Goodness-Of-Fit Tests

TIMATH.COM: STATISTICS

Activity Overview

In this activity, students test claims of whether given distributions "fit" theoretical distributions. The activity begins with an introduction to the goodness of fit test and a review of the chi-square distribution. Students then work through two problems, one in which the theoretical proportions of each category are the same and one in which they are not.

Topic: Sampling

- Use a χ^2 test to test the hypothesis that an observed frequency distribution fits an expected frequency distribution.

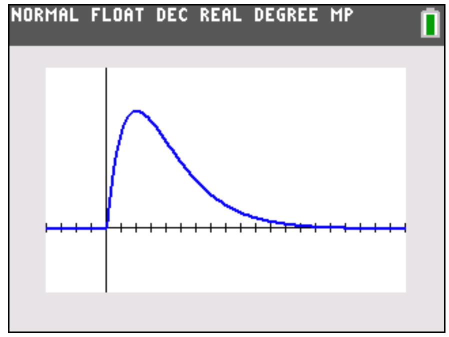
Teacher Preparation and Notes

- Students should already be familiar with the basic concepts involved in hypothesis testing.
- The problems in this activity use both the critical value method and the P-value method of hypothesis testing.
- Students will need to reference the Chi-Square distribution table during this activity.
- **To download the student worksheet, go to education.ti.com/exchange and enter "10285" in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Candy Pieces (TI-Nspire technology) — 10039
- Chi-Square Distributions (TI-Nspire technology) — 9738
- Testing Goodness-of-Fit and Two-Way Table Chi-Square (TI-84 Plus family) — 4590
- Chi-Square Test for Independence and Homogeneity (TI-84 Plus with TI-Navigator) — 1986



This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- GoodnessOfFit_Student.pdf
- GoodnessOfFit_Student.doc

Click HERE for Graphing Calculator Tutorials.

Problem 1 – The Test Statistic

Introduce the goodness-of-fit test and the test statistic that will be used.

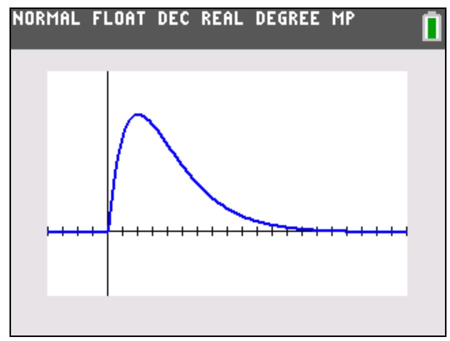
A **goodness of fit test** tests the hypothesis that observed frequencies in different categories fit a theoretical distribution. The test statistic is χ^2.

$\chi^2 = \sum \frac{(O-E)^2}{E}$, where O is the observed frequency of an outcome and E is the expected frequency of an outcome.

k: number of different categories
n: total number of trials
p: probability for a category
degrees of freedom: $k - 1$

When students graph the function χ^2 **pdf (X, 4)**, they may recognize the positively skewed chi-square distribution from earlier lessons on estimating population variance and standard deviation.

They can change the degrees of freedom and observe how the distribution changes.



Have students discuss the answers to the first three questions on their worksheet. Based on the formula for χ^2, the test statistic will be large when the observed and expected values are very different and small when the differences are small.

A small χ^2 would suggest that the values are a good fit with the theoretical distribution.

All goodness-of-fit tests are right-tailed because the leftmost value, 0, occurs when there are no differences.

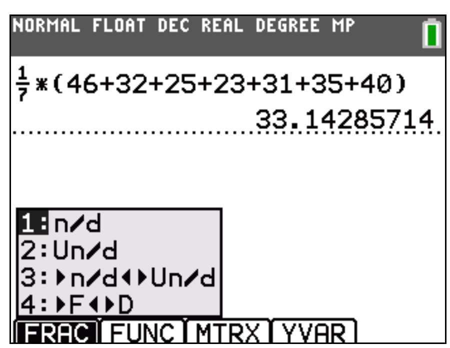
Problem 2 – Same Proportions Throughout

The scenario for Problem 2 tests a claim that every category has the same proportion. Students are to write H_0 and H_a on their worksheets.

H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7$

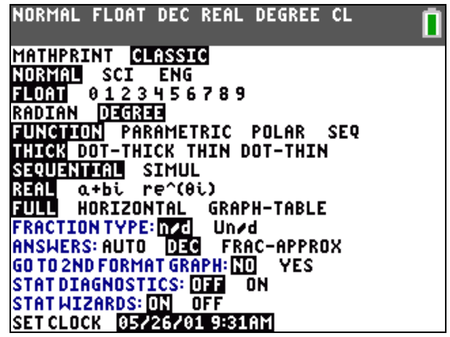
H_a: At least one of the proportions is different than the others.

After students read the scenario, they are to use the Home screen to find E, the expected value for each day. Assuming the null hypothesis is correct, the probability of a delay on any given day is $\frac{1}{7}n$. The value of n is the total number of trials, the sum of the delays.



Note: To enter a fraction, students can press [ALPHA] [F1] and select **n/d**. Enter the value of the numerator, press [▼] to move to the denominator, and enter the value of the denominator. Then press [▶] to move out of the fraction template.

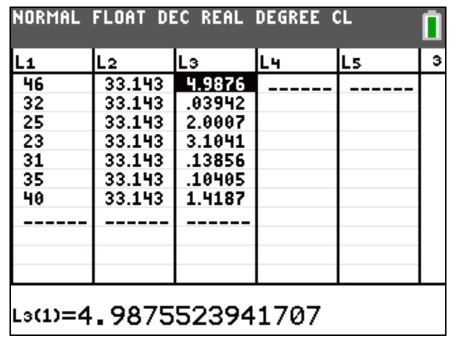
Note that the entry runs off the right side of the screen. The arrow indicates that the entry continues. If students wish to see the entire entry on the screen, they can switch to classic view by pressing [MODE] and pressing [ENTER] on **CLASSIC**. They will NOT be able to use the fraction template in this mode.



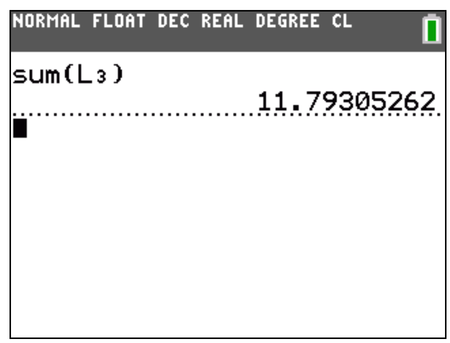
Students need to enter the observed frequencies in list **L1** and the expected frequency in list **L2**. Before they do this, they will need to clear all lists.

In the formula bar for **L3**, students are to enter **(L1-L2)^2/L2**.

They should see that this represents the $\frac{(O-E)^2}{E}$ part of the χ^2 formula.



On the Home screen students will finish calculating the test statistic by finding the sum of list **L3**.



Students will now need to use a Chi-Square distribution chart to find the critical value for the chi-square value with 0.95 of the area to the left or 0.05 of the area to the right.

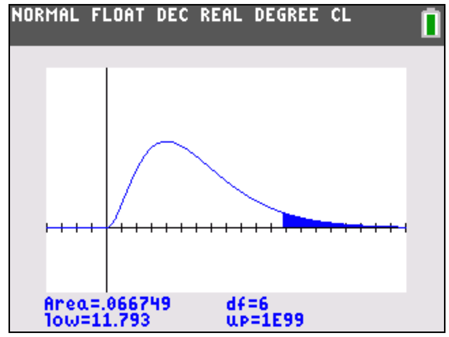
Remind students that the number of degrees of freedom is one less than the number of categories, so it is 6 for this problem.

The critical value is 12.5916. Ask students if they should reject or fail to reject the null hypothesis and why.

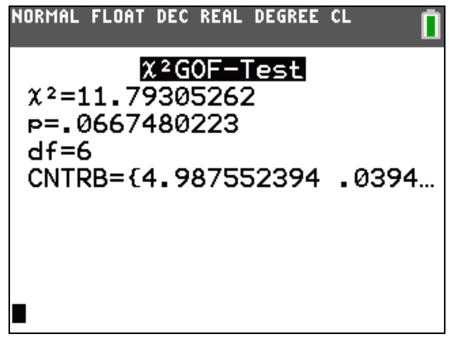
(Fail to reject; if the test statistic is less than the critical value, it is not to the right of the critical value (not in the critical region).)

Students will use the **Shade** χ^2 command from the Draw menu to find the *P*-value. First they should use the **ClrDraw** command to clear any existing drawings.

The *P*-value is about 0.066749. Have students discuss what this means and why it confirms their decision based on the critical value. (*P*-value is greater than 0.05.)



Students can verify their calculation by using the χ^2 **GOF–Test**. They enter select **L1** for Observed, **L2** for the Expected and **6** for Degrees of Freedom, df.



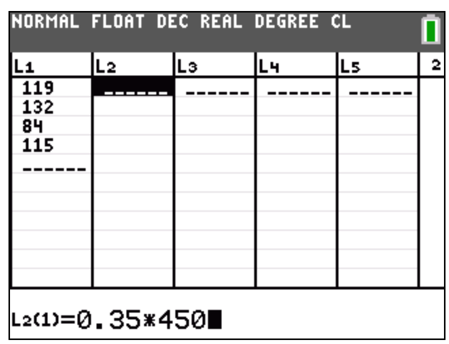
Problem 3 – Different Proportions Throughout

Unlike Problem 2, the expected values in Problem 3 will not all be the same. Students are to list H_0 and H_a on their worksheet.

H_0: $p_{\text{fruit}} = 0.35$, $p_{\text{nuts}} = 0.25$, $p_{\text{chocolate}} = 0.2$, $p_{\text{seeds}} = 0.2$

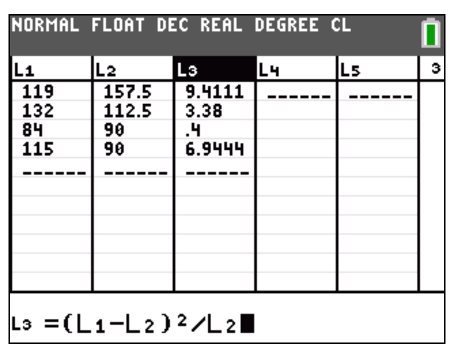
H_a: At least one of the proportions is different than what is listed above.

Before students enter the observed weights in **L1**, they need to clear all lists. Students are to use L1 and the percents to calculate the expected values in **L2**. For example, in cell **L2**(1), they should type **0.35 * 450**.



Students can then find each $\frac{(O-E)^2}{E}$ quotient in **L3** and the sum of the quotients as they did in Problem 2.

The test statistic equals 20.136.



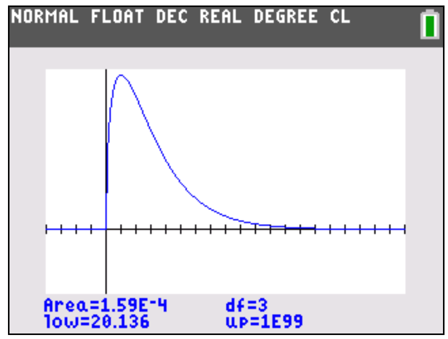
Students will now need to use a Chi-Square distribution chart to find the critical value for the chi-square value with 0.95 of the area to the left or 0.05 of the area to the right.

Remind students that the number of degrees of freedom is one less than the number of categories, so it is 3 for this problem.

The critical value is 7.815. Ask students if they should reject or fail to reject the null hypothesis and why.

(Reject; the test statistic is greater than the critical value; it is to the right of the critical value (in the critical region).)

Have students find the corresponding *P*-value on and interpret its meaning.



(*P*-value = 0.000159)

They should also answer the question on page 3.5 about why they should reject the null hypothesis. (Because the *P*-value is much less than 0.05.)

Students can then verify their calculations by using the χ^2 **GOF–Test** command.

Solutions – Student Worksheet

1. It would indicate a large discrepancy between observed and actual values, suggesting that the actual distribution does not fit the theoretical distribution.
2. It would indicate that observed values are very close to expected values, suggesting that the actual distribution closely fits the theoretical distribution.
3. All goodness-of-fit tests are right-tailed because the leftmost value, 0, occurs when there are no differences. The test statistic can only get larger as the actual distribution differs more and more from the expected, or theoretical, distribution.
4. H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7$
   H_a: at least one proportion is different from the others
5. 232
6. $\frac{1}{7}$
7. about 33.143
8. about 11.793
9. about 12.592
10. Fail to reject; the test statistic is less than the critical value; it is not to the right of the critical value (not in the critical region).
11. about 0.0667
12. about 0.0667
13. The *P*-value is greater than 0.05, the significance level.
14. H_0: $p_{\text{fruit}} = 0.35$, $p_{\text{nuts}} = 0.25$, $p_{\text{choc}} = 0.2$, $p_{\text{seeds}} = 0.2$
    H_a: at least one proportion is different than what is listed above
15. 450
16. fruit: 157.5, nuts: 112.5, chocolate: 90, seeds: 90
17. about 20.136
18. 7.815
19. Reject; the test statistic is greater than the critical value; it is to the right of the critical value (in the critical region).
20. about 0.000159