NUMB3RS Activity: Random Thoughts
Episode: "Burn Rate"

Topic: Probability   **Grade Level:** 9 - 12
Objective: Investigate random events
Time: 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator with Probability Simulation App
  installed (optional)

Introduction

In "Burn Rate," Charlie helps the FBI investigate a series of mail bombings with seemingly
unrelated targets. The FBI develops a likely suspect when Charlie analyzes the patterns
involved. However, he later realizes that the data is too perfect. Lines from the suspect's house
to each data point create equal angles—a result he thinks is too exact to be a coincidence. He
calls it "a perfect storm of data, an improbable event." Charlie explains the concept to Don by
laying out four bolts, forming a square. "If you want to create a square, you only need four
points, right?" Laying out two more bolts on each edge of the square, he continues "you don't
need eight more, but yet that's exactly what I have. More data than I need, all fitting together
perfectly." In this activity, students will investigate random events.

The first activity involves flipping a coin 25 times. This can be done with an actual coin, a
random number generator or the Toss Coins option on the Probability Simulation App on the
TI-83 Plus/TI-84 Plus graphing calculator. When using the Probability Simulation App for this
activity, choose **SET** to change the trial size to 25. Choose **TABL** to display the sequence of
flips. The Probability Simulation App can be downloaded from the Web site below:
http://education.ti.com/educationportal/sites/US/productDetail/
us_prob_sim_83_84.html

Question 2c asks students to identify a fake set of coin flips. To develop the fake sets ask a
group of students to fake the data. Most people will not include runs of four or more, feeling they
are not likely. Once the students investigate Question 2, they will have a better understanding of
the prominence of runs. The goal is to ask students to generate the fake sequence BEFORE
they have this knowledge. You may even consider asking students from a different class to
generate them. Mix the fabricated sets in with some that were generated randomly.

Student Page Answers:
1. *Each outcome has the same probability, $(1/2)^{25}$* **2a and 2b.** *Answers will vary* **2c.** *Students may be
able to identify the fabricated sequence by a lack of runs of four or more (though they should expect
some exceptions).* **3a.** $2^4=16$ **3b.** *2, all heads or all tails* **3c.** *2/16 = 12.5%* **4a.** *32* **4b.** *6*
4c. *6/32 = 18.75%* **5a.** *12/64 = 25%* **5b.** *40/128 = 31.25%* **5c.** *The probability of a run of at least four
increases as the number of flips increases.*

Name _____ Date _____

NUMB3RS Activity: Random Thoughts

In "Burn Rate," Charlie helps the FBI investigate a series of mail bombings with seemingly unrelated targets. The FBI develops a likely suspect when Charlie analyzes the patterns involved. However, he later realizes that the data is too perfect. Lines from the suspect's house to each data point create equal angles—a result he thinks is too exact to be a coincidence. He calls it "a perfect storm of data, an improbable event." Charlie explains the concept to Don by laying out four bolts, forming a square. "If you want to create a square, you only need four points, right?" Laying out two more bolts on each edge of the square, he continues "you don't need eight more, but yet that's exactly what I have. More data than I need, all fitting together perfectly." In this activity, you will investigate random events.

Coin Flips, Clusters and Randomness
What is random? Sometimes people mistake evenness for randomness. Consider the task of flipping a fair coin 25 times. Most people are confident that the resulting sequence will have a roughly equal number of heads and tails. But what will the distribution look like?

1. Consider first the probability of an exact sequence of 25 flips. Below are three possible sequences. Are some of these sequences more likely than others? How do the probabilities of each sequence below compare?

(1)   H T H T H T H T H T H T H T H T H T H T H H T H T

(2)   H T T H H T T H T T T T T H T H H T T H T H T H H

(3)   T T T T T H T T T T T T T T T T T T T T T T T T H

Now explore the general format of the sequences above. The first sequence alternates between heads and tails almost exclusively and the last sequence has mostly tails. Different sequences with the same patterns are possible, so a sequence with those general formats may not be equally likely to occur. A sequence with exactly alternating flips or exclusively heads are not very likely and are examples of the "perfect" data Charlie refers to in this episode.

To explore the idea of randomness further, investigate runs of heads or tails within a sequence. Flip a fair coin 25 times and record the sequence, looking for runs of heads or tails.

2. a. Does your sequence have at least one run of four or more in a row?

   b. Repeat the experiment or compile data from the class. How often was there a run of at least four in a row?

   c. Examine the sequences your teacher provides. Some were generated by classmates and some were made by people trying to "fake" a random sequence of coin flips. Based on your results from 2b, can you identify the sequences that are likely to be fabricated?

Your experiments indicated that a run of four or more is a fairly common occurrence when a coin is flipped 25 times. Computing the theoretical probability involves a concept called Bernoulli trials, and a link is provided in the extension section. You can calculate the probabilities for a smaller number of coin flips by listing possible cases.

3. a. How many different sequences are possible when a coin is flipped four times?

   b. How many of the sequences generated in 3a have a run of four heads or four tails?

   c. Use the results of 3a and 3b to determine the probability of a run of four when a coin is flipped four times.

4. a. How many different sequences are possible when a coin is flipped five times?

   b. How many of the sequences generated in 4a have a run of at least four heads or four tails?

   c. Determine the probability of a run of at least four when a coin is flipped five times.

5. a. Determine the probability of a run of at least four when a coin is flipped six times.

   b. Determine the probability of a run of at least four when a coin is flipped seven times.

   c. What does the general trend appear for the probability of a run of at least four when the number of flips increases?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Monkeys and Hamlet

A rather famous math problem asks you to place a monkey at a keyboard and allow him to randomly hit keys. Given enough time, the monkey will eventually type every combination of keys, producing every possible word and sentence. If you are very patient, the monkey will produce the entire text of Hamlet. Of course, you will have to be very patient. The Web site **http://www.heartofmath.com/resources/sample_explorations/nav/frames_5.html** provides an opportunity to explore this concept on a smaller scale. With fewer keys than a standard keyboard, you can enter a possible word and see how many random keystrokes are needed to produce your word.

Arranging Dots

To explain the "perfection" of the data he is analyzing, Charlie arranges twelve dots to form a square. Without peeking at his description, can you figure out how to do this? Peek when you are ready. The following questions are similar logic puzzles. Each one involves drawing dots on a paper. Hints and solutions are available at the Web site:
http://mathforum.org/k12/k12puzzles/rosebush.puzzle1.html

- Draw seven dots in six lines with three dots in each line.
- Draw ten dots in five lines with four dots in each line.
- Draw nineteen dots in nine lines with five dots in each line.

Additional Resources

The coin tossing experiment is one example of a Bernoulli trial. More information about Bernoulli trials and distributions can be found at the two Web sites below.
- **http://mathworld.wolfram.com/BernoulliDistribution.html**
- **http://www.mathpages.com/home/kmath341.htm**