Permutations & Combinations

Name ____________________

Class ____________________

Problem 1 – Arranging Letters

Would you believe there are over 20 different arrangements of the letters A, B, C and D, when selecting three of the four letters? Record all the arrangements you can below.

Note: Each arrangement of letters should be different than the previous arrangement. For example, ABC is different than BCA.

Each of these arrangements is known as a permutation. To find permutations of a set, objects are arranged in a way such that the order of the objects matters.

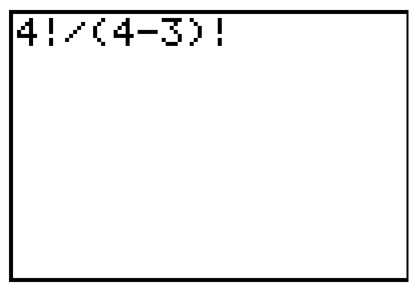
The notation for permutations is ${}_nP_r$, where n is the total number of objects and r is the number of objects selected. You can use the following formula to find the total number of permutations: ${}_nP_r = \frac{n!}{(n-r)!}$.

Use this formula to find out how many ways you can select 3 letters from 4 letters when the order matters.

Using n = 4 and r = 3, press [4] [MATH], select **!** from the **PRB** menu.

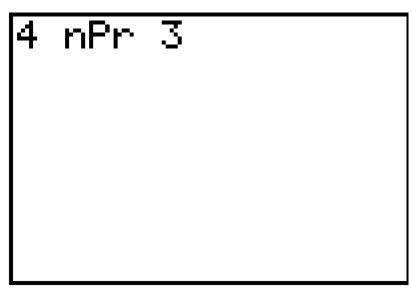
Next press [÷] [(] [4] [−] [3] [)] [MATH], select **PRB** and choose **!**.

Press [ENTER] to observe the answer.



To arrive at this answer much quicker, use the built-in **nPr** function of the graphing calculator.

To do this, press [4] [MATH], select **2:nPr** from the **PRB** menu, and then press [3].



Do these values match the number of arrangements you found above earlier?

Problem 2 – Arranging Letters in a Different Way

Now, let's arrange the letters in the same way, but this time the order of the letters does ***not*** matter. Record all the arrangements you can below.

For example, ABC is ***not*** different than BCA.

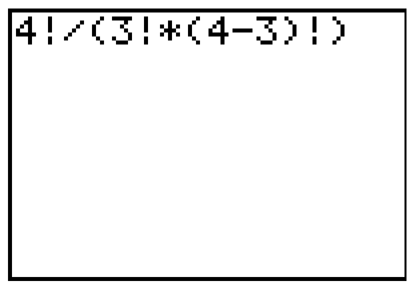
Each of these arrangements is known as a <u>combination</u>. To find combinations of a set, objects are arranged in a way such that the order of the objects does not matter.

The notation for permutations is ${}_{n}C_{r}$, where *n* is the total number of objects and *r* is the number of objects selected. You can use the following formula to find the total number of permutations:

$${}_{n}C_{r} = \frac{n!}{r!\cdot(n-r)!}.$$

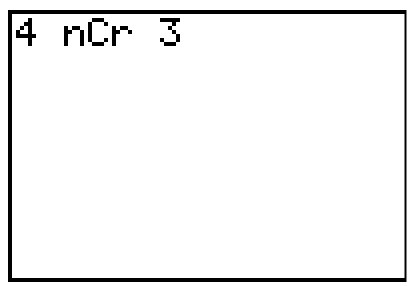
Use this formula to find out how many ways you can select 3 letters from 4 letters when the order does not matter.

Using *n* = 4 and *r* = 3, match the calculator screen to the right. Press [ENTER] to observe the answer.



To arrive at this answer much quicker, use the built-in **nCr** function of the graphing calculator.

To do this, press [4] [MATH], select **3:nCr** from the **PRB** menu, and then press [3].



Does this match the number of arrangements you found earlier?

Problem 3 – Permutation versus Combination

Answer the following questions.

- Selecting five cards from a standard deck of cards is an example of a combination.
  True or false?

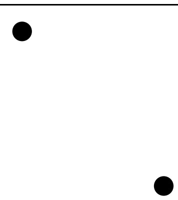
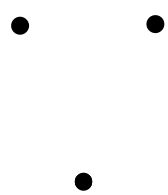
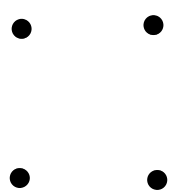
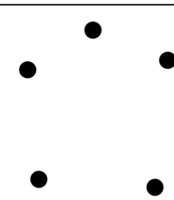
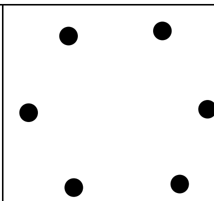
- Selecting three letters for a license plate is an example of a combination.
  True or false?
-
- Which expression has a larger value, ${}_5C_3$ or ${}_5P_3$?

- Why are there more arrangements when calculating a permutation than a combination?

Extension – Handshake Problem

Suppose each person in a group shakes hands with every other person in the group. How many handshakes occur?

On the pictures below, connect pairs of points (representing people) with a segment until all points are connected to each of the other points. The number of segments equals the number of handshakes. Record your results in the chart.

People					
n	2	3	4	5	6
Number of Handshakes					

- Is this situation a combination or permutation? Why?

- How many handshakes occur if there are *n* people?