EXPLORATIONS

Activity 13

How Close is Close?

Teacher Notes

Concepts

- Measures of variability
- Stem-and-leaf plots
- Mean absolute deviation
- Frequencies
- Standard deviation

Calculator Skills

- Statistics variables: $\bar{x}$, σx, Σx, Σx^2
- Frequency distributions: FRQ

Materials

- TI-30X IIS
- Student Activity pages (p. 132-133)
- Transparency
- Stopwatch
- 3 x 5 index cards (one per student)

Objective

- In this activity, students will calculate measures of variability, including standard deviation and variance, and will learn how these measures of variability can be interpreted.

Topics Covered

- Constructing charts and tables that summarize data from real-world situations and drawing inferences from the results
- Understanding and applying the measures of central tendency and variability
- Interpreting data and predicting results

Introduction

When any event occurs, many people may observe it. If these people were asked to give an account of what they saw, it is likely that each individual would report very different things. If two students were asked to measure the length of a garden to the nearest inch, they probably would get two different measurements. The results of a science experiment conducted by different individuals would also likely produce different results. In nearly

everything that occurs in nature is variability. There are many different measures of variability. We will explore some of these measures in this activity.

Investigation

1. Tell the students that you are going to conduct an experiment to see how well the class does with predicting elapsed time.
2. Look at a stopwatch and say "start" as the watch is clicked. After about 20 to 30 seconds, click the watch and say "stop." For this experiment, students are not allowed to look at their watches!
3. Have each student record on a 3 x 5 index card an individual estimate of how many seconds passed between "start" and "stop."
4. Tell the class exactly how many seconds it actually was. Each student then will record how far off their estimate was on the 3 x 5 card. Do not use any negative numbers! An error of 4 seconds too much and an error of 4 seconds too little will both be recorded as 4.
5. Collect the 3 x 5 cards, and have a student record the error data in a stem-and-leaf plot on the transparency.
6. Find the average error of the estimates for all the students in the class. (The table below is a sample of error data collected from a class and recorded in a stem-and-leaf plot. The actual time passed between "start" and "stop" for the sample data was 22 seconds.)

Stem	Leaf
0	0,0,1,1,1,2,3,3,3,4,4,4, 4
0	5,5,7,7,7,7,8,9, 9
1	10 11, 13, 13, 13
1	15

Have the students use mental mathematics to find the sum of the errors and then find the average.

7. Tell students to use their calculators to check their answers. The result is called the *mean absolute deviation*. For the sample above, they would enter the following:

Press:	The calculator shows:
CLEAR 0 × 2 + 1 × 3 + 2 + 3 × 3 + 4 × 4 + 5 × 2 + 7 × 4 + 8 + 9 × 2 + 10 + 11 + 13 × 3 + 15 ENTER	0*2+1*3+2+3→ 169. DEG
÷ 28	Ans/28 6.035714286 DEG

8. Repeat the experiment again with the class, using a different length of time. Have the class do the calculations again.

   Is the new average error *larger* or *smaller* than before?

   The new average error should be smaller than before.

   Why is this the case?

   Students will have some experience with the experiment so that estimates should be improved.

9. Explain that the standard deviation is another way of measuring the amount of variation about the mean in a set of data. The standard deviation is found by calculating the average of the *square* of the distance between each data point and the mean of the data set. The *square root* of this number is computed to be the standard deviation.

   For a data set $x_1, x_2, x_3, \ldots x_n$, where $\bar{x}$ (read "x-bar") is the mean of the data values and n is the number of data values, the formula for calculating the standard deviation for a population is σ (the lower-case Greek letter read "sigma") and is:

$$\sigma_x = \sqrt{\frac{\left(x_1 - \bar{x}\right)^2 + \left(x_2 - \bar{x}\right)^2 + \left(x_3 - \bar{x}\right)^2 + \ldots + \left(x_n - \bar{x}\right)^2}{n}}$$

10. Show the students how to calculate the mean and standard deviation for the amount of fat (in grams) found n fast foods. This can be done by completing a table of using a calculator.

    Note that nutritional information about fast foods can be found on the Internet or from your local fast food establishments. Total calories, amount of fat in grams, amount of cholesterol in milligrams, and amount of sodium (salt) in milligrams are all included in this information. The following data represent the amount of fat (in grams) per serving for 23 different fast food entrees. The entrees include hamburgers, chicken, and roast beef.

    Hamburgers: 12, 15, 21, 28, 19, 36, 39, 35, 63

    Chicken: 9, 15, 7, 20, 29, 30, 18, 43, 12, 15, 21

    Roast beef: 22, 19, 27

11. Find the mean and standard deviation of a set of data by using a table.

    STEP 1:
    Use the calculator to find the mean of the data using the formula:

    $$\bar{x} = \frac{\Sigma x}{n}$$

    The $\bar{x}$ indicates the mean. The Σ is the upper case Greek letter for "sigma" and is the mathematical shorthand for indicating *the sum of all the data points x.* Again, the letter *n* indicates the number of data points used in the calculations.

Press:	The calculator shows:
CLEAR 12 + 15 + 21 + . . . + 19 + 27 ENTER	12+15+21+28→ 555. DEG
÷ 23 ENTER	Ans/23 24.13043478 DEG

STEP 2:
Subtract the mean from each data value in the first column and record the result in the middle column in Student Activity Part 1. Square each difference in the middle column and record in the third column.

Press:	The calculator shows:
STO▶ ENTER	Ans→A 24.13043478 DEG
12 − 2nd [RCL] ENTER ENTER	12−24.13043→ ⁻12.13043478 DEG
x^2 ENTER	Ans^2 147.147448 DEG
15 − 2nd [RCL] ENTER ENTER	15−24.13043→ ⁻9.130434783 DEG K
. . .	

STEP 3:
Sum the squares in the third column.

3626.608695

STEP 4:
Find the standard deviation as follows:

$$\sigma_x = \sqrt{\frac{3626.608696}{23}}$$

Answer: 12.55701553 grams of fat

12. Find the mean and standard deviation of the data set using the statistics capabilities of the TI-30X IIS.

Press:	The calculator shows:
CLEAR 2nd [STAT]	1- VAR 2- VAR DEG
ENTER	STAT DEG
DATA	X_1= STAT DEG
12 ⊙	FRQ=1 STAT DEG
2 ⊙	X_2= STAT DEG
15 ⊙ 3 ENTER	X_3= STAT DEG
Continue until all data values are entered.	X_{19}= STAT DEG
STATVAR	**n** $\bar{x}$ Sx σx 23. STAT DEG
⊙	**n** $\bar{x}$ Sx σx 24.13043478 STAT DEG
⊙	**n** $\bar{x}$ Sx σx 12.83923089 STAT DEG
⊙	**n** $\bar{x}$ Sx σx 12.557101553 STAT DEG
⊙	Σx Σx^2 555. STAT DEG
⊙	Σx Σx^2 17019. STAT DEG

13. Compare the calculated results to those found using the table.

    Note: In the activities in this book, σx is used for the standard deviation of the entered data, treating the data as a population. In higher statistics courses, students will learn when and how to use the S*x* variable. This variable is the predicted standard deviation of the population, treating the entered data as a sample from the population.

14. Explain to the class that statisticians often describe a distribution of data by the percent of the data that falls within a certain parameter. The percent of data that falls within one, two, or three standard deviations of the mean usually determine such parameters. In the first example, the parameters for one standard deviation on either side of the mean are calculated to be:

    $\bar{x} \pm 1\sigma = 24.13043478 \pm 12.55701553$

    The number of data points out of the 23 in the example that fall between 11.57341926 and 36.68745031 is 18. Therefore, 78.26% of the data falls within one standard deviation of the mean. In a standard normal distribution, statisticians expect to find 68% of the data within one standard deviation of the mean. From this, we may conclude that there is not a great variation in the number of fat grams in the 23 entrees selected.

15. Have the students complete Student Activity Part 2.

Wrap-Up

To make the calculations in Student Activity Part 1 go more quickly, divide the work up among groups of four. Have students work in groups of two or three to complete Student Activity Part 2. Have the groups share their results with the class.

Extension

Use the data from Student Activity Part *2* for this problem. The population of a state determines the number of electoral votes allocated to each state. Compare the standard deviation of the states west of the Mississippi to the standard deviation of the states east of the Mississippi. What does this comparison tell you about the populations of the states that are east and those that are west?

Solutions Part 1

1.

Fat grams	(Fat grams – mean)	(Fat grams – mean)^2
12	-12.13043478	147.147448
15	-9.130434783	83.36483932
21	-3.130434783	9.799621928
28	3.869565217	14.97353497
19	-5.130434783	26.32136106
36	11.86956522	140.8865784
39	14.86956522	221.1039698
35	10.86956522	118.147448
63	38.86956522	1510.8431
9	-15.13043478	228.9300567
15	-9.130434783	83.36483932
7	-17.13043478	293.4517958
20	-4.130434783	17.06049149
29	4.869565217	23.71266541
30	5.89565217	34.45179584
18	-6.130434783	37.58223062
43	18.86956522	356.0604915
12	-12.13043478	147.147448
15	-9.130434783	83.36483932
21	-3.130434783	9.799621928
22	-2.130434783	4.538752363
19	-5.130434783	26.32136106
27	2.869565217	8.234404537

2. The mean is

   24.13043478

3. The sum of squares is

   3626.608695

4. The standard deviation is

   12.57701553

Solutions Part 2

The number of electoral votes allotted to each of the 50 states is listed below.

EAST of the Mississippi River: 9, 8, 3, 25, 13, 22, 12, 8, 4, 10, 12, 18, 7, 4, 15, 33, 14, 21, 23, 4, 8, 11, 3, 13, 5, 11

WEST of the Mississippi River: 3, 8, 6, 54, 8, 4, 4, 7, 6, 9, 10, 11, 3, 5, 4, 5, 3, 8, 7, 3, 32, 5, 11, 3

For *each* data set, use the calculator to find:

		East	West
1.	n	$n = 26$	$n = 24$
2.	$\bar{x}$	$\bar{x} = 12.15384615$	$\bar{x} = 9.125$
3.	Σx	$\Sigma x = 316$	Σx 219
4.	Σx^2	$\Sigma x^2 = 5294$	$\Sigma x^2 = 4893$
5.	σx	$\sigma x = 7.476590686$	$\sigma x = 10.98222996$

6. Explain the meaning of each of the symbols in problems 1 through 5.

*The symbol **n** is the number of data elements in the set.*

The symbol $\bar{x}$ is the mean of the set of data elements.

The symbol Σx is the sum of the data elements in the set.

The symbol Σx^2 is the sum of the squares of the data elements in the set.

The symbol σx is the standard deviation of the set of data elements.

Student Activity 13

Name _____

Date _____

Dealing with Data - How Close is Close?

Objective: *In this activity, you will calculate measures of variability, including standard deviation and variance, and will learn how these measures of variability can be interpreted.*

Part 1: Finding Mean and Standard Deviation with a Table

1.

Fat grams	(Fat grams – mean)	(Fat grams – mean)^2
12	-12.1304347	147.1474480
15	-9.1304347	83.3648393
21	-3.1304347	9.7996219
28	3.8695652	14.9735349
19		
36		
39		
35		
63		
9		
15		
7		
20		
29		
30		
18		
43		
12		
15		
21		
22		
19		
27		

2. The mean is _____.

3. The sum of squares is _____.

4. The standard deviation is _____.

Part 2: Finding Statistical Results with the Calculator

The number of electoral votes allotted to each of the 50 states is listed below.

EAST of the Mississippi River: 9, 8, 3, 25, 13, 22, 12, 8, 4, 10, 12, 18, 7, 4, 15, 33, 14, 21, 23, 4, 8, 11, 3, 13, 5, 11

WEST of the Mississippi River: 3, 8, 6, 54, 8, 4, 4, 7, 6, 9, 10, 11, 3, 5, 4, 5, 3, 8, 7, 3, 32, 5, 11, 3

For *each* data set , use the calculator to find:

		East	West
1.	n		
2.	$\bar{x}$		
3.	Σx		
4.	Σx^2		
5.	σx		

6. Explain the meaning of each of the symbols in problems 1 through 5.

TRANSPARENCY
Stem-and-Leaf Plot

STEM	LEAF