Absolute Value Function

Student Worksheet

7 8 9 10 11 **12**

TI-Nspire™ How To Student 30 min

Introduction

The absolute value of a function is defined as the 'unsigned' portion of the number.

$$|x| = \begin{cases} x & x \le 0 \\ x & x \ge 0 \end{cases}$$

The sign or signum (Latin for sign) is defined as:

$$sign(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

The above definitions are related by $|x| = x \cdot sign(x)$

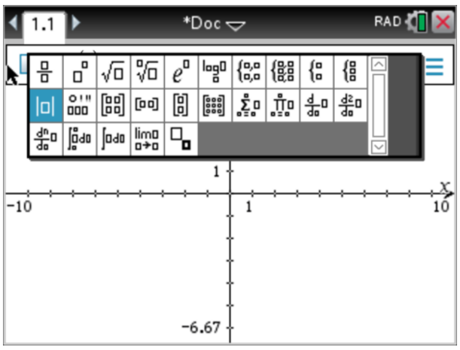
Exploring Graphs

Open a new TI-Nspire Document and insert a **Graph Application**.

Sketch the graphs of $y = x$ and $y = |x|$ on the same set of axes.

The equations template contains the absolute value notation or enter:

abs(x)



Question: 1.

Comment on the relationship between the graphs of $y = x$ and $y = |x|$.

Students should note that the when $x < 0$ the graph is reflected in the x axis.

Question: 2.

Graph and compare each of the following:

a. $y = x^2 - 4$ and $y = |x^2 - 4|$

b. $f(x) = x^3 - 3$ and $|f(x)| = |x^3 - 3|$

c. $g(x) = \sqrt{(2-x)} - 2$ and $|g(x)|$

d. $h(x) = x^3 - 2x^2 - 4x + 1$ and $|h(x)|$

e. $k(x) = \dfrac{1}{(x-2)^2} - 3$ and $|k(x)|$

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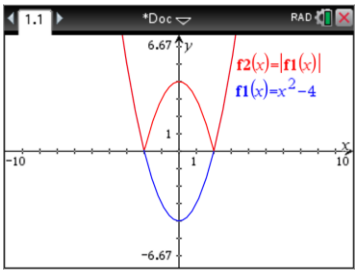
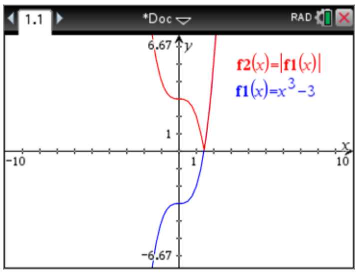
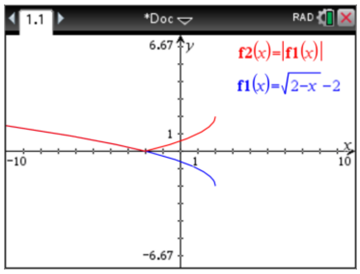
Question 2a)



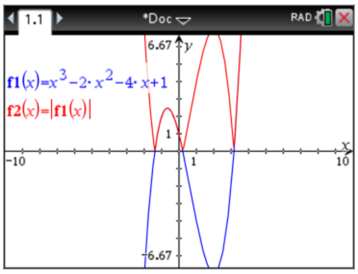
Question 2b)



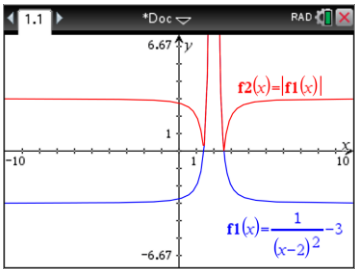
Question 2c)



Question 2d)



Question 2e)



These examples should help students identify the reflection in the x axis associated with $y = f(x)$ and $y = |f(x)|$.

Question: 3.

Generalise your findings with regards to what happens to the graph of $f(x)$ when we want to sketch the graph of $|f(x)|$.

Where $f(x) < 0$ the function is reflected in the x axis

Question: 4.

Graph and compare each of the following:

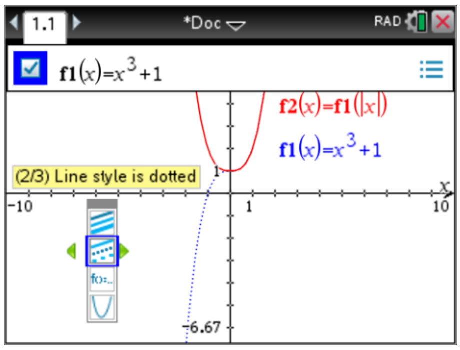
a. $f(x) = x^2 - 2x + 3$ and $f(|x|) = |x|^2 - 2|x| + 3$

b. $g(x) = x^3 + 1$ and $g(|x|) = |x|^3 + 1$

c. $h(x) = 2^x - 3$ and $h(|x|) = 2^{|x|} - 3$

d. $k(x) = \dfrac{1}{x-1}$ and $k(|x|) = \dfrac{1}{|x|-1}$

e. $p(x) = \log_e(x)$ and $p(|x|) = \log_e|x|$



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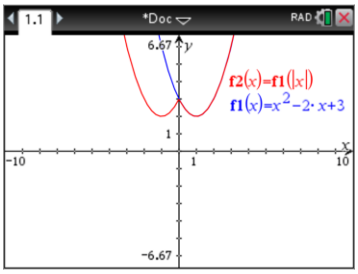
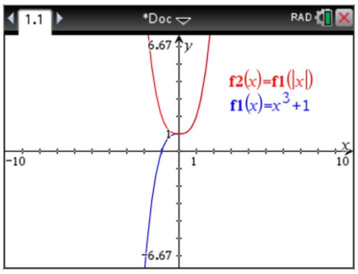
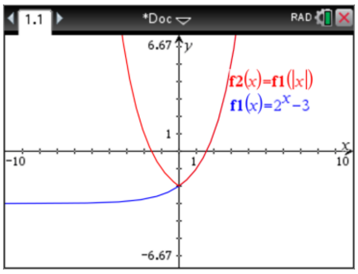
Question 4a)



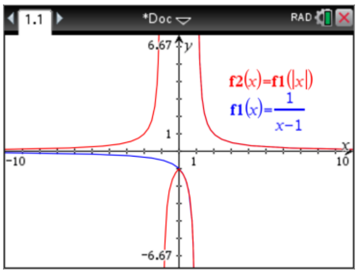
Question 4b)



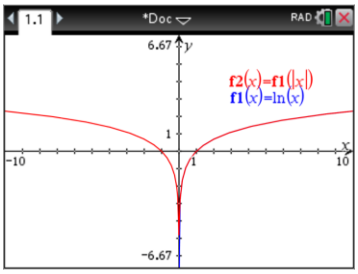
Question 4c)



Question 4d)



Question 4e)



These examples should help students identify the following:

Where x < 0 in the original function, this region ceases to exist and is replaced by a reflection of the function in the y axis for the region where x > 0.

In the case of 4(e) students should take special note of the significant change in the domain of the function.

Calculator Tip!

Time Saving Tip:

Enter the original equation in: $f_1(x)$ and then use: $f_2(x) = f_1(|x|)$

Attributes:

Attributes refers to some of the features or qualities of objects such as graphs. With your mouse over a graph press: **Ctrl + Menu** and select **Attributes**. Change the original function to a dotted

Question: 5.

Generalise your findings with regards to the graphs of $f(x)$ and $f(|x|)$.

Where x < 0 in the original function, this region ceases to exist and is replaced by a reflection of the function in the y axis for the region where x > 0.

Question: 6.

The graph of the function: $f(x) = 2^{|x|}$ can be generated by defining a piece-wise function rather than using the absolute value function. (Refer to the definition of $|x|$ in the introductory section of this activity.)

The function: $f(x) = 2^{|x|}$ can be defined as: $f(x) = \begin{cases} 2^{-x}, & x \le 0 \\ 2^{x}, & x > 0 \end{cases}$

Use your graphics calculator to sketch this piecewise function using the piecewise function entry. Use the absolute value function to generate a second graph to check your answer. Are the two graphs the same?

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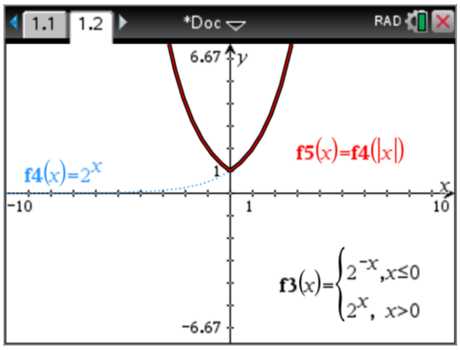
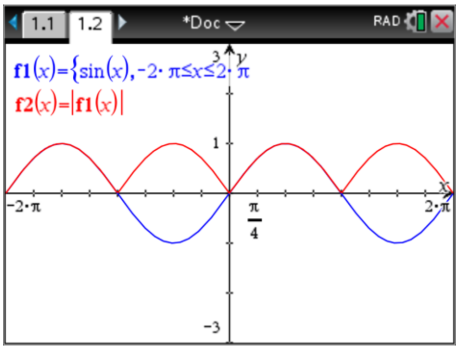
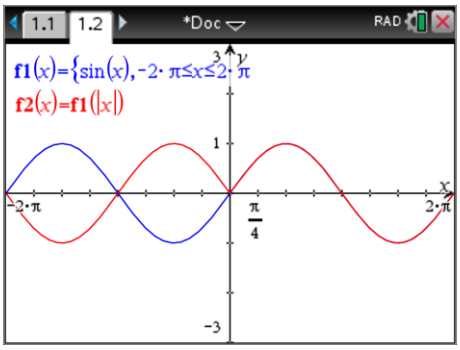
Drawing three functions may help students understanding of the equivalent transformations and also the piece-wise definition for the absolute value function.
The original function drawn in light blue (shown opposite) is clearly no longer represented in the function: $f\left(|x|\right)$.
The piecewise function drawn in black has been made bold so that it is clear that it is equivalent to the graph of $f\left(|x|\right)$. Students may also like to press Ctrl + T to produce a table of values to see how/why this relationship exists.



Question: 7.

Given the graph of: $f(x)=\sin x,\ -2\pi\le x\le 2\pi$, sketch the graphs of $|f(x)|$ and $f\left(|x|\right)$ without a calculator. Check your answers using your calculator.

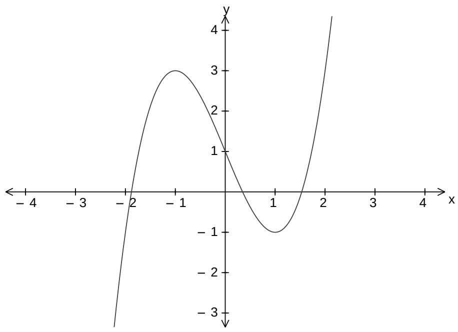




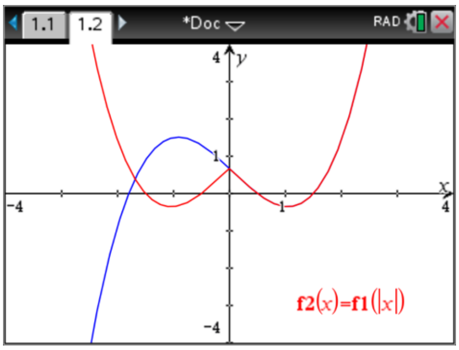
While no new concepts are included here, it is the first time students have been asked to do both $|f(x)|$ and $f\left(|x|\right)$ in the same question and on the same graph for comparative purposes.

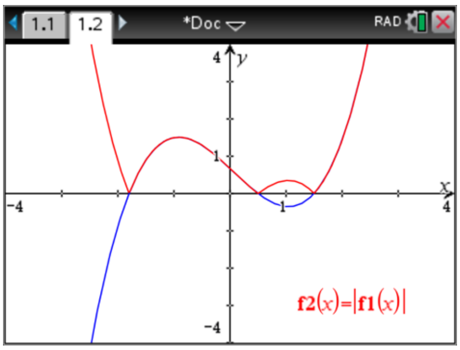
Question: 8.

For the graph of $f(x)$ shown opposite, sketch a graph of $f\left(|x|\right)$ and $|f(x)|$.



In the absence of an equation students must either determine an approximate equation that resembles the one shown or use their understanding to sketch the final results of the transformations.





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