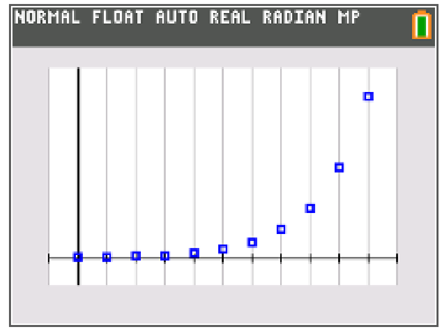
Lesson Files:

- Spreading_Doom_Student.pdf
- Spreading_Doom_Student.doc

Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing [2nd] [ZOOM] to change the [FORMAT] settings. If your students are using TI-84 Plus, they could use GridDot.

Modeling the Spread of a Virus

At 8 AM on January 26, 2004, the Mydoom virus was released. Six days later, over 1,000,000 people reported that their computers were infected and Mydoom was named the fastest spreading computer virus ever. In this activity, students explore how the virus spread so quickly and derive an equation to model its spread.



Before beginning, discuss as a class how the virus spread.

1. Users received an email with an attachment, often from a familiar email address.
2. If they opened the attachment, the virus infected their computer.
3. Then the virus sent copies of itself to all the email addresses in their contact list. The virus would even guess additional email addresses to send itself to. For example, if it found *bob@xyz.com* in a contact list, it would also send emails to *jim@xyz.com*, *alex@xyz.com* and so forth.
4. Those users would receive the emails and the cycle would repeat.

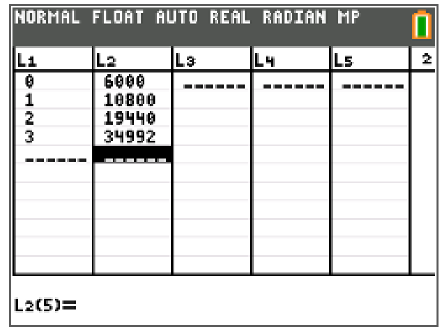
An example that you be more familiar with is a chain email: when you get the email, you forward it to your 5 closest friends. Then 1 + 5 = 6 people have seen the email. Then each of your 5 closest friends forwards it to their 5 closest friends. Now 31 (1 + 5 + 25) people have seen the email, and the chain continues.

The Mydoom virus was like a chain email that forwarded itself to your closest friends and thousands of other people you didn't even know!

No one knows exactly how fast Mydoom spread. The table shows one approximation of what we do know. The first column gives the number of hours since the virus was released. The second column gives the number of emails sent by the virus in that hour.

hours	email
0	6000
1	10800
2	19440
3	34992

Press [stat] than [enter] to open the List Editor. Enter the data from the table in lists **L1** and **L2**.



1. In your own words, describe how the virus spread.

Sample Answer: The virus searched through a user's contact list and sent out a copy of itself to everyone on that user's contacts.

The number of emails sent by Mydoom forms a sequence: 6000, 10800, 19440, 34992…

A geometric sequence is formed by multiplying a term in the sequence by a fixed number to find the next term. The fixed number is called the common ratio.

- How can you find and check the common ratio?
- What is the common ratio?

How can we find a rule for this sequence?

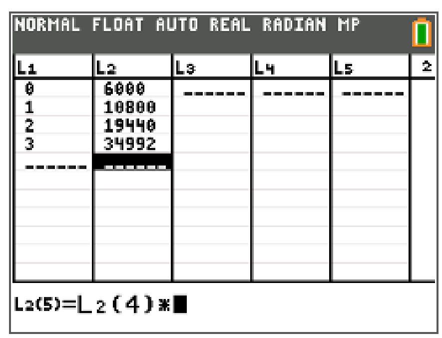
2. What is the common ratio for the sequence 6000, 10800, 19440, 34992, …? How do you know?

Answer: 1.8; by dividing a term by the term preceding it

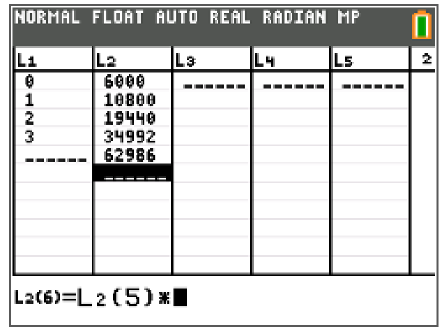
3. What is the rule to find the next term of this sequence?

Answer: Multiply by 1.8

Once you have a rule for this sequence, you can use it to find the next few terms. Each entry in a list is numbered. Arrow down to L2(5). Type **L2** by pressing [2nd] + [2]. Type **(4)** to represent the previous entry in the list, 34992. Then multiply L2(4) by the common ratio you found.



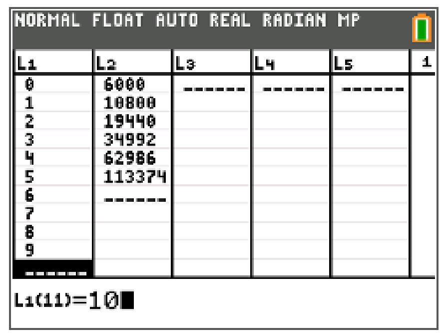
Repeat this process until you have found the first 10 terms of the sequence.



4. How many emails were sent by Mydoom in the 10th hour after it was released?

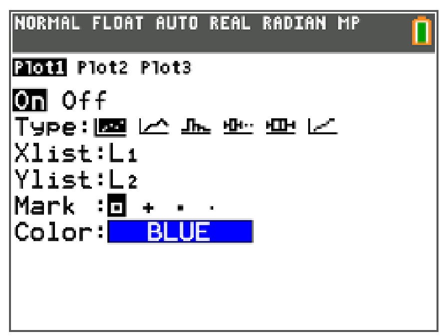
Answer: 2,142,280 emails

Make sure to fill in the number of hours in L1 as well.



Make a scatterplot of the sequence. Clear out or turn off any functions in the [y=] screen. Decide which variable, hours or emails, is independent and which is dependent.

Then press [2nd] + [y=] and open Plot1. Turn the plot **On**, choose **Scatter** as the type, and enter the appropriate **X** and **Y Lists**. Go to **Zoom > ZoomStat** to view the plot in an appropriately sized window.



Describe the shape of the scatterplot on your worksheet.

The virus continued to spread at this rate for 6 days or 144 hours. How many emails were sent in the 144th hour after it was released?

It would be too much to try to fill 144 entries in the list. We need a function that gives the number of emails $f(x)$ sent in the xth hour. Complete the table on your worksheet to find the function.

5. Describe the shape of the scatter plot.

Answer: It is an increasing curve that increases more rapidly as time increases.

6. Complete the table to find a function to model the spread of the Mydoom virus.

Answers:

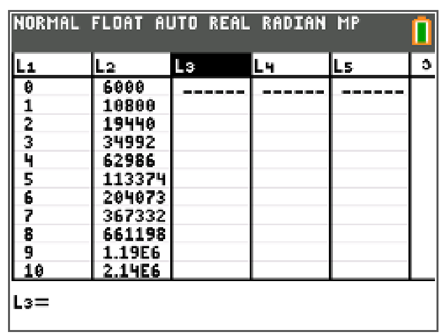
x	*f*(*x*)	Exponential Expression
x = 0	*f*(0) = 6000	*f*(0) = 6000 * 1.8^0
x = 1	*f*(1) = *f*(0) * 1.8 = 6000 * 1.8	*f*(1) = 6000 * 1.8^1
x = 2	*f*(2) = *f*(1) * 1.8 = 6000 * 1.8 * 1.8	*f*(2) = 6000 * 1.8^2
x = 3	*f*(3) = *f*(2) * 1.8 = 6000 * 1.8 * 1.8 * 1.8	*f*(3) = 6000 * 1.8^3
x = 4	*f*(4) = *f*(3) * 1.8 = 6000 * 1.8 * 1.8 * 1.8 * 1.8	*f*(4) = 6000 * 1.8^4

7. Write a function that gives the number of emails *f*(*x*) sent by the virus in the *x*th hour after its release.

Answer: $f(x) = 6000 \cdot 1.8^x$

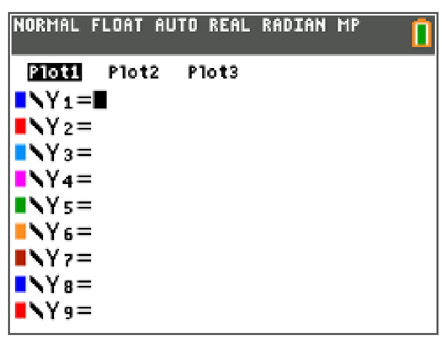
Check the function by entering it in the very top of L3 in the List Editor. You should type **L1** instead of *x*.

How do the values compare with the entries in L2?



Plot the function on top of the scatterplot by entering it in Y1 and pressing graph. How does the function compare with the data points in the scatter plot?

Use the function to answer the questions on your worksheet.



An exponential function has the form $f(x) = a \cdot b^x$, where *a* is a nonzero constant, *b* is greater than 0 and not equal to 1, and *x* is a real number. *a* represents the starting amount, and *b* is the growth factor.

What do *a* and *b* represent in this case? Discuss with your class.

8. This function is an exponential function of the form $f(x) = a \cdot b^x$. In your own words, explain what the values of *a* and *b* represent.

Answer: The values of *a* represents the starting amount of emails, the number of emails sent out initially; the value of *b* is the number of new infections per infected computer per hour.

9. Use the function to find the number of emails sent by the virus in the 144th hour after its release.

Answer: $f(144) = 3.446608742 \times 10^{40}$ or 34,466,087,420,000,000,000,000,000,000,000,000,000,000 emails

10. Models often work well only for a limited range of inputs. What are the real-world limitations of the exponential model in this case? Qualitatively, how would the real-world shape of the scatter plot look if the horizontal scale were extended to 144 hours? How about in the chain mail example? Can you think of other examples of exponential models, and how they might have limited applicability? Justify your answers.

Sample Answer: The number of emails can't keep growing forever. Eventually, people will start receiving the infected emails more than once, and won't open them; the number of new people receiving the email is limited by the number of people in the world with internet access. In the real world, the curve would have to become flat long before 144 hours had passed. The same is true in the chain mail example. Another example is population growth. If the population is low, growth can be exponential, but eventually food and other resources will become a limitation and the population will level off.