EXPLORATIONS

Chapter 6

Features Used

[GRAPH], **solve( ), expand( ), getDenom( ), zeros( ), NewProb,** [Y=], **getNum( ), factor( ),** [I], **3D graph, abs( ),** [WINDOW], **cFactor( ), NewData, cZeros( ), real( ), imag( )**

Setup

[♦]1, NewFold laplace

Laplace Analysis: The s-domain

This chapter demonstrates the utility of symbolic algebra by using the Laplace transform to solve a second-order circuit. The method requires that the circuit be converted from the time-domain to the s-domain and then solved for V(s). The voltage, v(t), of a sourceless, parallel, RLC circuit with initial conditions is found through the Laplace transform method. Then the solution, v(t), is graphed.

This chapter also shows how to find and plot the poles and zeros of a circuit's transfer function H(s) to gain insight to the frequency response.

Topic 27: RLC Circuit

Given the circuit shown in Figure 1, find v(t) for t>0 when v(0)=4 V and i(0)=1 A.

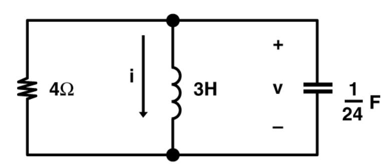


Figure 1. Simple parallel RLC circuit

Convert the components to their s-domain equivalents. Remember, the time-domain components map to their s-domain counterparts as shown in Figure 2.

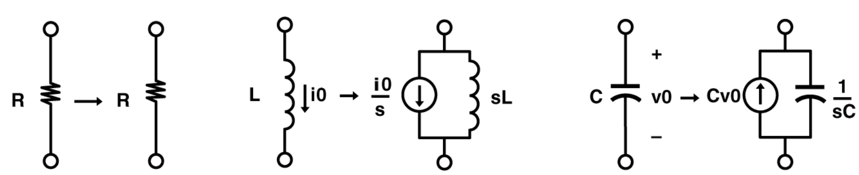


Figure 2. Time-domain to s-domain mappings

Note that the initial voltage and current transform into equivalent sources in the s-domain. The circuit in the s-domain is shown in Figure 3.

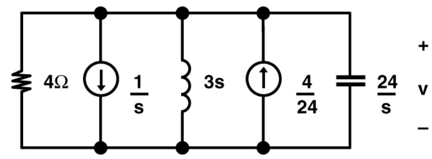


Figure 3. s-domain equivalent of the circuit in Figure 1

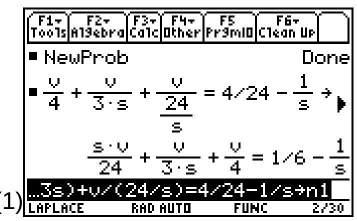
Using Kirchhoff's current law to sum the currents out of the top node, the equation is

$$\frac{v}{4}+\frac{v}{3s}+\frac{v}{\frac{24}{s}}=\frac{4}{24}-\frac{1}{s}\rightarrow n1$$

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

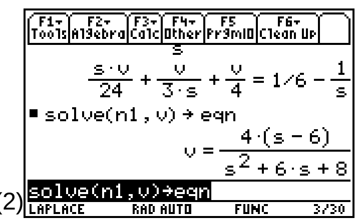
2. Enter the equation above (screen 1).

   v [÷] **4** [+] **v** [÷] [(] **3s** [)] [+] **v** [÷] [(] **24** [÷] **s** [)] [=] **4** [÷] **24** [−] **1** [÷] **s** [STO▸] **n1**

(1) 

3. The s-domain voltage is found with **solve**(**n1**,**v**) as shown in screen 2.

   [CATALOG] **solve(** **n1** [,] **v** [)] [STO▸] **eqn**

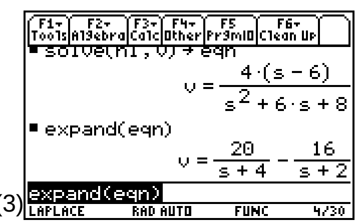
(2) 

4. Enter **expand**(**eqn**) to put **eqn** in a form for easy calculation of the inverse Laplace transform via a table lookup (screen 3).

   This must be an overdamped circuit since there are two real poles. The answer should contain two decaying exponents. From a Laplace transform table, the solution is

   $$v(t) = 20e^{-4t} - 16e^{-2t} \quad t \geq 0$$

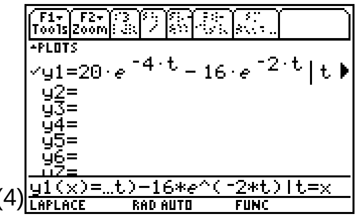
   This answer is in the expected mathematical form. How does v(t) appear as a function of time?

(3) 

5. To obtain a graph, press ◆ [Y=] and enter the expression for v(t) as shown in screen 4. Note that x is substituted for t using the “with” operator, [|], since the Y= Editor requires equations to be expressed as functions of x.

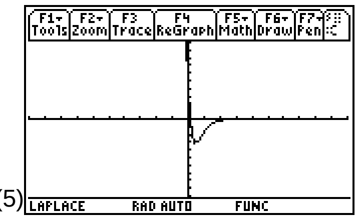
20 [×] [◆] [e^x] [(-)] **4t** [)] [−] **16** [×] [◆] [e^x] [(-)] **2t** [)] [|] **t** [=] **x**

Note: ***x*** *is substituted for* ***t*** *using the* ***with*** *operator, [|], since the* ***Y=*** *Editor requires equations to be expressed as functions of* ***x****.*



(4)

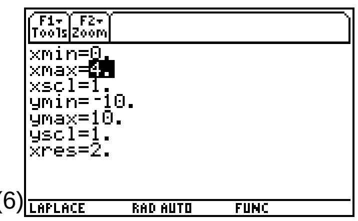
6. Now press [F2] **6:ZoomStd** to see the graph (screen 5).



(5)

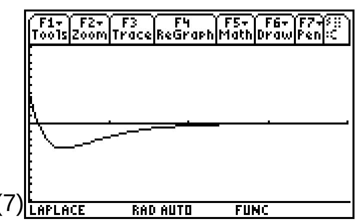
It appears to be a typical overdamped response!

7. To zoom in for a closer look, press ◆ [WINDOW] and set the range of **x** to be 0 to 4 (screen 6).



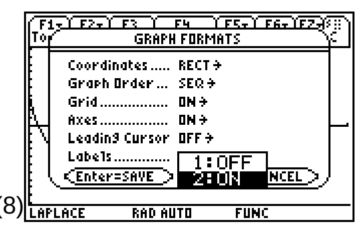
(6)

8. Now, press ◆ [GRAPH] to see the graph of v(t) as shown in screen 7.

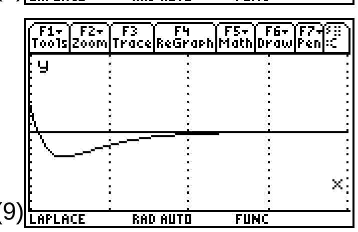


(7)

9. Press [F1] **9:Format** and specify **ON** for **Grid** and **Labels** (screens 8 and 9).



(8)



(9)

Topic 28: Critical Damping

Given the circuit of Topic 27, change the value of the resistor so that the circuit will be critically damped.

Assign the resistor value as **r** (see Figure 4) and let the TI-89 compute the node voltage **v**(**s**) in terms of **r**.

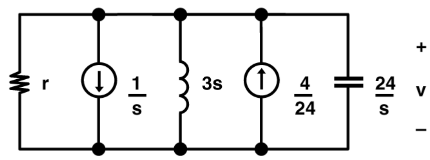


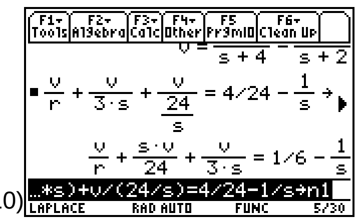
Figure 4. Circuit of Figure 3 with the 4 Ω resistor changed to **r**

The nodal equation in the s-domain is

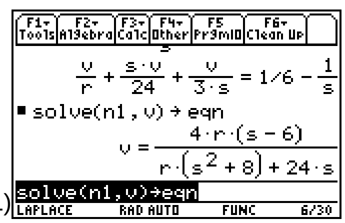
$$\frac{v}{r}+\frac{v}{3s}+\frac{v}{\frac{24}{s}}=\frac{4}{24}-\frac{1}{s}\rightarrow n1$$

1. Return to the Home screen, and enter this as shown in screen 10.

v ÷ **r** + **v** ÷ (**3s**) + **v** ÷ (**24** ÷ **s**) = **4** ÷ **24** − **1** ÷ **s** STO▸ **n1**

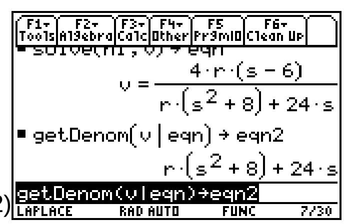
(10)


2. Solve for the node voltage with **solve**(**n1**,**v**) → **eqn** (screen 11).

(11)


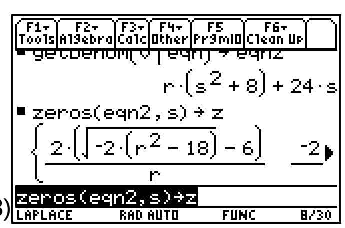
3. For critical damping, the time constants of the two exponentials of **v**(**s**) must be real and equal. To determine this condition, the two roots of the denominator of **v**(**s**) are found and set equal, and the resulting equation is solved for the required value of **r**. Get the denominator with **getDenom()** as shown in screen 12.

CATALOG **getDenom(v** | **eqn**) STO▸ **eqn2**

(12)


4. Solve for values of **s** which are the roots of the denominator using the **zeros()** command as shown in screen 13.

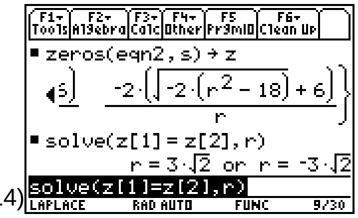
CATALOG **zeros(eqn2** , **s**) STO▸ **z**

(13)


5. Set the two roots equal to each other and solve for **r** as shown in screen 14.

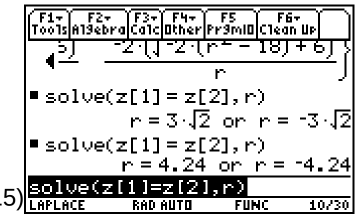
CATALOG **solve(z** 2nd [**1** 2nd] = **z** 2nd [**2** 2nd] , **r**)

Since negative resistances are not physically possible, the answer must be **r** = $3\sqrt{2}$.

(14)


6. To get the floating point approximation, press ◆ [≈] as shown in screen 15.

So **r** = 4.2 will give critical damping.

(15)


Topic 29: Poles and Zeros in the Complex Plane

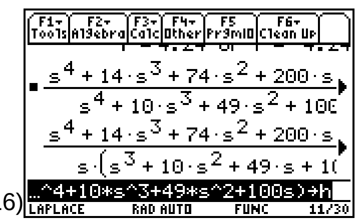
Given that

$$H(s) = \frac{s^4 + 14s^3 + 74s^2 + 200s + 400}{s^4 + 10s^3 + 49s^2 + 100s}$$

find and plot the poles and zeros.

1. Enter **h**(**s**) as shown in screen 16.

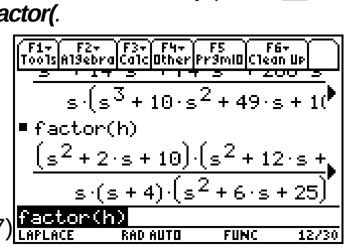
(**s** ^ **4** + **14s** ^ **3** + **74s** ^ **2** + **200s** + **400**) ÷ (**s** ^ **4** + **10s** ^ **3** + **49s** ^ **2** + **100s**) STO▸ **h**

(16)


2. A quick way to see the poles and zeros is to factor **h**(**s**) as shown in screen 17.

CATALOG **factor(h**)

***Note**: To enter **factor(**, press* F2 ***2:factor(**.*

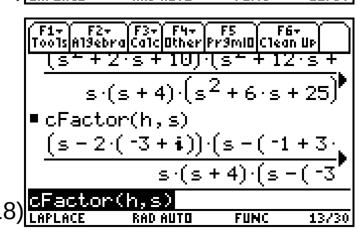
(17)


3. However, since **factor()** doesn't give complex factors, use **cFactor()** to get more information about **h**(**s**) (screen 18).

CATALOG **cFactor(h** , **s**)

Press ⊖ ▸ to see the rest of the terms of **h**(**s**). The complete answer is

$$\frac{(s-2(-3+i))(s-(-1+3i))(s+1+3i)(s+2(3+i))}{s(s+4)(s-(-3+4i))(s+3+4i)}$$

(18)


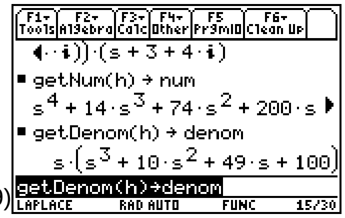
4. **getNum()** and **getDenom()** (screen 19) give the numerator and denominator, respectively.

[CATALOG] **getNum(h** [)] [STO▸] **num**

[CATALOG] **getDenom(h** [)] [STO▸] **denom**

The TI-89 automatically expands these terms, so **cFactor()** must be used again if you want to see the factors.

***Note**: To enter **getNum()** and **getDenom()** press [F2] **B:Extract**, then **1:getNum(** or **2:getDenom(**.*

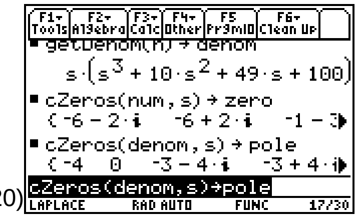


(19)

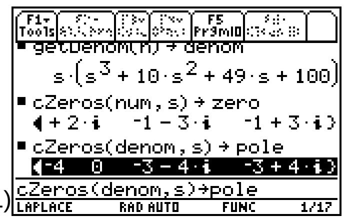
5. Once the numerator and denominator are separated, the zeros and poles are found by using the **cZeros()** command (screens 20 and 21).

[CATALOG] **cZeros(num** [,] **s** [)] [STO▸] **zero**

[CATALOG] **cZeros(denom** [,] **s** [)] [STO▸] **pole**



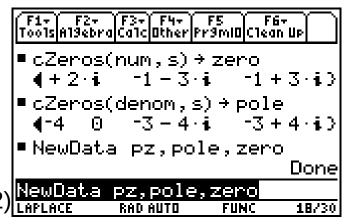
(20)



(21)

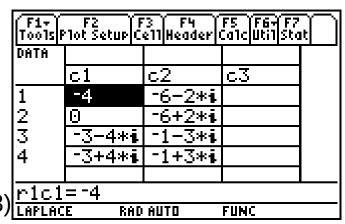
Plotting the poles and zeros takes a few steps.

a. Store the two lists in a data object called **pz** (screen 22).



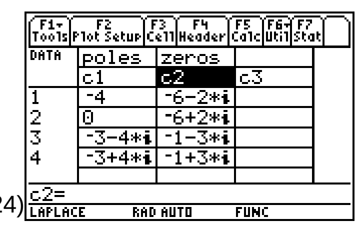
(22)

b. **pz** can't be displayed in the Home screen, but it can be edited by pressing [APPS] **6:Data/Matrix Editor 2:Open** (screen 23).



(23)

c. The first column lists the poles; the second column lists the zeros. To help remember this, add labels to each of the columns by pressing ⊙ ⊙ and typing **poles** [ENTER] followed by ⊙ ⊙ and typing **zeros** [ENTER] (screen 24).

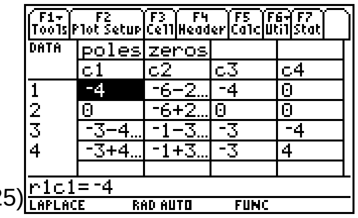


(24)

The real part of each pole (or zero) provides the x-component and the imaginary part, the y-component in the complex plane.

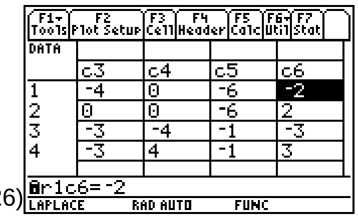
d. To separate the poles into their real and imaginary parts, first press ▶ and type **real(c1)** ENTER. This makes column **c3** the real part of column **c1**.

e. Then press ▶ ▲ **imag(c1)** ENTER to make column **c4** the imaginary part of **c1** (screen 25).

(25)


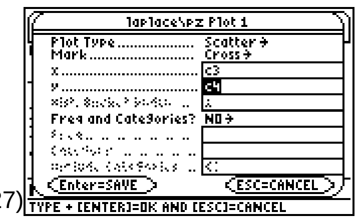
***Note**: Press ◆ | and select a cell width of 5 to see four columns.*

f. Repeat this process for the zeros making column **c5** the real part of **c2** (▶ ▲ **real(c2)** ENTER) and column **c6** the imaginary part of **c2** (▶ ▲ **imag(c2)** ENTER). Note that the screen scrolls to reveal **c5** and **c6** (screen 26).

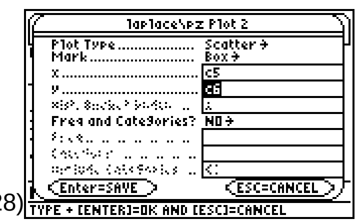
(26)


g. To plot the data, press F2 F1 and fill in the required data as shown in screen 27. Press ENTER.

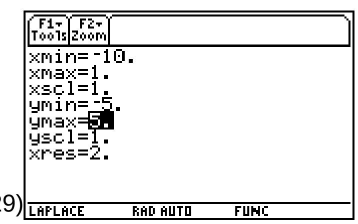
This will plot the real part of the poles (**c3**) versus the imaginary part of the poles (**c4**) as a cross.

(27)


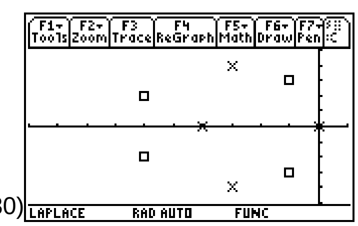
h. Press ▼ F1 to set **Plot 2** to plot the zeros with boxes (screen 28). Press ENTER.

(28)


i. Press ◆ [WINDOW] to set the plot ranges (screen 29). Turn **OFF Grid** and **Labels** with ◆ 1. Turn off the previous graph with ◆ |.

(29)


j. Finally, press ◆ [GRAPH] to see the poles and zeros graphed in the complex plane (screen 30). This representation is usually called the pole/zero constellation.

(30)


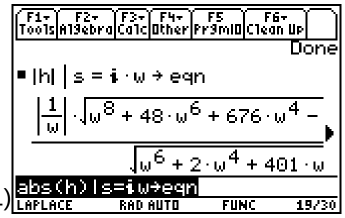
Topic 30: Frequency Response

The frequency response that corresponds to the pole/zero constellation in Topic 29 is graphed by noting that

$$|H(j\omega)| = |H(s)|_{s=j\omega}$$

1. To do this, press [HOME] and enter the equation as shown in screen 31.

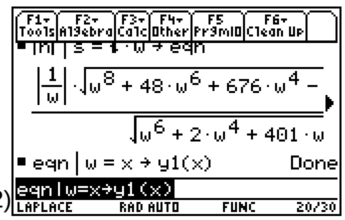
   [CATALOG] **abs(h** [)] [|] **s** [=] [2nd] [*i*] **w** [STO▸] **eqn**



(31)

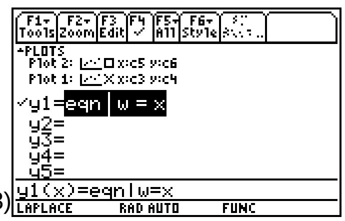
2. Enter **eqn** as the function **y1**(**x**) (screen 32).

   eqn [|] **w** [=] **x** [STO▸] **y1** [(] **x** [)]



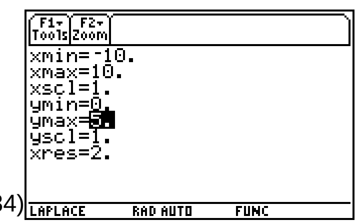
(32)

3. Press [◆] [Y=] to verify this. Be sure to deselect plots 1 and 2 in the Y= Editor using [F4] (screen 33).



(33)

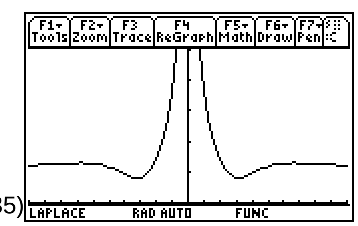
4. Press [◆] [WINDOW] to set the correct graphing parameters in the Window Editor (screen 34).



(34)

5. Press [◆] [GRAPH] to display the graph of frequency response (screen 35).

   Notice that the effects of the pole farthest from the axis can be seen as slight rises near the left and right sides of the graph. The zeros are causing the dips around **x**=±3, and the pole at the origin is causing the large peak in the middle.

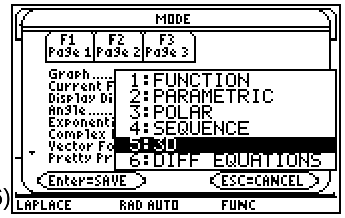


(35)

Topic 31: 3D Poles and Zeros

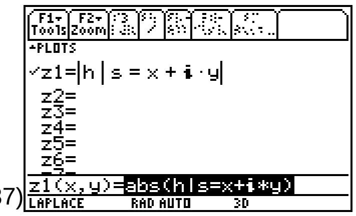
A different perspective of H(s) is gained from a 3D graph where the z-axis represents the magnitude of H(s).

1. To do so, press [MODE] ▶ and select **5:3D** (screen 36). Press [ENTER].

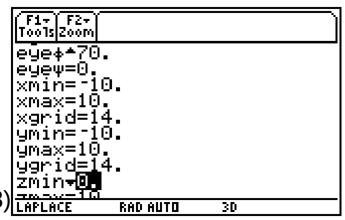
(36)


2. Press ◆ [Y=] and enter the function to be graphed (screen 37).

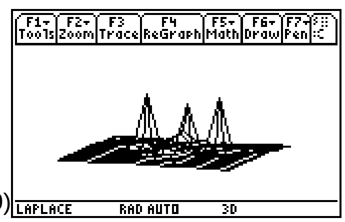
[CATALOG] **abs(** **h** [|] **s** [=] **x** [+] [2nd] [*i*] [×] **y** [)]

(37)


3. Press ◆ [WINDOW] and set the **x**, **y**, and **z** scales (screen 38). Note that these are the default values, except **zmin** has been set to 0.

(38)


4. Finally, press ◆ [GRAPH] (screen 39). It will take a few minutes for the graph to display. Once the graph is complete, press ◆ [|] and select **AXES** and turn **ON** the **Labels**.

(39)


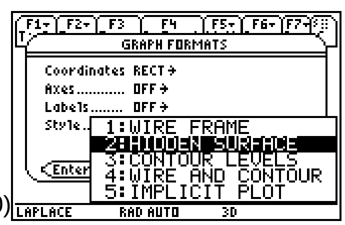
5. The three poles are clearly visible. Things to try:

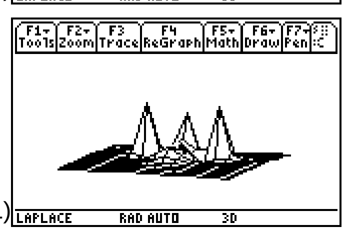
Press [X], [Y], or [Z] to look down the corresponding axis.

Use the cursor controls (◀ ▶ ▲ ▼) to spin the graph.

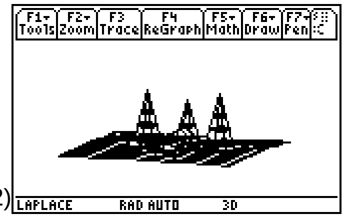
Press [0] to return to the original view.

6. Press ◆ [|] and change the **Style** to **HIDDEN SURFACE** (screens 40 and 41).

(40)


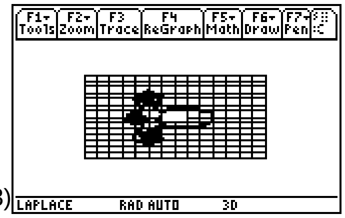
(41)


7. Press [◆] [|] and change the **Style** to **WIRE AND CONTOUR** to see contours highlighted on the graph (screen 42). This will take a few minutes to recalculate.



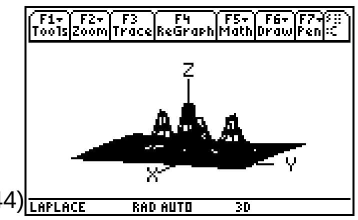
(42)

8. Press [Z] while in **WIRE AND CONTOUR** mode to view the contours from above (screen 43). Press [0] to return to the original view.



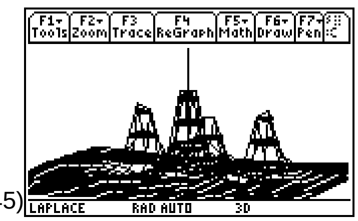
(43)

9. Press [◆] [WINDOW], set **xgrid** and **ygrid** to larger values (25 in this case), and press [◆] [GRAPH] to get a smoother graph (screen 44). This also takes a few minutes to recalculate.



(44)

10. To zoom in (screen 45), press the [×] key (the multiplication key, not the letter x).



(45)

Tips and Generalizations

The TI-89's symbolic math capability makes it a good choice for manipulating equations in the s-domain. The key step to plotting on the s-plane (real vs. imaginary) is to use the "with" operator ([|]) to replace **s** with **x** + [*i*] **y**. Although plotting |H(s)| is most common, the TI-89 can just as easily plot the angle of H(s) by entering

angle(h [(] **s** [)] [|] **s** [=] **x** [+] [*i*] **y** [)]**.**

Although these examples solved for a single node problem with only one equation, **v**(**s**), more complex circuits with more nodes (and therefore more equations) also can be solved.

The TI-89 assisted the conversion from the s-domain to the time-domain by doing the partial fraction expansion. Chapter 7 shows how to find a system's response by staying in the time-domain and using convolution.