Angles in Polygons

ID: 9055

Time required
45 minutes

Activity Overview

In this activity, students measure interior angles in convex polygons and use the ***Calculate*** *tool to find the sum of the angle measures. They keep a record of the sums and make and test a conjecture about the sum of the angle measures in an n-sided polygon. Students also find the measure of one interior angle of a regular polygon. Finally, they measure exterior angles in convex polygons, find their sum, and write a two-column proof for the sum of the exterior angles in a convex polygon.*

Topic: Quadrilaterals & General Polygons

- *Construct a polygon of n sides and conjecture a theorem about the total measures of its interior and exterior angles.*
- *Prove that the sum of the measures of the exterior angles of a polygon of n sides is 360°.*
- *Deduce that the sum of the interior angles of a polygon of n sides is (n – 2) × 360°.*

Teacher Preparation and Notes

- *This activity is designed for a high school geometry classroom. It assumes previous knowledge of the definition of a polygon as well as polygon classifications by number of sides (e.g., pentagon, hexagon, n-gon).*
- *Students should know the difference between a concave and convex polygon. Introduce or review these terms as needed.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9055" in the keyword search box.***

Associated Materials

- *AnglesInPolygons_Student.doc*
- *AnglesInPolygons.tns*

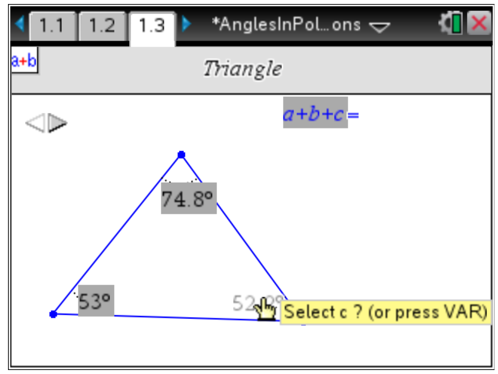
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Exterior and Interior Angle Theorem (TI-84 Plus family) — 4613*
- *What's the Angle? (TI-89 Titanium) — 1285*
- *Interior Angles of Polygons (TI-Nspire technology) — 9441*

For this activity, students will work with convex polygons. In a convex polygon, a line that contains a side does not pass through the interior of the polygon. In other words, any side, when extended, does not go through the polygon. This is not true for concave polygons.

Problem 1 – Interior Angles

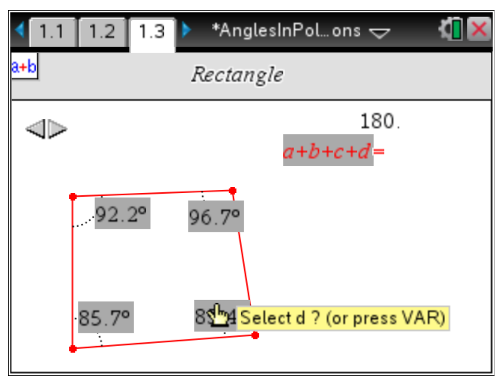


Students are shown a triangle on page 1.3. They need to complete the equation shown by selecting **MENU > Actions > Calculate**, clicking on the expression $a + b + c$, and then clicking on each of the three angle measures. After the third measure is chosen, the sum 180 will appear. Students can place it to the right of the equals sign.

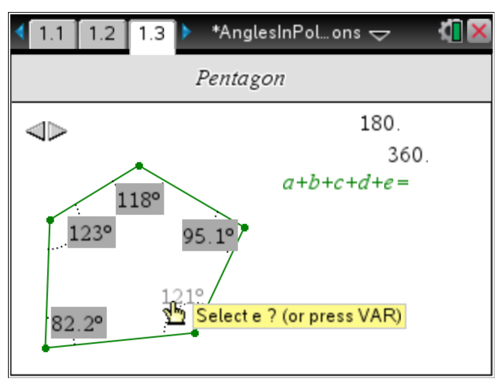
Students can drag a vertex of the triangle to change the shape of the triangle and see that the sum remains 180°. Students should record this sum in the table on their worksheet.

TI-Nspire™ Navigator™ Opportunity: *Live Presenter* and *Class Capture*

See Note 1 at the end of this lesson.

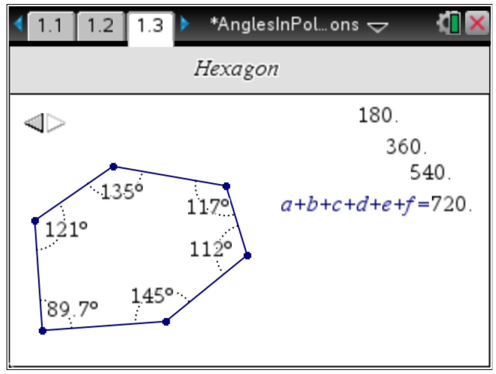


After students click the slider, a quadrilateral appears. They again use the **Calculate** tool to find the sum of the angles in the quadrilateral. As with the triangle, they can alter the shape of the quadrilateral and observe that the sum remains 360°. They should record this sum on their worksheet.



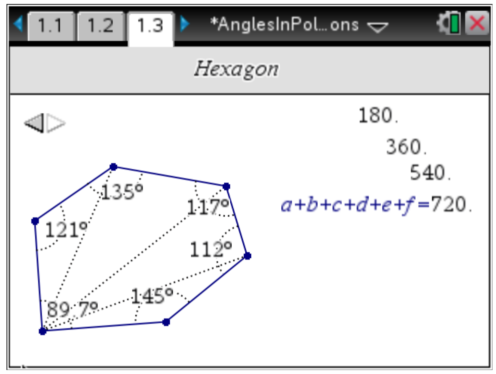
After students click the slider, a pentagon appears. They again use the **Calculate** tool to find the sum of the angles in the pentagon. As with the others, they can alter the shape of the pentagon and observe that the sum remains 540°. They should record this sum on their worksheet.

After students click the slider, a hexagon appears. They again use the **Calculate** tool to find the sum of the angles in the hexagon. As with the others, they can alter the shape of the pentagon and observe that the sum remains 720°. They should record this sum on their worksheet.



They should now have the sums of the interior angles measures for triangles, quadrilaterals, pentagons, and hexagons recorded on their worksheets.

Ask students to describe a pattern in the sums of the measures. (Increasing by one side increases the sum by 180°.) Then have them click the slider so that every student is not displaying the same polygon. Have the students draw all diagonals from any one vertex using the **Segment** tool from the **Points & Lines** menu. Students can now share their results and complete the third column of the table on their worksheets.



TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

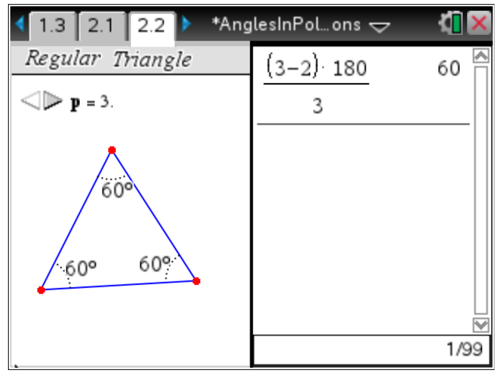
Prompt students to think about how the number of triangles created by the diagonals is related to the sum of the angle measures. (The sum is equal to the product of 180° and the number of triangles.) Now ask how the number of triangles is related to the number of sides in the polygon. The number of triangles is two less than the number of sides. Show that this can be written as $I = (n - 2) \cdot 180°$, where n is the number of sides of the polygon and I is the sum of its interior angle measures.

Problem 2 – Angle Measures in Regular Polygons

Remind students that a regular polygon is a polygon in which each interior angle has the same measure. Ask them to find a formula for the measure of one angle in a regular polygon. $\frac{(n-2)\cdot 180°}{n}$

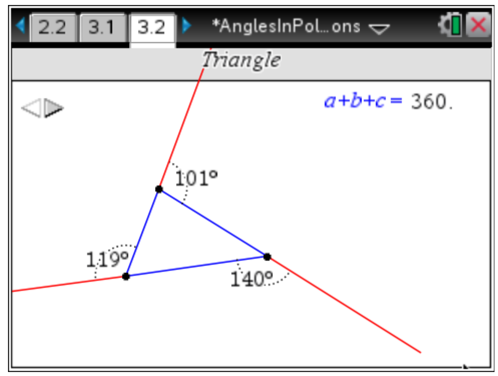
Students should use the formula to find the measure of one interior angle for the regular polygon they drew, and then compare their calculation to the displayed angle measures.

Changing the size of their polygon by dragging a vertex, students will observe that the angle measures remain the same.



Problem 3 – Exterior Angles

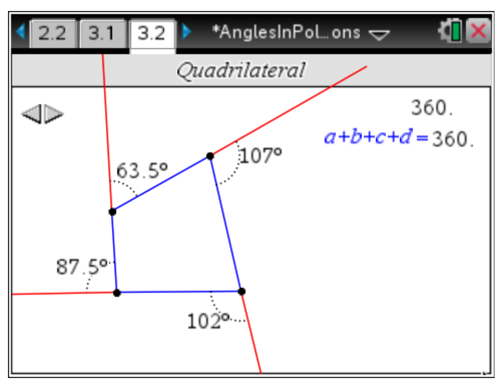
On page 3.2, the sides of a triangle have been extended to form exterior angles. Using the **Calculate** tool, students will find the sum of the measures of the exterior angles of a triangle is 360°.



TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

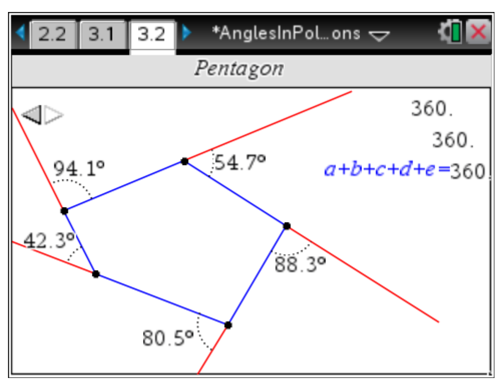
See Note 3 at the end of this lesson.

Then students click the slider and find the sum of the exterior angles of a quadrilateral is also 360°.

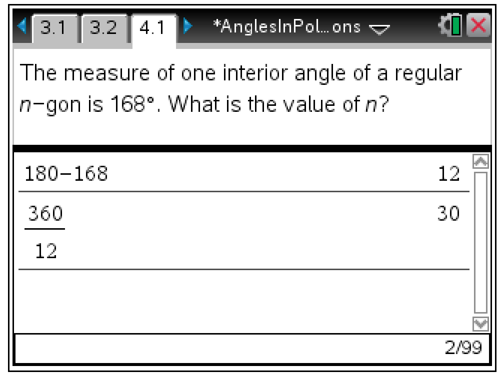


Then students click the slider and find the sum of the exterior angles of a pentagon is also 360°.

Discuss with students that the sum of the exterior angles of any convex polygon is 360° and that this can be written as E = 360°, where E is the sum of the exterior angles.



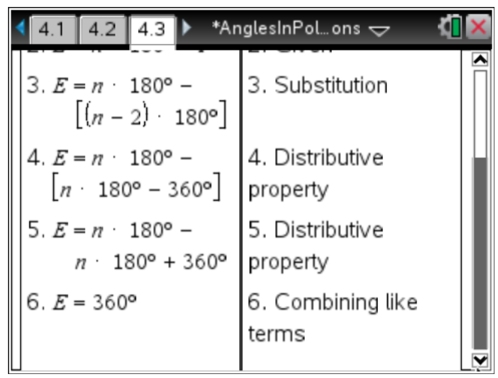
Extension – Relationship Between Interior and Exterior Angles



On page 4.1, students are asked to find the value of n in a regular n-gon if one interior angle measures 168°. If needed, give them this hint: *Each interior and corresponding exterior angle form a linear pair*.

The polygon is a 30-gon because the sum of the exterior angles divided by the measure of each exterior angle is 30.

Review that the sum of the interior angles of a convex polygon: $I = (n - 2) \cdot 180°$. Show that since each interior and exterior pair of angles sums to 180°, that the sum of the interior angles (I) plus the sum of the exterior angles (E) is equal to $n \cdot 180°$. This can be written as $I + E = n \cdot 180°$. Subtracting I from each side gives $E = n \cdot 180° - I$.



On page 4.3, students are to complete the proof that $E = 360°$. To simplify matters, the information discussed above is listed as *Given*.

The degree symbol may be accessed by pressing [?!▸], and variables may be italicized by selecting **MENU > Format > Format text**. Alternatively, the equations may be inserted into math expression boxes (**MENU > Insert > Math Box**), preserving mathematical formatting.

TI-Nspire Navigator Opportunities

Note 1
Question 1, *Live Presenter* and *Class Capture*
Use Live Presenter to demonstrate how to calculate the angle sum as well as how dragging a vertex of the polygon does not change the angle sum. Make sure students keep the polygons convex when dragging a vertex. Otherwise, the sum of the angles will not follow the proper pattern. Use Class Capture to monitor student progress as they work through Problem 1 and calculate the angle sums for the other polygons.

Note 2
Question 1, *Class Capture*
Use Class Capture to display all the different polygons showing their diagonals from one vertex to help students see the pattern of 180 * number of triangles.

Note 3
Question 3, *Quick Poll*
Before beginning Problem 3, use Quick Poll to ask students if they believe the exterior angle sum will depend upon the number of sides a polygon has- as the interior angle sum does.