# Under Pressure:
Inverse Relationship

Let's take a sample of air in a closed container, and keep it at room temperature. If you change the volume of the container, what will happen to the air pressure inside? You can feel this by squeezing a small balloon in your hand. As the balloon gets smaller, you have to push harder. That is, as the volume decreases, the pressure increases. Two quantities that change in this way could be *inversely* related. If pressure and volume are inversely related, even if both quantities change, then their product stays the same.

Suppose that x and y represent the quantities that are inversely related. Then

$$xy = k, \quad \text{or} \quad x = \frac{k}{y}$$

where k is a constant in both equations. Maybe you can think of some other quantities that also behave this way. For air and other gases, this relation has a name: *Boyle's law*.

In this activity, you will use a pressure sensor to investigate the relationship between pressure and volume for air contained within a closed syringe.
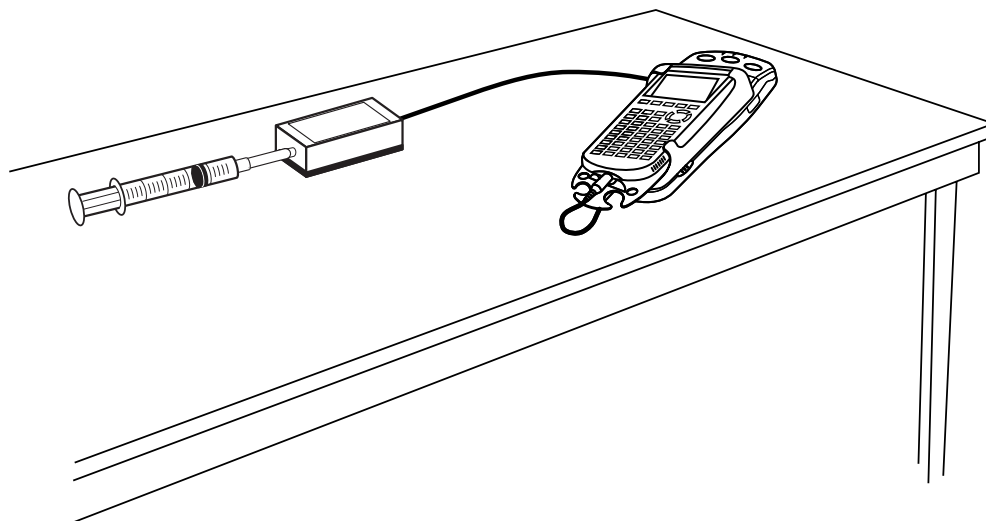
OBJECTIVES

- Record pressure versus volume data for a sample of air.
- Fit an inverse function model to the data.
- Use the table calculation feature of the TI graphing calculator.
- Re-plot the data using linearization.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator   EasyData application
Vernier Pressure Sensor with included syringe   data-collection interface

PROCEDURE

1. Prepare the Pressure Sensor and an air sample for data collection.

   a. Turn on the calculator. Connect the Pressure Sensor, data-collection interface, and calculator.
   b. With the syringe disconnected from the Pressure Sensor, move the piston of the syringe until the leading edge of the inside black ring is positioned at the 10.0 mL mark.
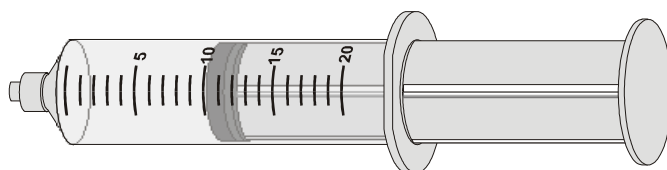
   c. Attach the syringe the Pressure Sensor. The air in the syringe/Pressure Sensor system is now sealed, so that as you move the piston, the air volume and pressure both change.

2. To obtain the best data possible, you will need to correct the volume readings from the syringe. Look at the syringe; its scale reports its own internal volume. However, that volume is not the total volume of trapped air in your system since there is a little bit of space in the tubing and internal to the pressure sensor.

   To account for the extra volume in the system, you will need to add a small and constant volume to your syringe readings. The correction is 0.8 mL. For example, with the 0.8 mL correction and a 5.0 mL syringe volume, the total volume would be 5.8 mL. *It is that total volume that you will need for the analysis.*

3. Set up EasyData for data collection.

   a. Start the EasyData application, if it is not already running.
   b. Select ⌈**File**⌉ from the Main screen, and then select **New** to reset the application.
   c. Select ⌈**Setup**⌉ from the Main screen, and then select **Events with Entry**.

4. You are now ready to collect pressure and volume data. It is best for one person to handle the gas syringe and for another to operate the calculator.

   a. Select ⌈**Start**⌉ to begin data collection.
   b. Move the piston so the front edge of the inside black ring is positioned at the 5 mL line on the syringe. Hold the piston firmly in this position until the pressure value displayed on the calculator screen stabilizes.
   c. Select ⌈**Keep**⌉. (The person holding the syringe can relax after ⌈**Keep**⌉ is selected.) Enter **5.8**, the total gas volume (in mL) on the calculator. Remember, you are adding the constant volume to the apparent volume of the syringe. Select ⌈**OK**⌉ to store this pressure-volume data pair.

d. To collect another data pair, move the piston to 7.0 mL. When the pressure reading stabilizes, select ⌈**Keep**⌉, enter the total volume, remembering to add the correction to the syringe volume, and then select ⌈**OK**⌉.

e. Continue with this procedure using volumes of 10.0, 12.0, 15.0, 17.0, and 20.0 mL.

f. Select ⌈**Stop**⌉ when you have finished collecting data. A graph of pressure versus volume will be shown.

5. In order to work directly with the data, you need to leave the EasyData application. Select ⌈**Main**⌉ to return to the Main screen. Exit EasyData by selecting ⌈**Quit**⌉ from the Main screen and then selecting ⌈**OK**⌉.

ANALYSIS

1. Redisplay the graph outside of EasyData.

   a. Press ⟨ **2nd** ⟩ [STAT PLOT].
   b. Press ⟨**ENTER**⟩ to select Plot1 and press ⟨**ENTER**⟩ again to select On.
   a. Press ⟨**ZOOM**⟩.
   b. Press ⟨▾⟩ until ZoomStat is highlighted; press ⟨**ENTER**⟩ to display a graph with the *x* and *y* ranges set to fill the screen with data.
   c. Press ⟨**TRACE**⟩ to determine the coordinates of a point on the graph using the cursor keys.

2. Examine the data pairs on the displayed graph. As you move the cursor right or left, the volume (*x*) and pressure (*y*) values of each data point are displayed below the graph. Record the pressure (round to the nearest 0.1 kPa) and volume data values in the Data Table on the *Data Collection and Analysis* sheet.

3. To test Boyle's law you can plot the inverse function $y = k/x$ with your data. You will see this equation with the data, and can then adjust the value for *k* to improve the fit. First, enter the inverse function, and then in the next step, explore values of *k*.

   a. Press ⟨ **Y=** ⟩.
   b. Press ⟨**CLEAR**⟩ to remove any existing equation.
   c. Enter k/x in the Y₁ field.
   d. Press ⟨◁⟩ until the icon to the left of Y₁ is blinking. Press ⟨**ENTER**⟩ until a bold diagonal line is shown in order to display your model with a thick line.
   e. Press ⟨ **2nd** ⟩ [QUIT] to return to the home screen.

4. Set a value for the parameter *k*, and then look at the resulting graph. To obtain a good fit, you will need to try several values. Use the steps below to store different values to the parameter *k*. Start with $k = 500$. Experiment until you find one that provides a good fit for the data.

   a. Enter a value for the parameter *k*. Press ⟨**STO▸**⟩ K ⟨**ENTER**⟩ to store the value in the variable K.
   b. Press ⟨**GRAPH**⟩ to see your data with the model graph superimposed. The model line is bold.
   c. Press ⟨ **2nd** ⟩ [QUIT] to return to the home screen.

⟹ Answer Question 1 on the *Data Collection and Analysis* sheet.

5. Find the products of the data coordinates and record them in the third column of the Data Table.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

6. The modeling equation you have determined in this activity can be used to predict syringe pressure values for given volumes. The second Data Table on the *Data Collection and Analysis* sheet lists a number of volumes. Fill in the corresponding pressures. Although it would not take long to do this by hand, try using the table feature of your calculator to do the calculation.

   a. Press ⬯2nd⬯ [TBLSET] to display Table Setup.
   b. Use the cursor keys to highlight Ask on the Indpnt: line, and press ⬯ENTER⬯ to select this mode.
   c. Use the cursor keys to highlight Auto on the Depend: line, and press ⬯ENTER⬯ to select this mode.
   d. Press ⬯2nd⬯ [TABLE] to display the table.
   e. Enter **2.5** for the first *x* value, and finish the entry with ⬯ENTER⬯. Note how the calculator has determined the *y* value from the model. Complete the Data Table for the remaining three entries.

⇒ Answer Questions 3 and 4 on the *Data Collection and Analysis* sheet.

7. Another way to see if two quantities are inversely related is to plot the one quantity versus the inverse of the other. For this pressure activity, that would be pressure versus the inverse of the volume. If the new plot shows a line of proportionality, then the original data is inversely related. You can see this by noting the simple rearrangement of the equation $y = k/x$ as $y = k \times (1/x)$. Written this way, you can see that y is proportional to the quantity $(1/x)$. The process of graphing calculated quantities to obtain a straight-line graph is sometimes called *linearization*.

   To see if the data is inversely related using this test, you need to define a new column containing the inverse of the volume data. The volume data is in L1; L3 is available for the inverse volume information. L2 will continue to hold the pressure data.

   a. Press ⬯2nd⬯ [QUIT] to return to the home screen.
   b. Press 1/ ⬯2nd⬯ [L1] ⬯STO▸⬯ ⬯2nd⬯ [L3] ⬯ENTER⬯. L3 will now hold the inverse of the values in L1.

8. You can plot the pressure versus inverse volume data on a new graph using the following commands.

   a. Press ⬯2nd⬯ [STAT PLOT] and press ⬯ENTER⬯ to select Plot 1.
   b. Change the Plot1 settings to match the screen shown here. Press ⬯ENTER⬯ to select any of the settings you change.
   c. Press ⬯ZOOM⬯ and then select ZoomStat (use the cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.
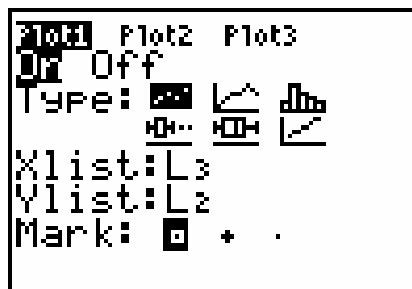
⇒ Answer Question 5 on the *Data Collection and Analysis* sheet.

EXTENSIONS

1. An alternate way to find a model for the data you collected involves a process known as *regression analysis*. The calculator will select the best parameters for the power function $y = ax^b$.

   Note: Before performing this extension, the Stat Plot 1 settings must first be restored to graph L1 and L2 using the plotting steps of Step 8, with L1 replacing L3.

   a. Press (STAT) and use the cursor keys to highlight CALC.
   b. Press the number adjacent to PwrReg to copy the command to the home screen.
   c. Press (2nd) [L1] (,) (2nd) [L2] (,) to enter the lists containing your data.
   d. Press (VARS) and use the cursor keys to highlight Y-VARS.
   e. Select Function by pressing (ENTER).
   f. Press (ENTER) to copy Y1 to the home screen.
   g. On the home screen, you will now see the entry PwrReg L1, L2, Y1. This command will perform a power regression with L1 as the *x* and L2 as the *y* values. The resulting regression line will be stored in equation variable Y1. Press (ENTER) to perform the regression.
   h. Press (ZOOM) and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

   Does the power law model provide a good fit to your pressure versus volume data? How many parameters are adjusted to achieve a good fit compared to the inverse model?

2. It is possible to deduce the internal volume of the sensor rather than using the supplied values. If you assume Boyle's law, you can determine the internal volume from the *y*-intercept value. For example, if the internal volume of the sensor is *d*, then the total volume of air in the system is $V + d$. From Boyle's law, we have $P*(V + d) = k$, which can be rearranged to $V = k*(1/P) - d$. From the form of this equation, we can see that the *y*-intercept of a *V* (*y*-axis) versus $1/P$ (*x*-axis) graph is the negative of the internal volume.

   Note that this is not the same graph setup as used in the activity. Collect a new data set, but do not correct the volumes; just enter the syringe volume directly. Create a graph of *V* versus $1/P$. Fit a straight line to the data, and from the *y*-intercept determine the internal volume of your particular sensor. For comparison to your results, the internal volume of the Vernier Gas Pressure Sensor is about 0.8 mL.

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLES

Volume in mL (x values)	Pressure in kPa (y values)	Product (x∗y)

Volume in mL (x values)	Pressure in kPa (y values)
2.5	
17.8	
520	
0.0012	

QUESTIONS

1. In the space below, use the value for *k* to record the model equation that gives the best fit to the data.

2. Notice that the values in the third column of the first Data Table are closely related to the value of *k* you found in Step 4. Explain why this is so.

3. Could the volume ever be zero? Why or why not? What would be the corresponding pressure?

4. Complete the following statement:
   As the volume of a gas sample decreases, its pressure _____.

5. Based on your graph of pressure versus inverse volume, are pressure and volume inversely proportional?