E X P L O R A T I O N S ™

Activity 3

Angles and Transversals

Introduction

Euclidean geometry depends on the supposition that parallel lines exist. This exploration investigates angle relationships when two parallel or non-parallel lines are cut by a transversal.

This activity makes use of the following definitions:

**Transversal**—a line that intersects two coplanar lines at two different points

Corresponding angles—pairs of angles that lie in relative positions when two lines are cut by a transversal (for example, ∠ 4 and ∠ 8)

Alternate interior angles—pairs of angles that lie on opposite sides of a transversal and between the two lines cut by the transversal (for example, ∠ 3 and ∠ 6)

Alternate exterior angles—pairs of angles that lie on opposite sides of a transversal and are outside the two lines cut by the transversal (for example, ∠ 2 and ∠ 7)

Same-side interior angles—pairs of angles that lie on the same side of a transversal and are between the two lines cut by the transversal (for example, ∠ 3 and ∠ 5)

Same-side exterior angles—pairs of angles that lie on the same side of a transversal and are outside the two lines cut by the transversal (for example, ∠ 1 and ∠ 7)
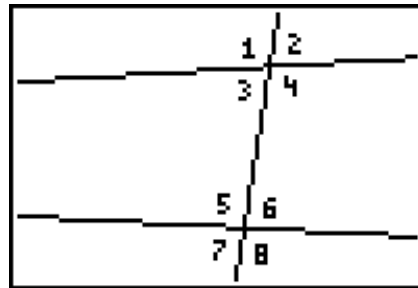
Part I: Corresponding Angles

Construction

Draw two non-parallel lines cut by a transversal.

[icons] Draw a non-horizontal line $\overleftrightarrow{AB}$ on the top half of the screen.

[icons] Draw a horizontal line $\overleftrightarrow{CD}$ on the bottom half of the screen.

[icons] Construct $\overleftrightarrow{YZ}$ as a transversal that intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ as shown.

[icons] Construct point W at the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{YZ}$.

[icons] Construct point X at the intersection of $\overleftrightarrow{CD}$ and $\overleftrightarrow{YZ}$.

[icon] Measure ∠AWX and ∠CXZ.
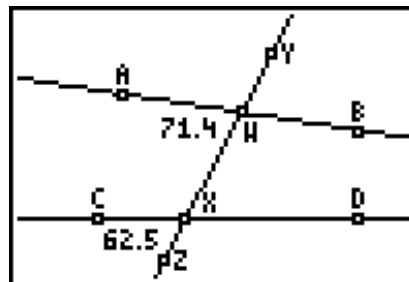
Exploration

[icon] Drag each line or a point on the line to find conditions when the measures of ∠AWX and ∠CXZ are equal. Note the relationship of the three lines each time the condition is met.

[icon] Investigate the relationships between the following pairs of angles when the measures of ∠AWX and ∠CXZ are equal.

- ∠AWX and ∠WXD
- ∠AWX and ∠CXW
- ∠CXZ and ∠YWB
- ∠CXZ and ∠AWY

Questions and Conjectures

1. For each angle pairing mentioned in the exploration, determine the angle relationship of the pair. For example, which pair of angles are alternate exterior angles?

2. Make a conjecture about the relationship among the three lines when the measures of ∠AWX and ∠CXZ are equal.

3. Make a conjecture about the relationships between the angles listed below when the measures of ∠AWX and ∠CXZ are equal.

- ∠AWX and ∠WXD
- ∠AWX and ∠CXW
- ∠CXZ and ∠YWB
- ∠CXZ and ∠AWY

Part II: Parallel Lines Cut by a Transversal

Construction

Draw two parallel lines cut by a transversal.

⌫	Clear the previous construction.
▱ Ⓐ	Draw a non-horizontal line $\overleftrightarrow{EF}$ on the top half of the screen.
▫ Ⓐ	Draw a point *G* not on $\overleftrightarrow{EF}$ on the bottom half of the screen.
▱ Ⓐ	Construct line $\overleftrightarrow{GH}$ parallel to $\overleftrightarrow{EF}$.
▱ Ⓐ	Construct line $\overleftrightarrow{ST}$ as a transversal that intersects $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$.
▱ Ⓐ	Construct point *M* at the intersection of $\overleftrightarrow{EF}$ and $\overleftrightarrow{ST}$.
▱ Ⓐ	Construct point *N* at the intersection of $\overleftrightarrow{GH}$ and $\overleftrightarrow{ST}$.

Exploration

◿ ◈	Measure two angles that are corresponding angles. Drag the lines and the points on the lines. Note the relationship between the two angles.
◿ ◈	Repeat the previous Exploration using a pair of angles that are:

- • alternate interior angles
- • alternate exterior angles
- • same-side interior angles
- • same-side exterior angles

Questions and Conjectures

Answer the following questions based on angles formed by two parallel lines cut by a transversal.

1. Name all pairs of corresponding angles and make a conjecture about their measurements.

2. Name all pairs of alternate interior angles and make a conjecture about their measurements.

3. Name all pairs of alternate exterior angles and make a conjecture about their measurements.

4. Name all pairs of same-side interior angles and make a conjecture about their measurements.

5. Name all pairs of same-side exterior angles and make a conjecture about their measurements.

Teacher Notes

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Angles and Transversals

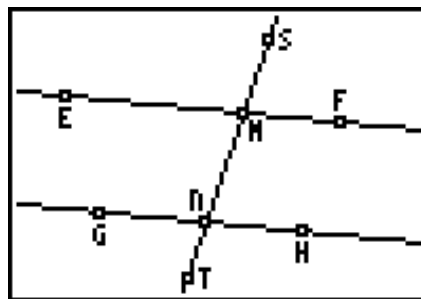## Part I: Corresponding Angles

Answers to Questions and Conjectures

1. For each angle pairing mentioned in the exploration, determine the angle relationship of the pair. For example, which pair of angles are alternate exterior angles?

   Corresponding angles: ∠*AWX* and ∠*CXZ*

   Alternate interior angles: ∠*AWX* and ∠*WXD*

   Alternate exterior angles: ∠*CXZ* and ∠*YWB*

   Same-side interior angles: ∠*AWX* and ∠*CXW*

   Same-side exterior angles: ∠*CXZ* and ∠*AWY*

2. Make a conjecture about the relationship among the three lines when the measures of ∠*AWX* and ∠*CXZ* are equal.

   When the measures of ∠*AWX* and ∠*CXZ* are equal, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel.
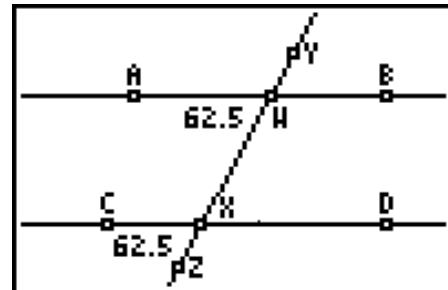
3. Make a conjecture about the relationships between the angles listed below when the measures of ∠*AWX* and ∠*CXZ* are equal.

   - ∠*AWX* and ∠*WXD*
   - ∠*AWX* and ∠*CXW*
   - ∠*CXZ* and ∠*YWB*
   - ∠*CXZ* and ∠*AWY*

When the measures of ∠*AWX* and ∠*CXZ* are equal,

- *m*∠*AWX* = *m*∠*WXD*.
- ∠*AWX* and ∠*CXW* will be supplementary.
- *m*∠*CXZ* = *m*∠*YWB*.
- ∠*CXZ* and ∠*AWY* will be supplementary.

Part II: Parallel Lines Cut by a Transversal

Answers to Questions and Conjectures

Answer the following questions based on angles formed by two parallel lines cut by a transversal.

1. Name all pairs of corresponding angles and make a conjecture about their measures.

    ∠*EMS* and ∠*GNM*, ∠*SMF* and ∠*MNH*, ∠*EMN* and ∠*GNT*, ∠*FMN* and ∠*HNT* are corresponding angles. When corresponding angles are formed by two parallel lines cut by a transversal, their measures are equal. They are congruent angles.
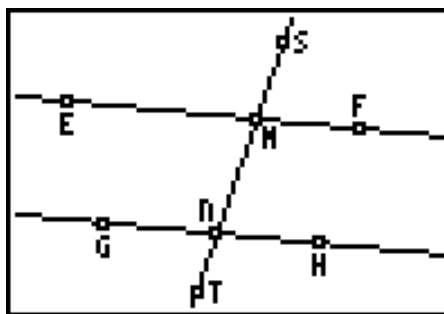
2. Name all pairs of alternate interior angles and make a conjecture about their measures.

    ∠*EMN* and ∠*HNM*, ∠*FMN* and ∠*GNM* are alternate interior angles. When alternate interior angles are formed by two parallel lines cut by a transversal, their measures are equal. They are congruent angles.

3. Name all pairs of alternate exterior angles and make a conjecture about their measures.

    ∠*SME* and ∠*TNH*, ∠*SMF* and ∠*TNG* are alternate exterior angles. When alternate exterior angles are formed by two parallel lines cut by a transversal, their measures are equal. They are congruent angles.

4. Name all pairs of same-side interior angles and make a conjecture about their measurements.

    ∠*EMN* and ∠*GNM*, ∠*FMN* and ∠*HNM* are same-side interior angles. When same-side interior angles are formed by two parallel lines cut by a transversal, the sum of their measures is 180°. They are supplementary angles.

5. Name all pairs of same-side exterior angles and make a conjecture about their measurements.

∠*SME* and ∠*TNG,* ∠*SMF* and ∠*TNH* are same-side exterior angles. When same-side exterior angles are formed by two parallel lines cut by a transversal, the sum of their measures is 180°. They are supplementary angles.