

In It To Win It!

ID: 11649

Time required
40 minutes

Activity Overview

In this activity, students will calculate the experimental probability of winning a basketball game by shooting two free-throws using a random integer simulation. They will also calculate experimental probabilities for two people having a shared birthday and a batter in a baseball getting walked to first base.

Topic: Probability

- *Experimental probability*
- *Simulations*

Teacher Preparation and Notes

- *The activity is intended to be a small group (3–4 students) activity. The student worksheet has directions for the handheld use in addition to inquiry questions. It may also be used a teacher led class activity. If it is teacher led, the teacher should be familiar with the questions on the student worksheet.*
- *Student directions are included in the TI-Nspire document, but the worksheet includes more detailed directions.*
- *The solution TI-Nspire document includes sample simulations for each problem with the correct formulas for each column in the spreadsheets.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “11649” in the quick search box.***

Associated Materials

- *InItToWinIt_Student.doc*
- *InItToWinIt.tns*
- *InItToWinIt_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *The Seven Dwarfs Giveaway (TI-Nspire technology) — 9859*
- *Birthday Buddies (TI-Nspire technology) — 8319*

Introduction

Have students consider the following questions before beginning the activity.

- A 70% free throw shooter is at the foul line with no time left on the clock. What is the probability that she will make both free throws to win the game?
- An airline knows that 85% of ticket holders will show up for a given flight. How many tickets should they sell for a flight to maximize the profit for that flight?
- There are 25 people in a room, all unrelated. What is the probability that two of them will share the same birthday?

Explain to students that they can investigate the answers to these and similar questions two ways, an experiment or a simulation.

An experiment could be conducted. For example, find a 70% free throw shooter and have her shoot pairs of baskets over and over again. After, let's say 50 times, have her stop and then calculate the experimental probability.

While this would be fun to watch, it is not practical. So, we will try a simulation. A **simulation** is a procedure for imitating a number of repetitions of an experiment where the constraints of the problem are reflected in the design of the simulation.

Problem 1 – Free throws

In this problem of the activity, students will simulate a 70% free throw shooter that needs to make both free throws to win the game.

Using the spreadsheet on page 1.3, students will find an experimental probability that the shooter will make both free throws to win the game.

Students will be using two different commands, **randInt** and **ifFn**. The **randInt** command returns an integer. It will be used to simulate the first and second shots.

randInt(lowBound, upBound, #Trials)

Columns A and C: **=randInt(0,9,100)**

This will generate 100 numbers, each representing a free throw. Each row of the spreadsheet will represent a trial or an attempt to win the game.

Discuss with students what other numbers could represent a miss and made shot. They should have three numbers for a miss and seven numbers for a made shot.

	A	B	C	D
	shot1	success1	shot2	succe
	=randint(0,9			
1	3			
2	4			
3	5			
4	2			
5	3			

The **ifFn** command evaluates the Boolean expression and produces one result if it is true and a different result if it is false. It will be used to evaluate if the shot is made and then evaluate if a trial is a win for the team.

ifFn(BooleanExpr , Value If true , Value If false)

Column B: **=ifFn(a[]>2,1,0)**

Column D: **=ifFn(c[]>2,1,0)**

	A shot1	B success1	C shot2	D success1
	=randint(0,9	=ifFn(a[]>2,		
1	3	1		
2	4	1		
3	5	1		
4	2	0		
5	3	1		

Formula bar: B success1: =ifFn(a[1:]>2,1,0)

Have students verify that the handheld output matches whether the free throw should be a miss or a made shot.

Students should then look at row 1 and determine how many baskets were made. Ask them if the team would win if row 1 happened? What number in column E would constitute a win? (2)

Now, students need to assess this for the 100 trials. They will calculate the number of baskets made during each attempt. To do so, they add the numbers in Columns B and D.

Column E: **= b[] + d[]**

Students will use another Boolean expression to determine if the trial is a win for the basketball team.

Column F: **=ifFn(e[]=2,1,0)**

	D success2	E total	F wins
	=randint(0,9	=ifFn(c[]>2,1	=ifFn(e[]=2,1
1	3	1	2
2	9	1	2
3	2	0	1
4	1	0	0
5	1	0	1

Formula bar: F wins: =ifFn(e[1:] = 2, 1, 0)

They should be able to explain that 1 represents a win and a 0 represents a loss.

The final step of the simulation is to determine the total number of wins for the 100 trials. Since a win is equal to 1, summing the column will give the total number of wins.

sum(starting cell: ending cell)

Cell G1: **=Sum(f1:f100)**

They should then divide that number by 100 to get the experimental probability of winning the game.

	D success2	E total	F wins	G
	=ifFn(c[]>2,1	=b[]+d[]	=ifFn(e[]=2,1	
1	1	2	1	49
2	1	2	1	
3	0	1	0	
4	0	0	0	
5	0	1	0	

Formula bar: G1: =sum(f1:f100)

After completing the simulation individually, encourage students to compare their results first with their group members and then with the entire class.

Note: After the simulation is completed once, it can be run a second time quickly by choosing **MENU > Actions > Recalculate** (⌘ + R). This command will generate a new set of random integers and fill the columns accordingly.

The theoretical probability is 49%, which is calculated by multiplying the probability of making each free throw (0.7*0.7).

To investigate this problem further, have students repeat the experiment with a lower free throw percentage and then again with a higher free throw percentage.

Another investigation could include a free throw percentage where the first shot is 70% and the second shot is 50%.

Problem 2 – Birthday Problem

This is the famous problem – If there are 25 unrelated people in a room, what is the probability that two of them will share a birthday?

A simulation using a random number generator will be used to investigate the solution. A good discussion can result from students discussing how random numbers can be used to represent this problem. This could be done in a group or as a class.

After the discussion, students are to assign an integer to each birthday, assuming that there are 365 days in a year. Since there are 25 people, students are to enter the command **randInt(1,365,25)** in Column A.

Each integer represents a different day of the year. Have students scroll down the column and compare the numbers to see if any are the same. Then, students can sort the list to make this inspection easier.

To sort a list, they highlight the list. (Scroll to the name of the list and then press the up arrow one more time. The list will darken.) Then choose **MENU > Actions > Sort**. When the dialogue box appears, they are to choose Column A and ascending order. This will overwrite the list.

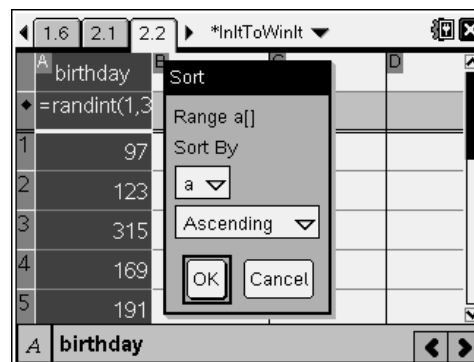
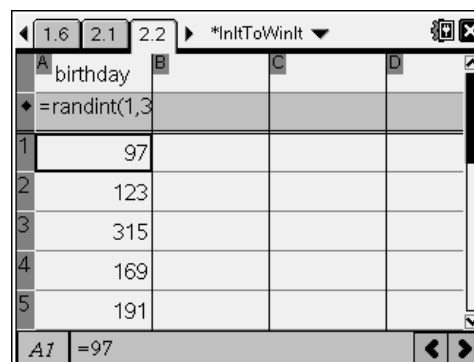
Ask students if they have two people with the same birthday. They are to repeat this procedure 9 more times and record each trial as yes or no, where 'yes' is for one or more shared birthdays and 'no' is for no shared birthdays. To calculate the experimental probability, students should divide the total number of yes's by 10. They can then compare their probability with those in their group and the class.

The theoretical probability is 57%.

$$1 - P(\text{no shared}) = P(\text{1 shared})$$

$$1 - \frac{365!}{(365 - n)! \cdot 365^n} = 1 - \frac{365!}{(365 - 25)! \cdot 365^{25}} = 0.5687$$

An extension to this problem could include changing the number of people in the room or including February 29th making the number of possible birthdays 366.



Extension – Casey at Bat

There is a famous poem titled Casey at the Bat by Ernest Lawrence Thayer. It can be found at the end of this document. You can choose to have students read it and then complete the problem, however it is not vital for the understanding of the problem.

Students to find the experimental probability that Casey walks to first base (because the pitcher threw 4 balls before 3 strikes). They are told that the pitcher throws strikes 40% of the time and Casey will not swing at bat.

Students may need more information about baseball, namely the specifics on a walk versus a strike out.

They need to create a simulation that represents the scenario. They will execute the simulation 25 times and record for each time yes or no, ‘yes’ meaning that Casey walked and ‘no’ meaning that Casey struck out. The experimental probability is the number of yes’s divided by 25.

Remind students to press **(ctrl) + (R)** to recalculate the simulation.

Further investigation could include determining what percentage of strikes the picture throws in order for Casey to have a 75% chance of walking to first base, etc.

Sample simulation:
 Column A: **randInt(0,9,6)**, where 0–3 represent a strike and 4–9 represent a ball. Maximum of six pitches.
 Column B: **ifFn(a[]<4,1,0)**. If four 0s occur before three 1s then Casey walks. If three 1s occur before four 0s, then Casey strikes out.

	A	B	C	D
pitch	eval	yes	no	
	randInt(0,9=ifFn(a[]<4,1			
1	3	1	14	11
2	8	0		
3	3	1		
4	7	0		
5	1	1		

Casey at the Bat
By Ernest Lawrence Thayer
Taken From the San Francisco Examiner - June 3, 1888

The outlook wasn't brilliant for the Mudville nine that day;
The score stood four to two, with but one inning more to play,
And then when Cooney died at first, and Barrows did the same,
A pall-like silence fell upon the patrons of the game.

A straggling few got up to go in deep despair. The rest
Clung to that hope which springs eternal in the human breast;
They thought, "If only Casey could but get a whack at that —
We'd put up even money now, with Casey at the bat."

But Flynn preceded Casey, as did also Jimmy Blake,
And the former was a hoodoo, while the latter was a cake;
So upon that stricken multitude grim melancholy sat;
For there seemed but little chance of Casey getting to the bat.

But Flynn let drive a single, to the wonderment of all,
And Blake, the much despised, tore the cover off the ball;
And when the dust had lifted, and men saw what had occurred,
There was Jimmy safe at second and Flynn a-hugging third.

Then from five thousand throats and more there rose a lusty yell;
It rumbled through the valley, it rattled in the dell;
It pounded on the mountain and recoiled upon the flat,
For Casey, mighty Casey, was advancing to the bat.

There was ease in Casey's manner as he stepped into his place;
There was pride in Casey's bearing and a smile lit Casey's face.
And when, responding to the cheers, he lightly doffed his hat,
No stranger in the crowd could doubt 'twas Casey at the bat.

Ten thousand eyes were on him as he rubbed his hands with dirt.
Five thousand tongues applauded when he wiped them on his shirt.
Then while the writhing pitcher ground the ball into his hip,
Defiance flashed in Casey's eye, a sneer curled Casey's lip.

And now the leather-covered sphere came hurtling through the air,
And Casey stood a-watching it in haughty grandeur there.
Close by the sturdy batsman the ball unheeded sped —
"That ain't my style," said Casey. "Strike one!" the umpire said.

From the benches, black with people, there went up a muffled roar,
Like the beating of the storm-waves on a stern and distant shore;
"Kill him! Kill the umpire!" shouted some one on the stand;
And it's likely they'd have killed him had not Casey raised his hand.

With a smile of Christian charity great Casey's visage shone;
He stilled the rising tumult; he bade the game go on;
He signaled to the pitcher, and once more the dun sphere flew;
But Casey still ignored it, and the umpire said "Strike two!"

"Fraud!" cried the maddened thousands, and echo answered "Fraud!"
But one scornful look from Casey and the audience was awed.
They saw his face grow stern and cold, they saw his muscles strain,
And they knew that Casey wouldn't let that ball go by again.

The sneer has fled from Casey's lip, the teeth are clenched in hate;
He pounds with cruel violence his bat upon the plate.
And now the pitcher holds the ball, and now he lets it go,
And now the air is shattered by the force of Casey's blow.

Oh, somewhere in this favored land the sun is shining bright,
The band is playing somewhere, and somewhere hearts are light,
And somewhere men are laughing, and little children shout;
But there is no joy in Mudville — mighty Casey has struck out.