Two-Dimensional Collisions – ID: 9678

By Charles W. Eaker

Time required
45 minutes

Topic: Momentum and Collisions

- *Compare kinetic energy and momentum.*
- *Classify collisions as elastic or inelastic.*
- *Compare the momentums of two objects before and after elastic collisions.*
- *Compare changes in kinetic energy during perfectly elastic and inelastic collisions.*
- *Solve two-dimensional collision problems using vector addition.*

Activity Overview

In this activity, students observe animations of two-dimensional elastic collisions of spheres. They adjust the radii, masses, and velocities. The details of the collisions are analyzed in terms of the velocity vectors before and after the collisions. Students then calculate linear momenta and kinetic energies and show that linear momentum and kinetic energy are conserved.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes

Teacher Preparation

Before carrying out this activity, review the calculations of linear momentum and kinetic energy for a moving particle. Make sure students understand the difference between velocity, kinetic energy, and momentum.

- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7 and 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9678" in the search box.***

Classroom Management

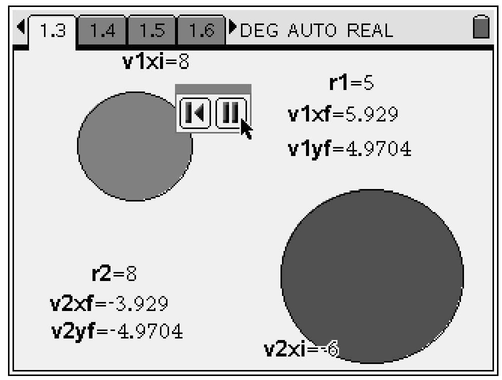
- *This activity is designed to be* ***teacher-led,*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- What are the effects of size, mass, and velocity on the motion of the particles after a two-dimensional collision?
- Is momentum conserved in a two-dimensional collision?
- How does the kinetic energy of a particle change during a two-dimensional collision?

In problem 1, students will vary the sizes (radii) and velocities of two spheres of the same mass. In problem 2, students will vary the masses and velocities of two spheres of the same radius. Student will observe relationships between the *x*- and *y*-components of the initial and final velocities. Students will analyze velocity vectors and calculate linear momentum and kinetic energy.

Problem 1 – Collisions of spheres with different radii and velocities



Step 1: Students should open the file **PhyAct_9678_2Dcollisions.tns** and read the first two pages. After pressing the arrow on page 1.3 to start the animation, students observe a simulation of a collision between two spheres of equal masses in two dimensions. Students can vary the radius of each sphere (**r1** and **r2**) and the initial *x*-velocity of each sphere (**v1xi** and **v2xi**). The simulation will display the *x*- and *y*-components of the final velocity for each sphere (**v1xf** and **v1yf** for sphere 1, **v2xf** and **v2yf** for sphere 2). Then, students should answer questions 1–4.

Q1. What is the *y*-component of each sphere's initial velocity?

A. *The y-components are equal to zero. If students struggle with this, remind them that the spheres are moving horizontally before they collide.*

Q2. What is the relationship between the *y*-components of the spheres' final velocities?

A. *They are equal in magnitude but opposite in sign.*

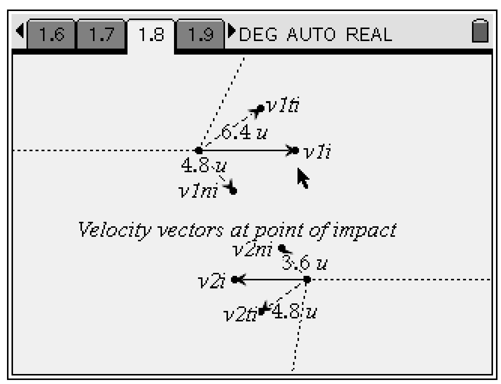
Q3. In order for a collision to occur, (**r1** + **r2**) > *b,* where *b* is a quantity called the impact parameter. What is the value of *b* for this problem? (Hint: Try entering different values for **r1** and **r2** and observing whether a collision will occur.)

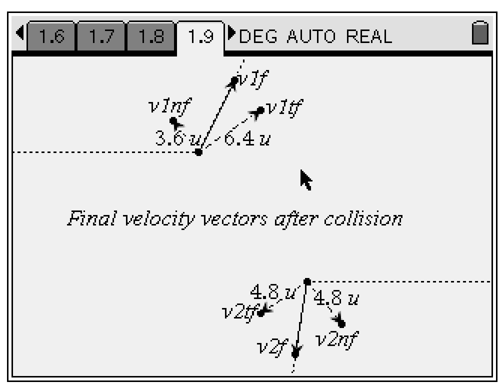
A. *12; encourage students to start answering this question by entering values for* ***r1*** *and* ***r2*** *that will definitely not result in a collision (e.g., 3 and 4). Have them discuss why the spheres will not collide under these conditions. (They will never touch.) Then, have students increase the radius of one of the spheres incrementally until the spheres will collide.*

Q4. For a collision of two spheres of the same size and mass with **v1xi** = 10 and **v2xi** = –6, which sphere will have the larger angle of deflection?

A. *the sphere with* ***v2xi*** *= –6 (i.e., the sphere with the smaller velocity magnitude)*

Step 2: Next, students should read the text on page 1.7 before moving to page 1.8. Page 1.8 shows a diagram of the velocity vectors just before the collision. The initial velocities are each separated into two components—one parallel to a line connecting the centers of the circles (**v1ni** and **v2ni**), and one perpendicular to this line (**v1ti** and **v2ti**). Page 1.9 shows the velocity vectors after the collision. Students should use the **Length** measurement tool (**Menu > Measurement > Length**) to measure the lengths of **v1ni**, **v2ni**, **v1ti**, **v2ti**, **v1nf**, **v2nf**, **v1tf**, and **v2tf**. Then, they should answer questions 5–9.





Q5. How do **v1ni** and **v1nf** compare?

A. *They point in opposite directions and have different magnitudes.*

Q6. How do **v1ti** and **v1tf** compare?

A. *They are identical. The components of the velocities perpendicular to the line connecting the centers are unaffected by the impulse of the collision.*

Q7. How do **v2ni** and **v2nf** compare?

A. *They point in opposite directions and have different magnitudes.*

Q8. How do **v2ti** and **v2tf** compare?

A. *They are identical.*

Q9. Measure the magnitudes of the initial velocities (**v1i** and **v2i**) and final velocities (**v1f** and **v2f**), and use these measurements to show that kinetic energy is conserved in the collision.

A. *Exact answers will vary depending on the initial conditions. If **v1xi** = 10, **v2xi** = –6, **r1** = 7, then **r2** = 6, **v1f** = 9.515, and **v2f** = 6.743. This leads to the following calculation (*m *is the mass of the spheres):*

$$KE_i = \frac{1}{2}mv_{1,i}^2 + \frac{1}{2}mv_{2,i}^2$$

$$KE_i = \frac{1}{2}m(10)^2 + \frac{1}{2}m(-6)^2$$

$$KE_i = 50m + 18m = 68m$$

$$KE_f = \frac{1}{2}mv_{1,f}^2 + \frac{1}{2}mv_{2,f}^2$$

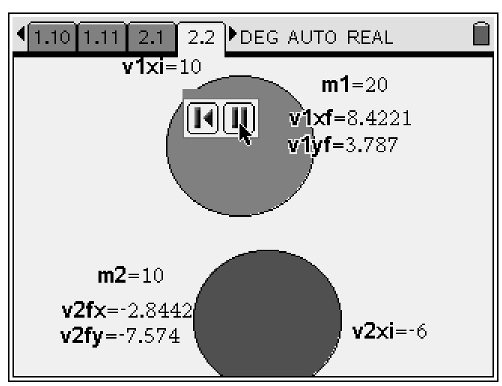
$$KE_f = \frac{1}{2}m(9.515)^2 + \frac{1}{2}m(6.743)^2$$

$$KE_f = 45.27m + 22.73m = 68m$$

*Students may wish to hide or delete the length measurements they made in step 2 before measuring the lengths of **v1f** and **v2f**, to avoid clutter on the screen.*

Problem 2 – Collisions of spheres with different masses and velocities

Step 1: Next, students should read the text on page 2.1 before moving to page 2.2. Page 2.2 shows an animation of the collision of two spheres of equal radius. Students should vary the masses and velocities of the spheres and observe the effects of these variations on the final velocities of the spheres. Pages 2.3 and 2.4 show the initial and final velocity vectors for the collision. Students should experiment with the animation, examine the velocity vectors on pages 2.3 and 2.4, and then answer questions 10–15.



Q10. In a collision of two spheres with *unequal masses* but velocities of the same magnitude, which sphere has the larger angle of deflection?

A. *the sphere with the smaller mass*

Q11. In a collision of two spheres with equal masses but *velocities of different magnitudes,* which sphere has the larger angle of deflection?

A. *the sphere with the smaller velocity magnitude*

Q12. What is the initial value of the *y*-component of the linear momentum?

A. *zero; if students struggle with this, remind them of the equation for momentum (p = mv). Because the initial* y*-velocity is zero, the initial* y*-component of the momentum must also be zero* ($m_1v_{1y,i} + m_2v_{2y,i} = m_1(0) + m_2(0) = 0$).

Q13. What is the *y*-component of the linear momentum after the collision? Show your work.

A. *zero; calculations will vary depending on the initial conditions of the collision. If* ***m1*** *= 20,* ***m2*** *= 10,* ***vx1i*** *= 6, and* ***vx2i*** *= –6, the calculations are as follows:*
$m_1v_{1y,f} + m_2v_{2y,f} = (20)(2.84) + (10)(–5.68)$
$= 56.8 + (–56.8) = 0$

Q14. Show that linear momentum is conserved in collisions of spheres of unequal masses and unequal speeds. Use a specific example, and show your work.

A. *Exact numbers will depend on the conditions the students choose. If* ***v1xi*** *= 10,* ***m1*** *= 20,* ***v2xi*** *= –6, and* ***m2*** *= 10, then* ***v1xf*** *= 8.42,* ***v2xf*** *= –2.84,* ***v1yf*** *= 3.787, and* ***v2yf*** *= –7.574. This leads to the following calculations:*
$p_{x,i} = m_1v_{1x,i} + m_2v_{2x,i} = (20)(10) + (10)(–6)$
$= 140$
$p_{x,f} = m_1v_{1x,f} + m_2v_{2x,f} = (20)(8.42) + (10)(–2.84)$
$= 140$
$p_{y,i} = 0$ (as shown in question 12)
$p_{y,f} = m_1v_{1y,f} + m_2v_{2y,f}$
$= (20)(3.787) + (10)(–7.574)$
$= 0$

Q15. Measure the magnitudes of the initial velocities (**v1i** and **v2i**) and final velocities (**v1f** and **v2f**), and use these measurements to show that kinetic energy is conserved in collisions of spheres of unequal masses and unequal speeds. Use a specific example, and show your work.

A. *Exact numbers will depend on the conditions the students choose. If **v1xi** = 10, **m1** = 20, **v2xi** = –6, and **m2** = 10, then **v1f** = 9.234, and **v2f** = 8.090. This leads to the following calculations:*

$$KE_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$$

$$KE_i = \frac{1}{2}(20)(10)^2 + \frac{1}{2}(10)(-6)^2$$

$$KE_i = 1{,}000 + 180 = 1{,}180$$

$$KE_f = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

$$KE_f = \frac{1}{2}(20)(9.234)^2 + \frac{1}{2}(10)(8.090)^2$$

$$KE_f = 852.7 + 327.2 = 1{,}179.9$$

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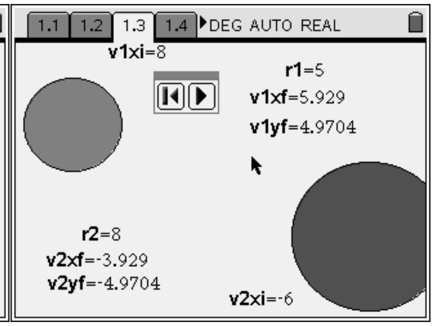
(Student)TI-Nspire File: *PhyAct_9678_2Dcollisions.tns*

TWO-DIMENSIONAL COLLISIONS

Physics

Momentum and Collisions

The next page shows an animation of a two–dimensional collision. Vary the radii of the spheres and the initial velocities **v1xi** and **v2xi**. (For the animation to work properly, **v1xi** must be positive, and **v2xi** must be negative.) In this problem, the masses of the spheres are the same.



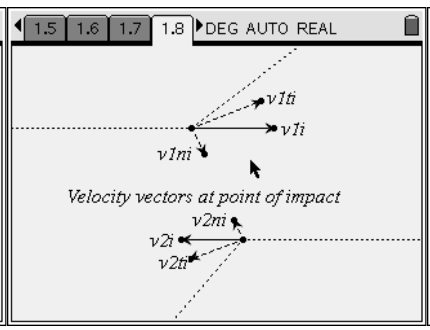
1. What is the *y*–component of each sphere's initial velocity?

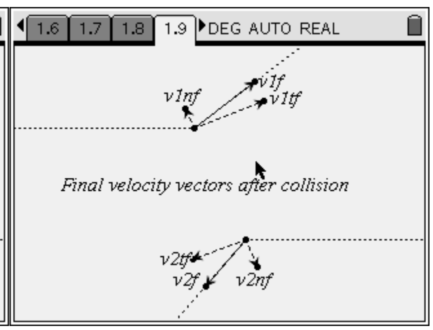
2. What is the relationship between the *y*–components of the spheres' final velocities?

3. In order for a collision to occur, (**r1** + **r2**) > *b*, where *b* is a quantity called the impact parameter. What is the value of *b* for this problem? (Hint: Try entering different values for **r1** and **r2** and observing whether a collision will occur.)

4. For a collision of two spheres of the same size and mass with **v1xi** = 10 and **v2xi** = –6, which sphere will have the largest angle of deflection?

Page 1.8 shows the initial velocity vectors **v1i** and **v2i** at the point of impact. Page 1.9 shows these vectors after impact. Upon impact, an impulse is produced along the line connecting the centers of the spheres. **v1ni** and **v2ni** are the velocity components parallel to this line. **v1ti** and **v2ti** are the velocity components perpendicular to the line.





5. How do **v1ni** and **v1nf** compare?

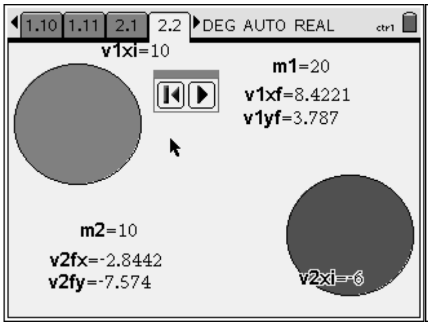
6. How do **v1ti** and **v1tf** compare?

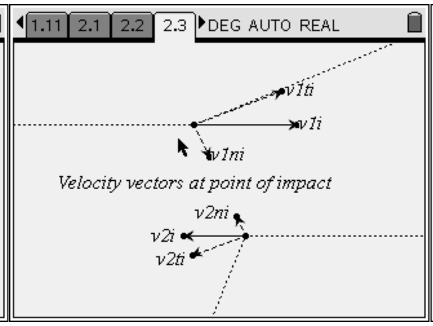
7. How do **v2ni** and **v2nf** compare?

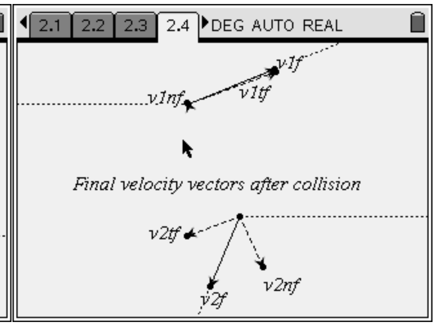
8. How do **v2ti** and **v2tf** compare?

9. Measure the magnitudes of the initial velocities (**v1i** and **v2i**) and final velocities (**v1f** and **v2f**), and use these measurements to show that kinetic energy is conserved in the collision.

In the next simulation, you can explore how changing masses and initial velocities affect the dynamics of two–dimensional collision. The spheres have the same radius. Observe the results of mass and velocity changes on pages 2.2–2.4.







10. In a collision of two spheres with *unequal masses* but velocities of the same magnitude, which sphere has the larger angle of deflection?

11. In a collision of two spheres with equal masses but *velocities of different magnitudes*, which sphere has the larger angle of deflection?

12. What is the initial value of the *y*-component of the linear momentum?

13. What is the *y*-component of the linear momentum after the collision? Show your work.

14. Show that linear momentum is conserved in collisions of spheres of unequal masses and unequal speeds. Use a specific example, and show your work.

15. Measure the magnitudes of the initial velocities (**v1i** and **v2i**) and final velocities (**v1f** and **v2f**), and use these measurements to show that kinetic energy is conserved in collisions of spheres of unequal masses and unequal speeds. Use a specific example, and show your work.