Mr. Taylor, I Presume?

ID: 10096

Time required
45 minutes

Activity Overview

Students will explore Taylor polynomials graphically and analytically, as well as graphically determine the interval where the Taylor polynomial approximates the function it models.

Topic: Series & Taylor Polynomials

- *Display in a spreadsheet the first few terms of a Taylor series approximation to f(x) for a given value of x and compare the value of the Taylor approximation with the value of f(x).*
- *Graph a function and its Taylor polynomials of various degrees to show their convergence to the function.*

Teacher Preparation and Notes

- *Students should be familiar with basic differentiation prior to beginning this activity.*
- *This activity requires the use of CAS technology.*
- *This activity is designed to be teacher-led.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "10096" in the quick search box.***

Associated Materials

- *MrTaylorIPresume_Student.doc*
- *MrTaylorIPresume.tns*
- *MrTaylorIPresume_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *MacLaurin Polynomials (TI-Nspire CAS technology) — 10129*
- *Exploring Taylor Series (TI-84 Plus family or TI-89 Titanium) — 5525*
- *Taylor Polynomials (TI-84 Plus family) — 4375*

Introduction

This activity begins with students finding a polynomial given only the value of its derivatives at a specific *x*-value. This is at the heart of finding Taylor polynomials. If students have difficulty understanding that this method works, it may be necessary to reverse the process and have the students find the derivatives and observe that their values at the given *x*-value are the given values of the derivative.

At the conclusion of the introductory problem, the Taylor polynomial form is found when centered at zero. At this time, place the general form of the Taylor polynomial when $x = 0$ on the board:

$$P_n(x) = \frac{f^{(0)}(a)}{0!}(x-a)^0 + \frac{f'(x)}{1!}(x-a)^1 + \frac{f''(x)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(x)}{n!}(x-a)^n$$

Explain that when $a = 0$, this matches the form $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

Taylor Polynomial Centered at Zero

Students first work through an example to find a polynomial $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ if $P(0) = 1,\ P'(0) = 1,\ P''(0) = 6$ and $P'''(0) = 9$. Next, students are asked to find the 4th degree Taylor polynomial that approximates $f(x) = \ln(x + 5)$ at $x = 0$. The derivatives of the function are:

$$f(0) = \ln(5) \approx 1.609$$

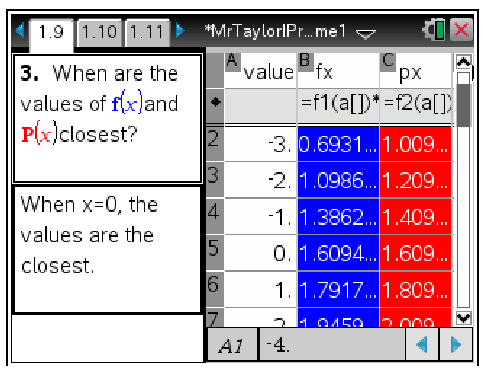
$$f'(x) = \frac{1}{x+5} \to f'(0) = \frac{1}{5}$$

$$f''(x) = \frac{-1}{(x+5)^2} \to f''(0) = -\frac{1}{25}$$

$$f'''(x) = \frac{2}{(x+5)^3} \to f'''(0) = \frac{2}{125}$$

$$f^{(4)}(x) = \frac{-6}{(x+5)^4} \to f^{(4)}(0) = -\frac{6}{625}$$

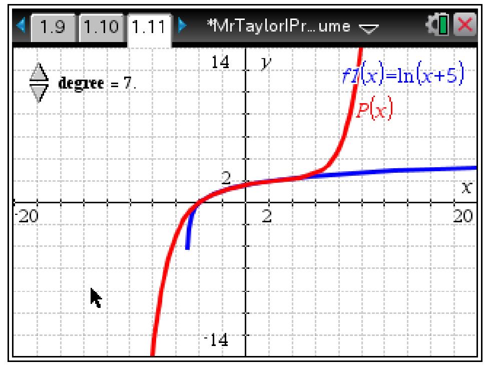
As the students compare the values of the Taylor polynomial with the values of the original function, they will notice that they are the closest at the center (the *x*-value of where the derivatives were found) and become farther apart the further the *x*-values are from the center (the graph demonstrates this point even further). Point out to the students that not all Taylor polynomials have such a large interval as this one.



TI-Nspire Navigator Opportunity: Class *Capture*

See Note 1 at the end of this lesson.

Students will also have a chance to explore different degrees of a Taylor polynomial. Students should notice that the interval for approximation does not change with the degree. If time permits, have a few students find the 10th degree Taylor polynomial and graph it on the same axis with the fourth and original function. This will demonstrate that a Taylor polynomial will only approximate the function over a given interval, no matter how large the degree of the Taylor polynomial.



Students are given another example to complete on their own.

Taylor Polynomial Not Centered at Zero

When the students work the problem of where the Taylor polynomial is not centered at zero, they will observe that the polynomial follows the original function for a much smaller interval. Remind students that they need to use $(x - a)^n$ instead of x^n.

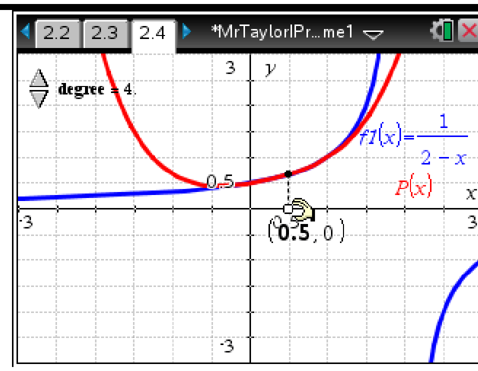
The derivatives for this problem are

$$f'(x) = \frac{1}{(2-x)^2} \qquad f''(x) = \frac{-2}{(2-x)^3} \qquad f'''(x) = \frac{6}{(2-x)^4} \qquad f^{(4)}(x) = \frac{24}{(2-x)^5}$$

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Students are given the chance to adjust the center of the polynomial on page 2.4. They should notice that the interval increases as the center gets closer to zero.



Solutions

1. When $x = 0$, the values are closest.
2. Answers may vary slightly. A possible answer may be $-2 < x < 3$.
3. The interval does not change with the degree.
4. $P(x) = 1 + (x-1) + (x-1)^2 + (x-1)^3 + (x-1)^4$
5. The interval increases as the center approaches zero.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Class Capture*
Use Class Capture to verify that students are able to manipulate the sliders correctly and to elicit class discussion.

Note 2
Problem 1, *Quick Poll*
Use Quick Poll to assess student understanding. Questions 1 though 5 can be used for possible questions to ask.