

**Features Used**  
`cSolve()`, `[∠]`, `[°]`, `[□]`,  
`abs()`, `angle()`, Numeric  
 Solver, `when()`, `log()`,  
`DelVar`, `DrawFunc`, `DrawInv`,  
`NewProb`, `[□]`, `[Y=]`,  
`[□][GRAPH]`, `[□][WINDOW]`

**Setup**  
`[♦]1`, `NewFold steady`,  
`setMode("Angle", "Degree")`

**Steady-State  
 Circuit Analysis  
 And Filter Design**

This chapter shows how the TI-89 implements *phasors* to perform sinusoidal steady-state analysis. The focus is on how to enter and display complex numbers. This chapter also shows a typical steady-state application—how to use the Numeric Solver to find the required order of lowpass Butterworth and Chebyshev filters in making a standard “handbook” filter design.

**Topic 13: Phasor Analysis**

Given the circuit shown in Figure 1, find *v*, the voltage across the current source.

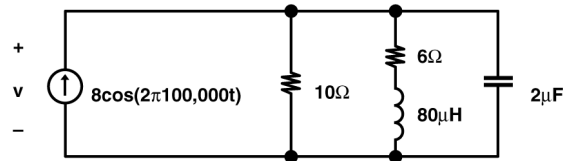


Figure 1. A circuit in steady-state

The first step is to convert the actual circuit to its phasor equivalent. The circuit shown in Figure 2 includes these conversions.

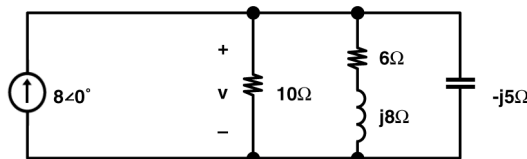


Figure 2. The phasor equivalent of the Figure 1 circuit

Only one nodal equation is needed to solve for *v*

$$-8\angle 0^\circ + \frac{v}{10} + \frac{v}{6 + j8} + \frac{v}{-j5} = 0$$

1. Clear the TI-89 by pressing  $\boxed{2nd} \boxed{F6} \mathbf{2:NewProb} \boxed{ENTER}$ .
2. Be sure the **Complex Format** mode is set to **Rectangular**. Be sure the **Graph** mode is set to **Function**.
3. Enter the equation as shown in screen 1.

$\boxed{(-)} \mathbf{8} \boxed{+} \mathbf{v} \boxed{\div} \mathbf{10} \boxed{+} \mathbf{v} \boxed{\div} \boxed{(} \mathbf{6} \boxed{+} \mathbf{2nd} \boxed{[i]} \mathbf{8} \boxed{)} \boxed{+} \mathbf{v} \boxed{\div} \boxed{(} \boxed{(-)} \mathbf{5} \boxed{)} \boxed{=} \mathbf{0} \boxed{STO} \mathbf{n1}$

4. Using **solve(n1,v)** will return “false” since it is valid for real solutions only. To get a complex solution, enter **cSolve()** as shown in screen 2.

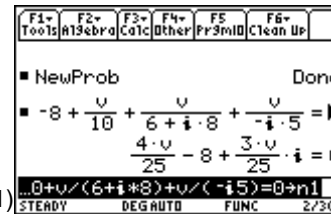
5. Phasors are expressed as a magnitude at an angle,  $M\angle\theta$ . There are a couple of ways to obtain this form.

The first way is to use the functions **abs()** (top of screen 3) and **angle()** (middle of screen 3). In this example,  $\boxed{\blacklozenge} \boxed{[=]}$  is used to get the approximate values for the second **angle()** command (bottom of screen 3).

This shows that the phasor form of the voltage is  $40\angle-36.87$  in **Degree** mode.

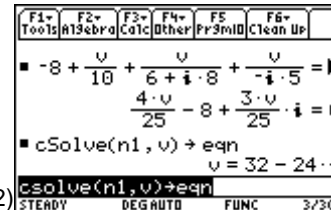
The second approach is to put the TI-89 in **Polar** mode. Press  $\boxed{MODE}$  and select **Complex Format 3:Polar** (screen 4).

6. Using **cSolve()** (and with  $\boxed{\blacklozenge} \boxed{[=]}$  for a second approximate solution) gives the same results in the polar mode as **abs()** and **angle()** in the rectangular mode, as shown in the bottom two lines of screen 5.



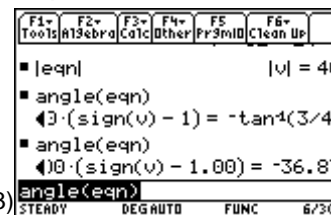
(1)

**Note:** The usual imaginary number  $j$  used in electrical engineering is entered as  $i$  which is  $\boxed{2nd} \boxed{[i]}$ .



(2)

**Note:** To enter **cSolve()**, press  $\boxed{F2}$  **A:Complex 1:cSolve()** or  $\boxed{CATALOG}$  **cSolve()**.



(3)

**Note:** To see the values on the right end of a solution line in the history section, press  $\boxed{\leftarrow}$  to get to the line and then press  $\boxed{\rightarrow}$  to move to the right.

**Note:** Press  $\boxed{MODE}$  to switch to degree mode if it isn't already set. If in radian mode the angle would have been given in radians.



(4)



(5)

### Topic 14: Graphing Frequency Response

It is easy to find the voltage across the current source as a function of  $\omega$  using symbolic expressions. Table 1 shows the variations of phasor circuit elements with radian frequency  $\omega$ .

Element	Element Equation	Phasor Result
$8\cos 10^5 t$	$v(t) = \text{Re}[V e^{j\omega t}]$	$V = 8\angle 0^\circ$
$10 \Omega$	$Z_{Ra} = R_a$	$Z_{Ra} = 10 \Omega$
$6 \Omega$	$Z_{Rb} = R_b$	$Z_{Rb} = 6 \Omega$
$80 \mu\text{H}$	$Z_L = j\omega L$	$Z_L = j\omega 80 \times 10^{-6} \Omega$
$2 \mu\text{F}$	$Z_C = \frac{1}{j\omega C}$	$Z_C = \frac{1}{j\omega 2 \times 10^{-6}} \Omega$

Table 1: Frequency Dependence of Phasor Circuit Elements

The nodal equation then becomes

$$-8\angle 0^\circ + \frac{v}{r_a} + \frac{v}{r_b + z_{ell}} + \frac{v}{z_{c1}} = 0$$

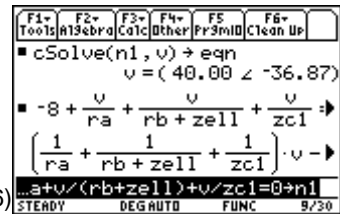
Note that **ra**, **rb**, and **zc1** are used because **r1**, **r2**, and **zc** are TI-89 system variables.

1. Switch back to **Rectangular Complex Format** mode and enter the equations as shown in screen 6.

$(\text{2nd}) 8 (+) v (\div) ra (+) v (\div) ( ( rb (+) zell ) ) (+) v (\div) zc1 = 0 \text{ (STO)} n1$

as shown in screen 6.

2. Define the element values from the table (screens 7 and 8). For convenience, **w** ( $\alpha w$ ) is used instead of  $\omega$  ( $\alpha w$ ).

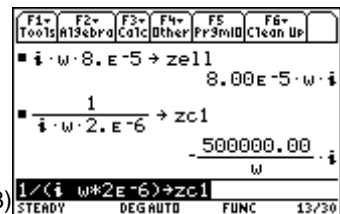


(6)

**Note:** **zell** is used to avoid confusing **z1** with **z1** (z followed by a 1), a reserved name.

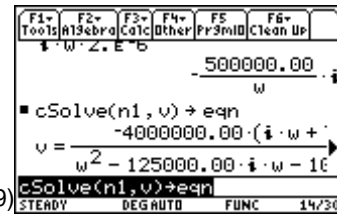


(7)



(8)

3. With **cSolve()**, the solution in screen 9 shows that the voltage varies with frequency.



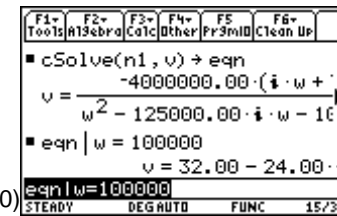
The complete answer is

$$v = \frac{-4000000.00(iw + 75000.00)}{w^2 - 125000.00iw - 10000000000.00}$$

The answer is a bit of a mess. To check it with the previous solution, enter **eqn | w=100000** (screen 10).

It's the same answer as Topic 13, screen 2.

4. To view the variation of the voltage magnitude versus frequency, graph **v** versus **w**. Since the original problem used **w=100,000**, graph from **w=0** to **w=200,000**. Press **[WINDOW]** and set **xmin** to 0, **xmax** to 200000, **ymin** to 0, and **ymax** to 50. Press **[Y=]** and set **y1** to graph the magnitude of **v** (screen 11).

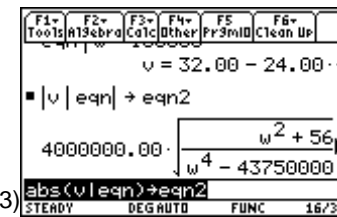
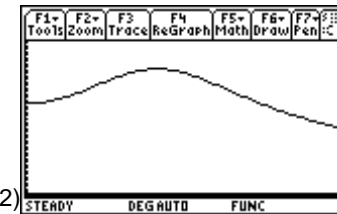
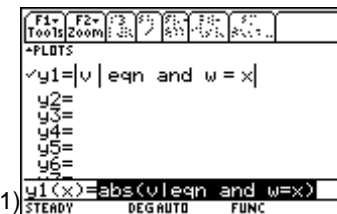


5. Press **[GRAPH]** to see the magnitude graph (screen 12).
6. This graph takes a long time to complete because the “with” substitutions are made over and over again for each pixel. One way to speed it up is to do the “withs” once before graphing and save the result in another variable name which is then graphed.

To do this, press **[HOME]** and enter the expression:

**[CATALOG]** **abs(v | eqn)** **[STO]** **eqn2** as shown in screen 13.

Then press **[Y=]**, deselect **y1**, and enter **y2** (screen 14).



(9)

(10)

(11)

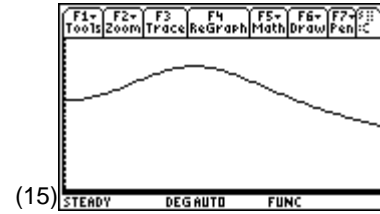
(12)

(13)

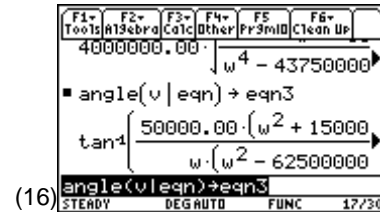
(14)

**Note:** Deselect equation **y1(x)** with **[F4]**.

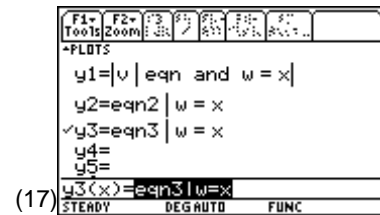
7. Press  $\square$  [GRAPH]. The same result as the previous graph appears much more quickly (screen 15).



8. The phase can be graphed defining the phase angle of the voltage on the Home screen as eqn3 (screen 16).

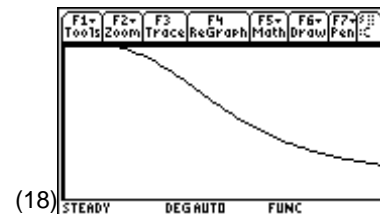


9. Press  $\square$  [Y=], deselect y2, and enter eqn3 as plot variable y3 (screen 17).



10. Press  $\square$  [WINDOW] and set ymin to -90 and ymax to 0 (since the calculations have been in the degree mode). xmin and xmax can remain the same.

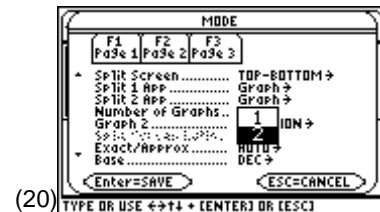
11. Press  $\square$  [GRAPH] to see the phase graph as shown in screen 18.



12. Usually the magnitude and phase plots are shown together. This can be done using the split screen mode. To do this, press [MODE] [F2]  $\downarrow$   $\downarrow$ . Screen 19 presents the Split Screen options.

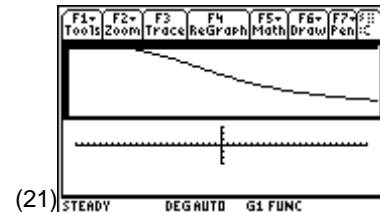


13. Press [ENTER]. Move down to Split 2 App, and select 4:Graph. Finally, set Number of Graphs to 2 (screen 20).

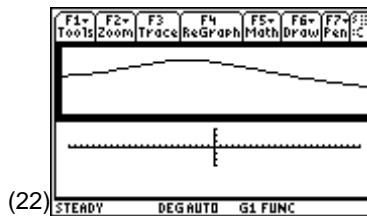


14. Press [ENTER] to view the split screen plots (screen 21).

The top graph is the phase plot shown before; the bottom graph contains no data yet.

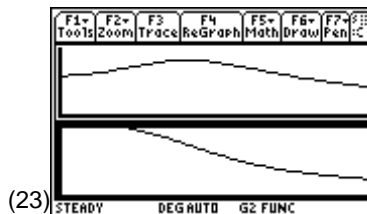


15. Convention says the magnitude plot should be on top. To do this, press  $\blacklozenge$  [Y=] and use [F4] to select **y2** and deselect **y3**. Next, press  $\blacklozenge$  [WINDOW] and set **ymin** to 0 and **ymax** to 50. Finally, press  $\blacklozenge$  [GRAPH] to see the magnitude plot in the upper graph as shown in screen 22.

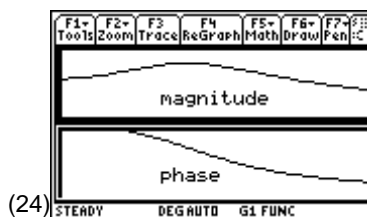


16. To set up the phase plot in the lower window, change to the other half of the screen by pressing [2nd][ $\oplus$ ] and set up the graph as before. The following operations will give the phase plot in the bottom window (screen 23).

$\blacklozenge$  [Y=], select **y3**,  $\blacklozenge$  [WINDOW], set **xmin** to 0, **xmax** to 200000, **ymin** to -90 and **ymax** to 0, and finally,  $\blacklozenge$  [GRAPH]



17. You can use [2nd][F7] **7:Text** to add magnitude and phase labels to the graphs. To do this, press [2nd][F7] **7:Text** and position the cursor where the text should start. The characters will appear below and to the right of the crosshairs. Be careful; once a letter is placed it can't be erased except by [2nd][F7] **2:Eraser**.



18. To return to a single screen, press [MODE][F2] and set **Split Screen** to **1:FULL**.

### Topic 15: Filter Design Overview

A class of realizable frequency responses for lowpass filters has the form

$$|H(f)|^2 = \frac{1}{1 + \epsilon^2 \Psi^2(f)}$$

where  $\Psi(f)$  is a polynomial in  $f$ . If

$$\Psi(f) = \left(\frac{f}{f_p}\right)^n$$

the filter is a Butterworth filter. An alternative is to make  $\Psi(f) = C_n(f/f_p)$  where  $C_n$  is a Chebyshev polynomial, the filter is a Chebyshev filter. The next topic deals with a Butterworth filter, the following topic with a Chebyshev filter.

The design of lowpass filters requires specification of passband and stopband responses often given in dB.  $|H(f)|^2$  in dB is calculated as

$$|H(f)|_{dB}^2 = 10 \log \left( \frac{1}{1 + \epsilon^2 \Psi^2(f)} \right)$$

which becomes

$$|H(f)|_{dB}^2 = 10 \log \left( \frac{1}{1 + \epsilon^2 \left( \frac{f}{f_p} \right)^{2n}} \right)$$

for Butterworth and

$$|H(f)|_{dB}^2 = 10 \log \left( \frac{1}{1 + \epsilon^2 C_n^2 \left( \frac{f}{f_p} \right)} \right)$$

for Chebyshev.

### Topic 16: Butterworth Filter

The performance specifications of a filter are often given in graphical form as shown in Figure 3. The design of a Butterworth filter with these performance specifications is described here.

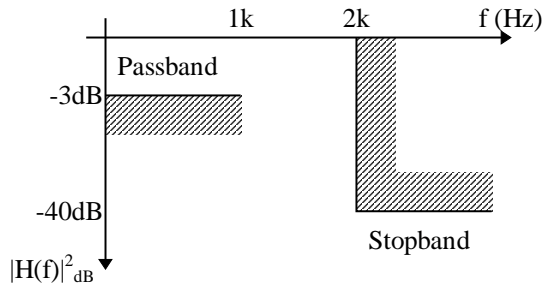


Figure 3. Filter design specifications for a Butterworth filter

Suppose a filter with the maximum passband ripple is -3 dB, and the passband edge is at  $f_p=1$ kHz is to be designed. Additionally, the stopband gain is to be no more than -40 dB with a stopband edge at  $f_s=2$ kHz.

- From the Home screen, press  $\blacklozenge$  1 [2nd] [F6] 2:NewProb to reset the TI-89 to a known state. Then enter the Butterworth equation as shown in screen 25.

10 [x] [CATALOG] log( 1 [÷] ( 1 [+ ] eps [^] 2 [x] ( f [÷] fp [)] [^] ( 2n [)] [)] [)] [STO] butter

- Press [APPS] 9:Numeric Solver exp=butter [ENTER] (screen 26).

- Note that the Numeric Solver listed each of the variables for values to be entered. Find the value of **eps** by entering the data for the passband edge with a -3 dB response at 1000 Hz. **n** is unknown but at the passband edge all values give the same result, so for now enter 1 for **n** as shown in screen 27.

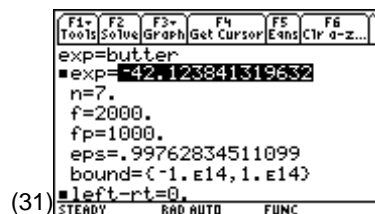
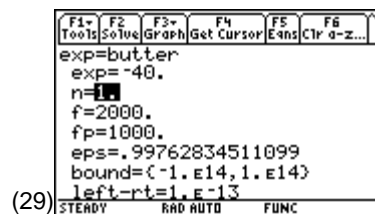
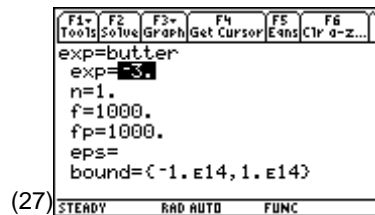
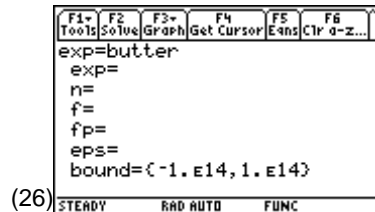
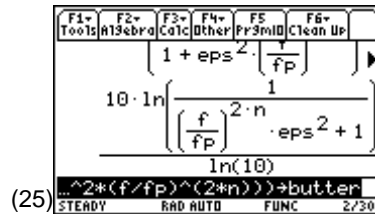
- Place the cursor on the **eps** line and press [F2] to solve for eps. After a second or two, the screen shows **eps** is about 1 (screen 28).

- Now, find the order of the filter by setting the stopband edge response (**exp**) to -40 dB and **f** to 2000 (screen 29).

- Solve for **n** by placing the cursor on **n** and pressing [F2]. After a couple of seconds the solution of **n**=6.6 is shown, as in screen 30.

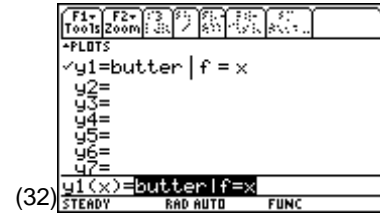
- Since **n** must be an integer, set **n** to the next larger integer value of 7 and solve for **exp** to find the stopband gain for this value of **n** (screen 31).

With a 7<sup>th</sup>-order Butterworth filter the stopband gain is -42 dB, a little better than the minimum needed.

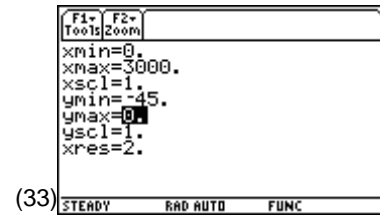




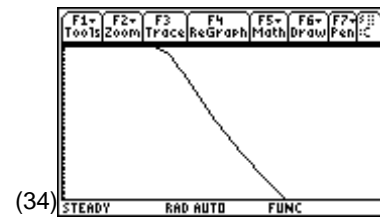
- Now, plot the Butterworth equation to see the frequency response. To do this, press  $\blacklozenge$  [Y=] and enter **butter** with **f** replaced by **x** (screen 32).



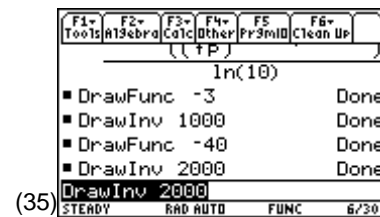
- Since the stopband edge of the filter is 2 kHz, plot **x** from 0 to 3000. The stopband value is -40 dB, so plot **y** from -45 to 0. Enter these values in the Window Editor ( $\blacklozenge$  [WINDOW]) as shown in screen 33.



- Press [MODE] to be sure the **Graph** mode is set to **FUNCTION**. Then press  $\blacklozenge$  [GRAPH] and wait a few seconds to see screen 34.

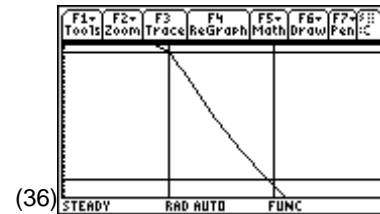


The response in the passband looks very flat, which is correct for Butterworth, but are the passband and stopband edges in the right places? These can be checked graphically by pressing  $\text{2nd}$  [F6] **2:DrawFunc** to draw horizontal lines at -3 and -40 dB and  $\text{2nd}$  [F6] **3:DrawInv** to draw vertical lines at 1000 and 2000 Hz (screen 35).



- Press  $\blacklozenge$  [GRAPH] to plot the results (screen 36).

The curve passes through the -3 dB point at 1000 Hz and passes below the -40 dB point at 2000 Hz. The filter meets the required specifications.



### Topic 17: Chebyshev Filter

This section shows how to design a Chebyshev lowpass filter with the same specifications as discussed in Topic 16 and shown in Figure 4.

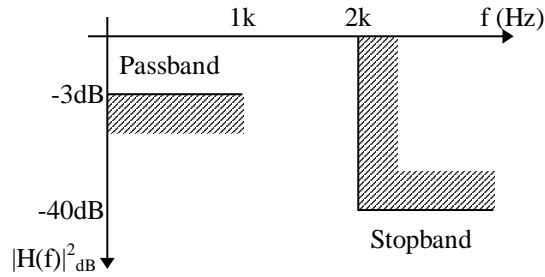


Figure 4. Filter design specifications for a Chebyshev filter

The Chebyshev equations are

$$C_n\left(\frac{f}{f_p}\right) = \cos\left(n \cos^{-1}\left(\frac{f}{f_p}\right)\right) \quad \left|\frac{f}{f_p}\right| < 1$$

$$C_n\left(\frac{f}{f_p}\right) = \cosh\left(n \cosh^{-1}\left(\frac{f}{f_p}\right)\right) \quad \left|\frac{f}{f_p}\right| > 1$$

where n is the order of the polynomial.

Therefore  $H_{dB}$  is

$$|H(f)|_{dB}^2 = 10 \log \left( \frac{1}{1 + \epsilon^2 C_n^2\left(\frac{f}{f_p}\right)} \right)$$

Follow these steps to enter these three equations.

1. From the Home screen, clear **f**, **fp**, **eps**, and **n** using **DelVar**.
2. Enter  $C_n$  for  $|f/f_p| < 1$  as shown in screen 37.

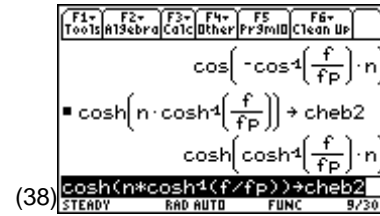
`[2nd] [COS] n [X] [♦] [COS-1] f [÷] fp [)] [)] [STO] cheb1`

**Note:** *DelVar* can be entered by pressing `[F4] 4:DelVar`.



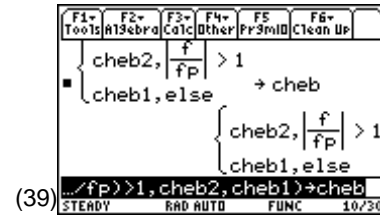
3. Enter  $C_n$  for  $|f/f_p| > 1$  as shown in screen 38.

`CATALOG` `cosh(n × CATALOG cosh-1(f ÷ fp) ) )` `STO>`  
**cheb2**



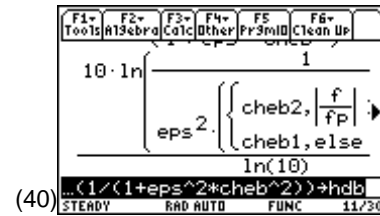
4. Define **cheb** to be **cheb1** for  $|f/f_p| < 1$  and **cheb2** for  $|f/f_p| > 1$ . This is done using the “when” function as shown in screen 39.

`CATALOG` `when(CATALOG abs(f ÷ fp) < 1, cheb2`  
`, cheb1) )` `STO>` **cheb**



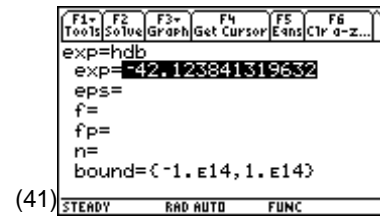
5. Define **hdb** to be  $H(f)^2_{dB}$  as shown in screen 40.

`10` `CATALOG` `log(1 ÷ (1 + eps ^ 2 × cheb ^ 2) )`  
`STO>` **hdb**

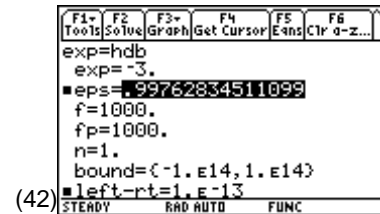


6. To design the filter, use the Numeric Solver. Press

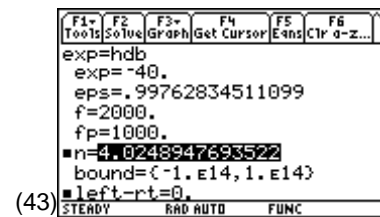
`APPS` `9:Numeric Solver` `exp=hdb` `ENTER` (screen 41).



7. Proceed as with Butterworth. Find **eps** for -3dB at the passband edge by entering the values as shown in screen 42. The result is the same as with Butterworth.

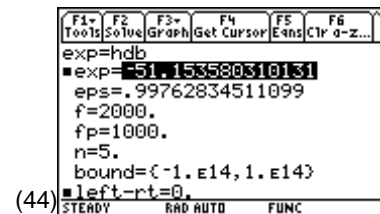


8. Now find **n** for -40 dB at 2000 Hz. It takes a few seconds to find the order of the filter (screen 43).

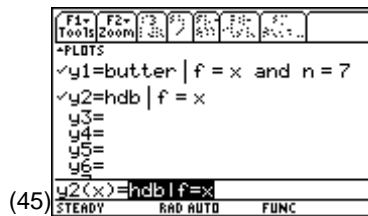


9. To meet the design specifications, **n** must be 5. Enter 5 and calculate the response. See what the gain is (screen 44).

This filter exceeds the design specifications by more than 10 dB with a lower order than the Butterworth.



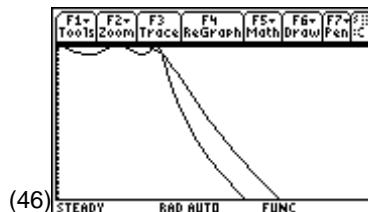
10. In the Y= Editor, enter the expressions for the two filters to compare them (screen 45). Note that  $n=7$  is added to  $y1$  so that the Butterworth filter uses the order computed for it instead of using the value of  $n=5$  used for the Chebyshev filter.



11. Press  $\blacklozenge$  [GRAPH] to see the graphs (screen 46).

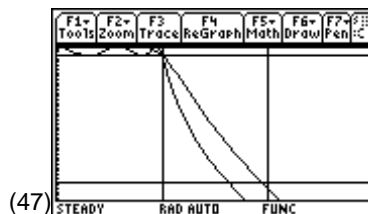
(See Topic 16 for a review of the instructions for a graphical comparison of the filters' performances.)

As expected, the Chebyshev filter has ripples in the passband, but it drops more rapidly in the transition band than the Butterworth.



12. Compare the filters' performances with the lines which represent the passband and stopband design specifications (screen 47). Press  $2^{nd}$  [F6] **2:DrawFunc** to draw horizontal lines at -3 and -40 dB. Press  $2^{nd}$  [F6] **3:DrawInv** to draw vertical lines at 1000 and 2000 Hz.

Both filters show the required -3 dB response at 1000 Hz; both filters exceed the specifications since they are below -40 dB at 2000 Hz.



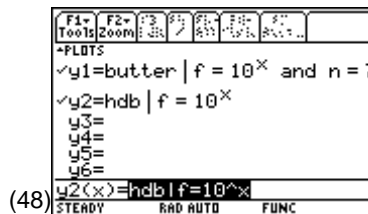
### Topic 18: Logarithmic Frequency Plots

Often frequency responses are plotted on a log frequency scale. Although the TI-89 doesn't directly support log plots, they are easy to do.

1. Return to the Y= Editor and alter the "with" operation to include the logarithmic relation of frequency with  $x$  as shown in screen 48.

**y1:** butter | f = 10  $\wedge$  x [CATALOG] and n = 7

**y2:** hdb | f = 10  $\wedge$  x

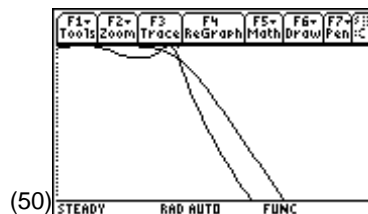


The values for  $x$  are linearly spaced, but the values of  $10^x$  are logarithmically spaced.

2. Press  $\blacklozenge$  [WINDOW] to adjust the range on  $x$ . Graph the functions for  $f=500$  and  $f=3000$ . To do this, enter **log(500)** for  $xmin$  and **log(3000)** for  $xmax$  as shown in screen 49.

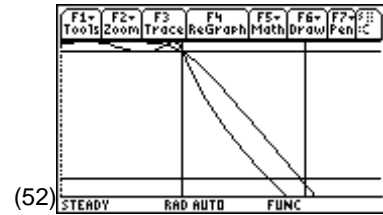
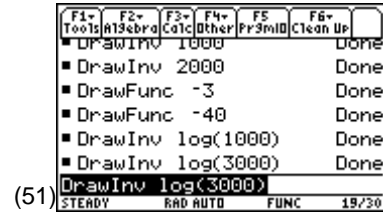


3. Press  $\blacklozenge$  [GRAPH] to display screen 50. The logarithmic plots take on a different appearance than the linear plots of screen 46.



- The passband and stopband lines are added by using **log()** of the **f** values in **DrawInv** (screen 51). Press **[2nd] [F6] 2:DrawFunc** for -3 and -40 dB. Press **[2nd] [F6] 3:DrawInv** for  $\log(1000)$  and  $\log(2000)$  Hz.

As expected, the plots in screen 52 show that both filters meet the design specifications.



### Tips and Generalizations

Topic 13 show how to enter, solve, and display equations with complex numbers. Multiple equations with multiple complex unknowns can be solved. Also the matrix approach of Topic 3 can be used with complex numbers.

The Numeric Solver works nicely for filter design but can easily solve any equation for an unknown value.

Once a steady-state response is known, the power dissipated by the various elements can be found. Chapter 5 explores this topic further.

