

It's To Be Expected

ID: 11858

Time required

20 minutes

Activity Overview

In this activity, students will use a tree diagram to find theoretical probabilities and use this information in a spreadsheet to find the expected value.

Topic: Expected Value

- Theoretical probability
- Expected value
- Binomial Distributions

Teacher Preparation and Notes

- This activity introduces tree diagrams and expected value. A homework assignment is included.
- Students should have knowledge of **nSolve** and combinations functions on the handheld.
- Students should know the difference between independent and dependent events. They should know the multiplication rule for independent events.
- To download the student and solution TI-Nspire™ documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11858" in the keyword search box.

Associated Materials

- *Its_To_Be_Expected_Student.doc*
- *Its_To_Be_Expected.tns*
- *Its_To_Be_Expected_Soln.tns*

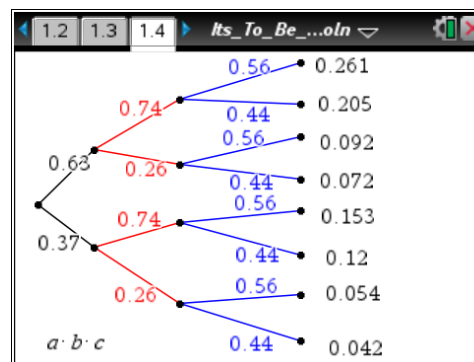
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Makin' It Through the Winter* (TI-Nspire™ technology) — 11937

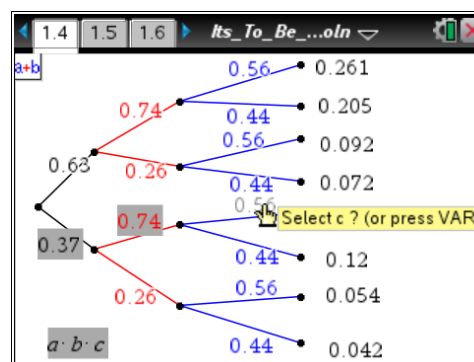
Creating a Tree Diagram

Students are given a scenario of three basketball players with different shooting percentages. First discuss with them whether the events are independent or dependent and why. Then on page 1.4, they need to change the labels of the tree diagram to show the percentages associated with each branch. To change a label, students need to move the cursor over the label and then double click. After editing the label, press **enter**.



The diagram given represents one way to calculate the probabilities. Discuss with students other ways to set up the tree diagram, such as with Bria or Carmen shooting first, and whether that would change the probabilities.

With the help of the diagram, students are to use the **Calculate** tool to calculate the eight probabilities. From these probabilities, they will determine the probabilities of none, one, two, or all three of the girls making their shots.



Discuss with students the multiplication rule that allows them to calculate the probabilities. Ask students what other probabilities they can calculate:

What is the probability that Aisha will make her shot and Bria will miss her shot?
 $((0.63)(0.26) = 0.1638$ or 16.38%)

Answers to questions:

1. (8 outcomes, ^m means miss)
ABC, AB^mC, ABC^m, AB^mC^m, A^mBC, A^mB^mC, A^mBC^m, A^mB^mC^m
2. **0.63; 0.37**
3. **0.74; 0.26**
4. **0.56; 0.44**
5. **0.042328**
6. **0.246416**
7. **0.450184**
8. **0.261072**
9. **\$12,080.80**

A screenshot of TI-Nspire page 1.5. The question is: "What is the probability that **none** of the girls make their shots?" The answer is 0.042328. The next question is: "What is the probability that **one** girl makes her shot? (Hint: Which of the 8 probabilities must be added together to find the answer?)" The answer is $0.072072+0.120472+0.053872=0.246416$.

Introducing Expected Value

Students are introduced to expected value with the continuation of the scenario. The results of the contest are associated with different dollar amounts. Students will use the spreadsheet on page 1.8 to determine what the girls can expect to earn for their charity.

Before entering the probabilities, discuss with students what amount they think the girls will earn. The payoff of the outcome with the highest probability should be the most likely choice.

- Step 1: Students will enter the probabilities of 0, 1, 2, and 3 shots made in Column B.
- Step 2: Then they are to enter the associated payoff in Column C.
- Step 3: To calculate the probability•payoff (Column D), students need to enter $=b1 \cdot c1$ in cell d1. Then copy and paste the formula into cells d2, d3, and d4.
- Step 4: The expected value is the sum of these cells. Students can use the sum command or add up the cells.

$=\text{sum}(d1:d4)$ or $=d1+d2+d3+d4$

The expected value may differ based on the rounding that occurs.

The solution TI-Nspire document contains a more in-depth spreadsheet on page 1.10 that encompasses the calculations of the entire activity.

outcome	prob	payoff	prob_pa...
0	0.042328		
1	0.246416		
2	0.450184		
3	0.261072		

outcome	prob	payoff	prob_pa...
0	0.042328	0	$=b1 \cdot c1$
1	0.246416	5000	
2	0.450184	12500	
3	0.261072	20000	

outcome	prob	payoff	prob_pa...
2	0.450184	12500	5627.3
3	0.261072	20000	5221.44
			12080.8

carmen	prob	succ...			
0.56	0.2610...	3	0.2610...	20000	5221.44
0.44	0.2051...	2	0.4501...	12500	5627.3
0.56	0.0917...	2			
0.44	0.0720...	1	0.2464...	5000	1232.08
0.56	0.1533...	2			
0.44	0.1204...	1			
0.56	0.0538...	1			
0.44	0.0423...	0	0.0423...	0	0.
					12080.8

Extension – Putting it All Together

Answers to questions:

1. See page 2.2 in the solution TI-Nspire document.
2. **42¢** (may differ based on rounding)
3. **58¢**
4. **\$1** (The break-even point is when the expected value is equal to what someone pays to play.)
5. **Approx. \$144,000,000** (See **nSolve** at right. Let the jackpot be x and solve the equation so that the expected value is 1.)

Note: The handheld uses E to display some of the numbers in scientific notation. On a Calculator page, students can see the entire number by highlighting the number and then pressing **enter**.

A	prob	payoff	=a[]*b[]
1	5.69115...	42000000	0.239028
2	2.56102...	250000	0.064025
3	0.000001	10000	0.014512
4	0.000065	150	0.009796
5	0.000073	150	0.010885
6	0.003268	7	0.022876
7	0.001185	10	0.011848
8	0.007092	3	0.021277
9	0.013333	2	0.026667
10			0.420914
11			0.181885 ev=c1
12			

D10 =sum(c1:c9)

1.10 2.1 2.2 Its_To_Be_Ex...oln

©Break even point for lottery

$$\text{nSolve}\left(0.181885 + \frac{1}{175771536} \cdot x = 1, x\right)$$

1.43801E8

©Number of different sets of lottery numbers

$$\text{nCr}(56, 5) \cdot 46 \quad 175711536$$

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