Inverses of Functions

ID: 11404

Time Required
25 minutes

Activity Overview

In this activity, students will explore three ways to find the inverse of a function. First, students graph two scatter plots and find the line of reflection. Then, they will graph a line and use the x- and y-intercepts to create the graph of the inverse. Students will also find the inverse of the line algebraically.

Topic: Sequences, Series & Functions

- *Reflection*
- *Inverse of data points*
- *Inverse of functions*

Teacher Preparation and Notes

- *Load the TI-Nspire document Inverse.tns onto student handhelds.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11404" in the quick search box.***

Associated Materials

- *Inverse_Student.doc*
- *Inverse.tns*
- *Inverse_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

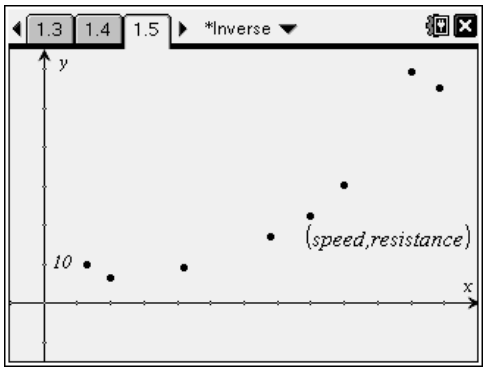
- *Inverse Variation (TI-Nspire technology) — 9840*
- *Powers and Roots as Inverses (TI-Nspire technology) — 9826*

Problem 1 – Inverse of data points

The wind tunnel data to be used in this part of the activity can be found on page 1.4.

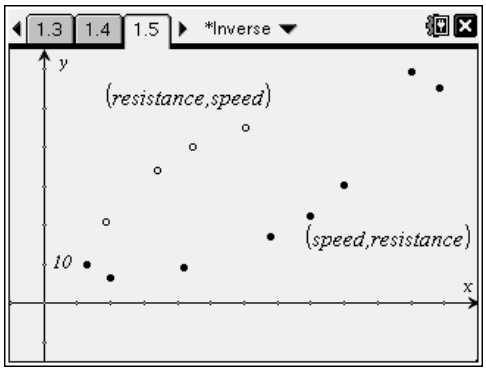
On page 1.5, students are to graph the data by pressing **MENU > Graph Type > Scatter Plot**. At the bottom of the graph screen, students will need to select **speed** for *x* and **resistance** for *y* for the original graph (*s1*).

The window settings are [–5, 65] for *x* and [–15, 65] for *y*.



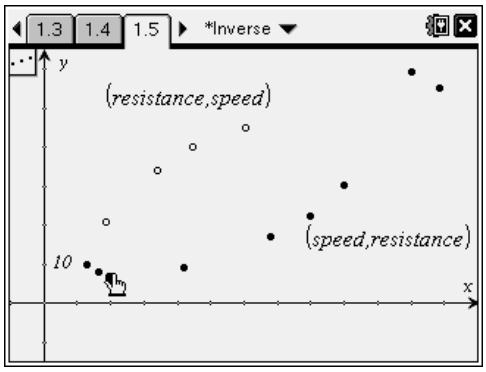
Instructions are given on page 1.6 for the second plot to be created with *s*1 on page 1.5. For this second scatter plot (s2), students will need to select **resistance** for *x* and **speed** for *y*.

Note: It is okay is students graph the scatter plots the opposite way described.



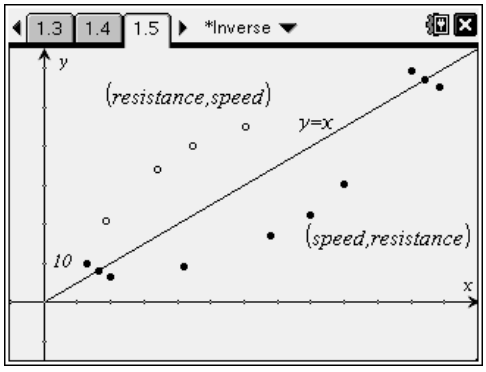
On page 1.7, students are asked to comment on their observations on the two scatter plots. They should notice that the points seem to be reflected across a line.

Students are instructed to find the midpoint between the first point on each of the scatter plots and the midpoint between the last point for each of the scatter plots on the graph. To do this, students need to press **MENU > Construction > Midpoint** and then select the two points described above.



Once both of these midpoints are found, students will connect these points with a line. They should press **MENU > Points & Lines > Line** and then select the two midpoints.

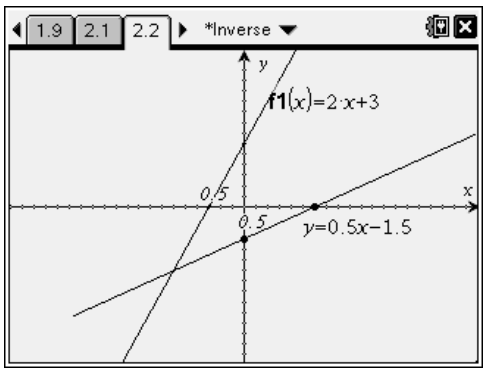
The line can be extended by grabbing the ends of the graph using the "grab hand" and moving the cursor farther out.



Students then can determine the slope of the line by pressing **MENU > Measurement > Slope** and selecting the line. They should be able to determine the *y*-intercept by looking at the graph.

Problem 2 – Inverse of a line

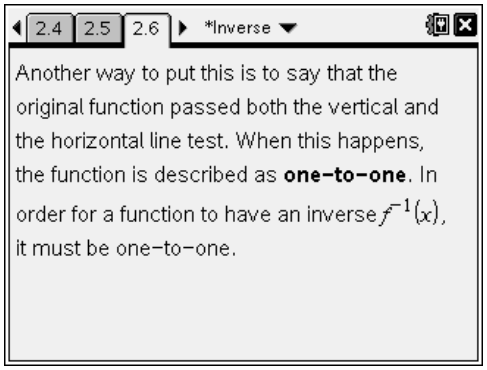
In this part of the activity, students will explore the inverse of a line instead of data points.



On page 2.2, students will graph the given line. Then, they will switch the ordered pairs for the *x*- and *y*- intercepts to obtain two new points. Students are to use those points to construct a new line and determine its equation.

Algebraically, students are to find the inverse by switching *x* and *y* in the given equation and to solve for *y*. They should see that the equation obtained graphically is the same obtained algebraically.

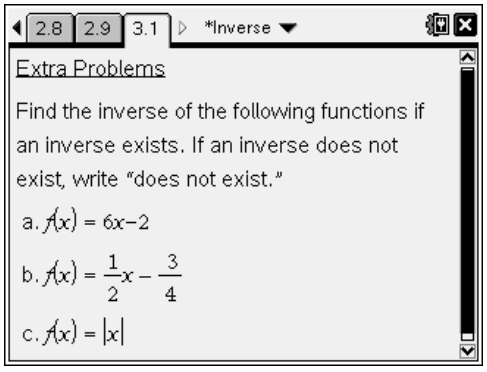
Pages 2.5 and 2.6 explain the necessity of a function being one-to-one in order to have an inverse.



Students are encouraged to explore the concept of inverses further on page 2.7. However, if you choose not to have them do this, delete this page before distributing the TI-Nspire document.

Questions are interspersed throughout this activity to aid the students in developing conceptual understanding. These questions may either be answered in the TI-Nspire document or the associated worksheet.

Extra problems to help students practice how to find an inverse are given on page 3.1. These problems can be done in class for students that finish early or they can be assigned as homework.



For each problem, students should find the inverse both graphically and algebraically to reinforce what was learned in the activity.

Solutions to the Student Worksheet

1. The points obtained by switching the domain and range appear to be a reflection of the original points across a line.
2. $y = x$; line of reflection
3. $y = \frac{x-3}{2}$ or $y = \frac{x}{2} - \frac{3}{2}$ or $y = 0.5x - 1.5$
4. It is the same.
5. Switch the domain and range, switch x and y in the equation, or reflect the graph of the function across the line $y = x$.
6. Agree
   Sample answer:
   In order for a function to have an inverse, it must be one-to-one. In other words, it should pass both the vertical and horizontal line tests, resulting in a reflection across $y = x$ that will also pass the vertical line test. It is therefore a function.
7. *a.* $f^{-1}(x) = \frac{x+2}{6}$

   b. $f^{-1}(x) = 2x + \frac{3}{2}$

   c. $f^{-1}(x)$ does not exist