Class: Calculus

Topic: Application of Maximum-Minimum Problems

Purpose: To find the minimum cost of installing a new power line from a power station to a point on an island.

Problem: A power line is to be constructed from a power station at point A to an island at point C, which is 1 mile directly out in the water from a point B on the shore. Point B is 4 miles down shore from the power station at point A. It costs $5000 per mile to lay the power line under water and $3000 per mile to lay the line under ground. At what point S down shore from A should the line come to the shore in order to minimize cost?

1. From the Home key (home) press 6 for a new document. If a screen pops up asking "Do you want to save 'Unsaved Document'? press ▶ to No and ⏎ or enter.

   If using a computer go to View and select Handheld Screen or TI-Nspire™ Handheld View.

   Press menu, 2 for View and 2 for Plane Geometry View.

2. For this problem we will use a scale drawing. If the scale is not on the screen press enter 2 7 to show the scale.

   Begin by constructing a rectangle.

   menu, 8 Shapes, 3 Rectangle. Move ↖ to top of and slightly to the left of center of screen. Press enter caps C to name the point. Move ↖ to the right keeping the segment as near horizontal as possible. Do not get to close to the scale. Press enter. Do not name the point. ▼ until you get a rectangle the size you want—about half way. Press enter and caps B to label point in bottom left corner. To label the point in the bottom right corner place ↖ on the vertex and press menu ▶ 6 enter caps A enter esc.

3. Change the scale from cm to miles(mi)

   Place the cursor ɪ on the scale and ⏎ ⏎. A flashing vertical bar should be located to right of m in cm. Press clear clear to remove cm and type M I for miles and press enter. See Figure 1.

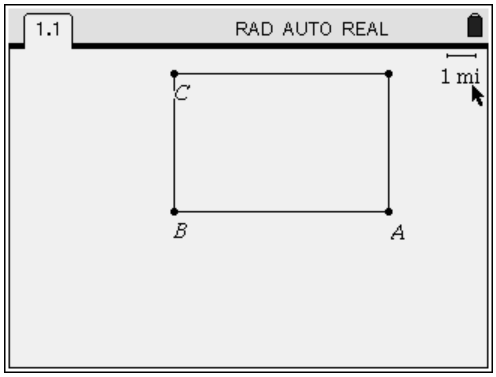

Figure 1

4. Measure BC and AB and move measurements.

Press (menu) (7) (1) for length. Place ↖ on side BC. If the rectangle flashes press (tab) to find the length of side BC. Press (enter) (enter). Move ↖ to side AB and repeat the process. See Figure 2 . Press (esc).

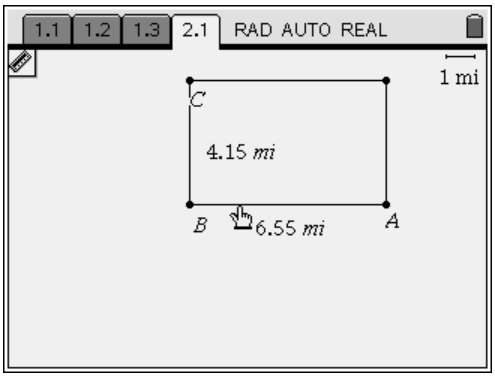

Figure 2

Move measurements below the rectangle and to the right side. Place the cursor on the measurement for AB until the measurement flashes and the word text appears. See Figure 3. Press(ctrl) (hand). Hand will close. Use arrows to move the measurement. Press (esc) . Repeat the process for side BC. Drag the measurement below the value for AB and press (esc).

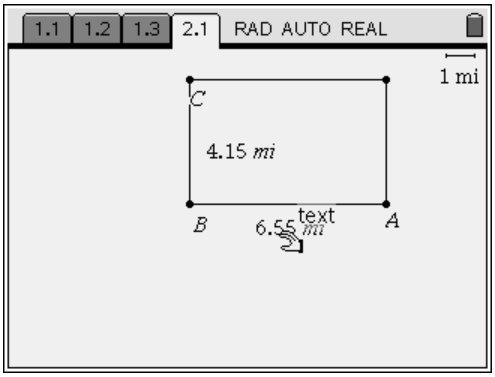

Figure 3

5. Scale the drawing. Note: We have already changed the scale from cm to mi.

a. Copy the horizontal measurement. Move the cursor to the horizontal measurement until the hand appears and (hand)(hand). Move the flashing vertical bar behind the number. Press and hold (shift) while pressing ◀ to highlight the number. Press (ctrl) (c) (enter) to copy the number.

b. Move ↖ over the scale (1 mi) until ↕ appears and (hand)(hand). Move the flashing vertical bar behind the number. Press (clear) until the (1) is deleted. Press (4) (the distance from A to B in the problem.) (÷) (ctrl) (v) (enter). See Figure 4. Note: The horizontal distance has changed to 4 miles.

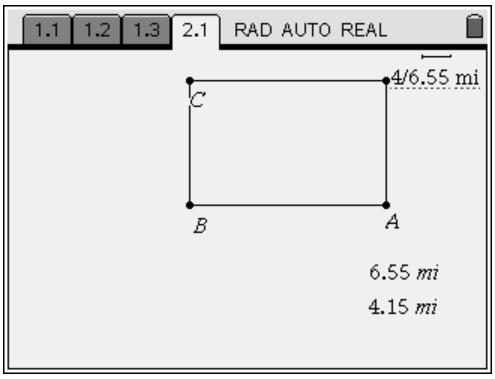

Figure 4

c. The vertical distance has also changed but not to 1 mile which is the distance in the problem. See Figure 5. Move the cursor ↖ over the vertical measurement and (hand)(hand). Move the flashing vertical bar behind the number. Press (clear) until the number is removed and press (1) (the distance from the island to shore)(enter). Note: This will change the size of the rectangle. See Figure 6 on the next page.

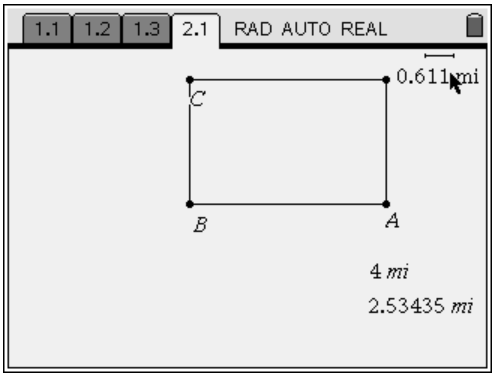

Figure 5

6. Construct segments BC and AB.

menu 6 5 Place the cursor on point B and press enter. Move cursor to point C and press enter. Repeat the process to get segment AB. Place the cursor on B. enter Move the cursor to A. Press enter esc.

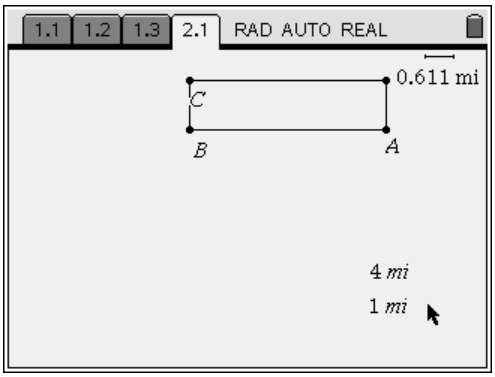

Figure 6

7. Hide the rectangle and the vertex not named.

Place the cursor on the rectangle until it flashes. If the segment flashes press tab. See Figure 7. Press menu 1 Actions 3 for Hide/Show and enter. Move the cursor to the unnamed vertex. Press enter and esc. See Figure 8.

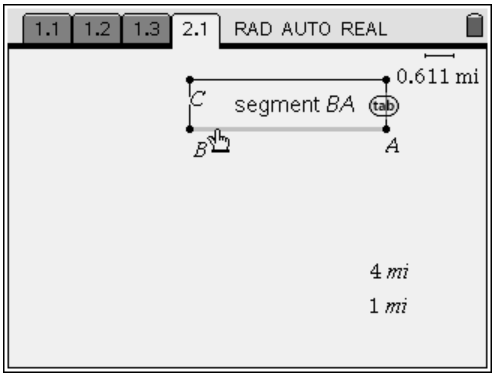

Figure 7

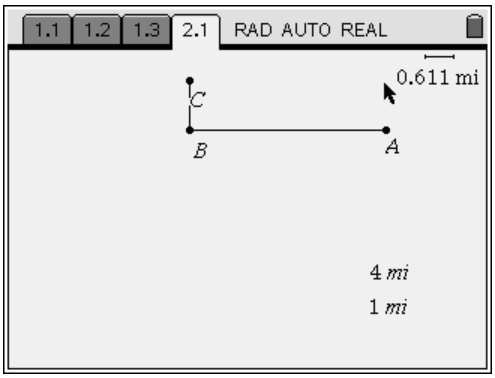

Figure 8

8. Construct segment CS where S is a point on AB.

menu 6 5. Move cursor to point C and enter. Move until you reach segment AB (about half way between A and B) and "point on" appears on the screen and press enter. See Figure 9. Be sure to label point S while in the segment mode and then esc.

9. Construct segments BS and SA.

menu 6 5 Place the cursor on point B and press enter. Move cursor to point S and press enter. Repeat the process to get segment SA. Place the cursor on S. enter Move the cursor to A. Press enter esc.

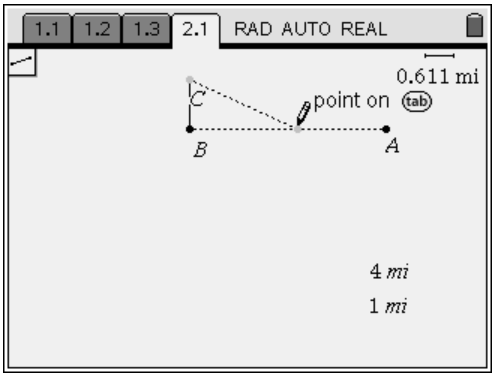

Figure 9

10. Measure segments BS and SA.

Press (menu) (7) (1) for length. Place ↖ on segment BS and press (enter) (enter). Move ↖ to segment SA and repeat the process. Press (esc).

Move measurements below the rectangle and to the right side.
Place the cursor on the measurement for BS until the measurement flashes and the word text appears. Press (ctrl) (☜). Hand will close. Use arrows to move the measurement.
Press (esc) . Repeat the process for segment SA. Drag the measurement above the value for BS and press (esc). See Figure 10.

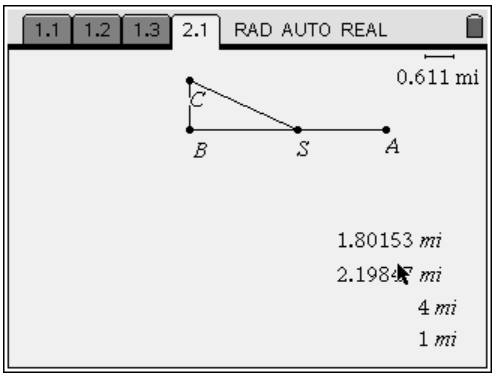

Figure 10

11. Label the lengths BS, SA, and BC as such.

Place ↖ on the measurement for BS and (☜). Press (var) (1). Hold down (⇧) while pressing (B) (S) (enter). Repeat the process for SA and BC. See Figure 11. With BC flashing press (ctrl) (☜) and move the value to the left of segment BC.

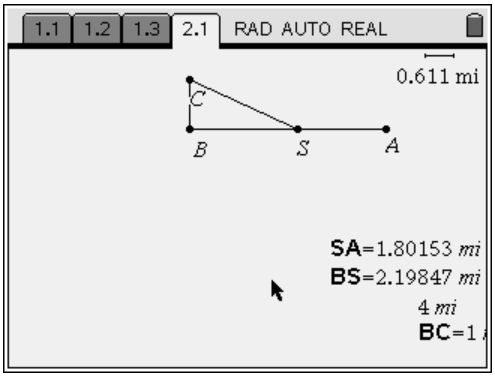

Figure 11

12. Use "text" to label CS and SA with d = CS and x = SA.

Place ↖ near segment CS and press (menu) (1) (6) (enter) (D) (enter). Move the d if necessary. Repeat the procedure for x and (esc). See Figure 12.

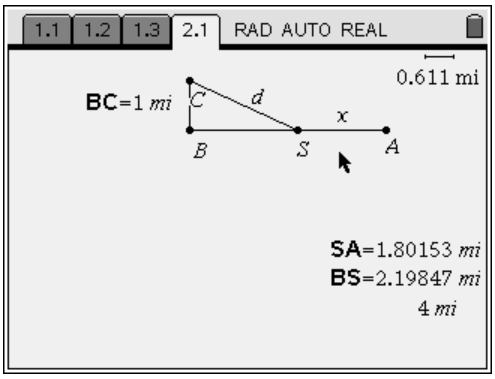

Figure 12

13. Write a formula for CS, calculate its current value, and assign a variable to the value.

a. Place ↖ below and to the left side of the drawing. (menu) (1) (6) (enter). Type $CS = \sqrt{CB^2 + BS^2}$ (enter) (esc).

b. To calculate the current value place ↖ on CS until the segment flashes. Press (menu) (7) (1) (enter) (enter) (esc). Move the value below the formula for CS and (esc).

c. To assign a variable to CS place ↖ on the value for CS. Press (☜) (var) (1) and type (D) (enter) and (esc). See Figure 13.

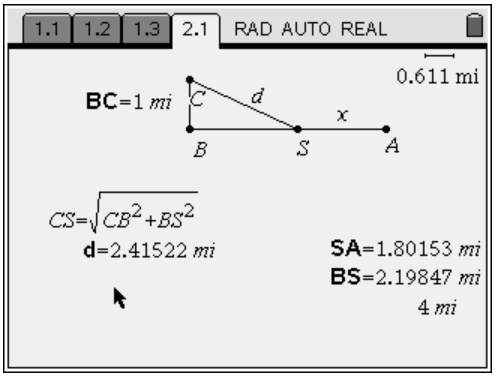

Figure 13

14. Determine a cost equation and assign a variable to the calculated value.
total cost = 5000d + 3000x

Place ▸ below the value for d and press (menu) (1) (6) (enter) and type (C)(O)(S)(T)(=)(5)(0)(0)(0)(D)(+)(3)(0)(0)(0)(X) (enter) (esc).

To calculate the total cost using the current values press (menu) (1) (8). Move ▸ to the cost equation until it flashes. Press (enter). Move ▸ to the value for d and value flashes. See Figure 14. Press (enter). Again you will get a text box asking you to select a value for x. Move ▸ to the value for SA (Rem: x = SA). Press (enter). Move the ▸ off SA and a number will follow you. This is the cost for the given values of d and x. Move the value under the cost equation and press (enter) (esc).

To assign a variable to the cost value place ▸ on the value until the hand appears and the number flashes. Press (ctrl) (stor var) (1) and type (T)(C) and press (enter) (esc). See Figure 15.

15. Open a Lists and Spreadsheet page. (home) (3).

▲ to A. Type (L)(A)(N)(D) and (tab) to B and type (W)(A)(T)(E)(R). (tab) to C and type (T)(O)(T)(A)(L)(C)(O)(S)(T) and (enter). ◂ to formula line under A and press (menu) (3) (2) (2) (to manually collect data later) and type (S)(A) and (enter). Press ▸ ▲ under B and press (menu) (3) (2) (2) and type (D). Press ▼ for the pull down menu and select Variable Reference and (enter). Press ▸ ▲ under C and press (menu) (3) (2) (2) and type (T)(C) and (enter). See Figure 16.

16. We are now ready to collect data for spreadsheet.
Press (ctrl) ◂ to return to the Graphs and Geometry page. Grab point S ((ctrl) (click)) and move it to the right until SA = 0. See Figure 17.

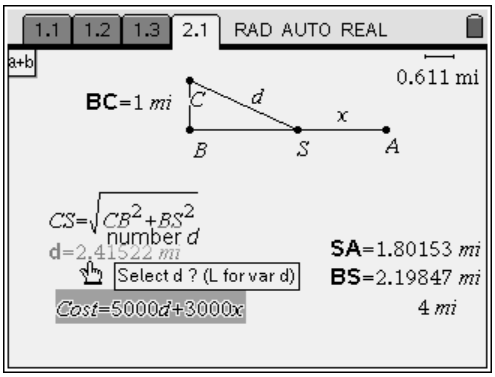

Figure 14

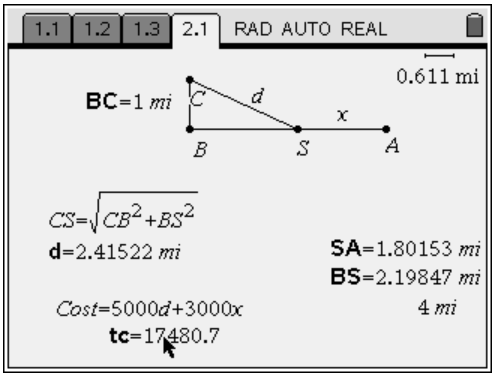

Figure 15

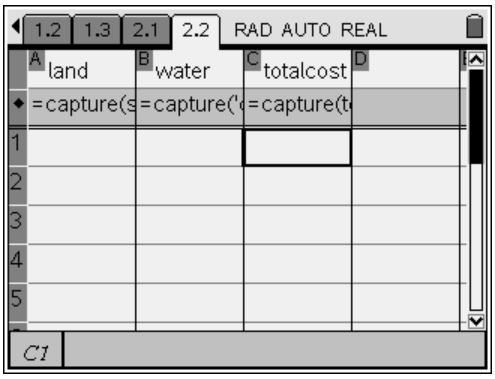

Figure 16

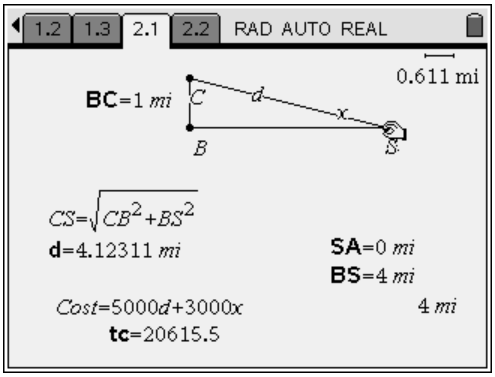

Figure 17

Press ctrl . to set the first point in the spreadsheet. Press ctrl ▶. See Figure 18.
Note: The value for d is 4.12311 or $\sqrt{17}$
As it should be if we traveled by water only.

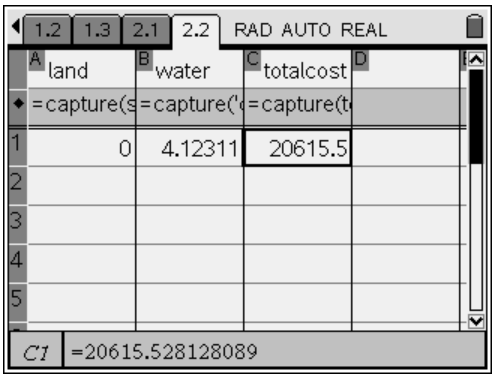

Figure 18

17. Press ctrl ◀ to return to the previous page. Place the cursor on Point S. Press ctrl ☜ to get a closed hand. Move the closed hand to the left a few clicks (I use 5.) and press ctrl . Continue this process until Point S lies on top of Point B. Press ctrl . one last time and esc. See Figure 19.

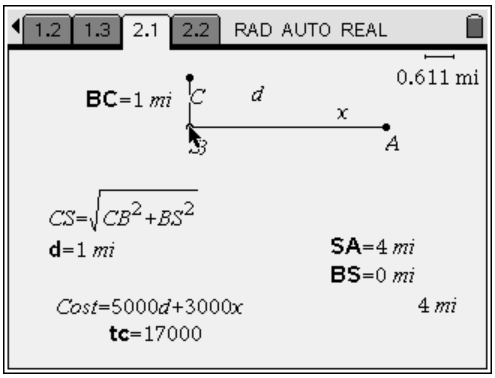

Figure 19

18. Press ctrl ▶ to look at the spreadsheet.
Note: The last value in the spreadsheet should be sa = 4, d = 1, and tc = 17000 (See Figure 20.) which represents the distance traveled along the legs of the triangle.

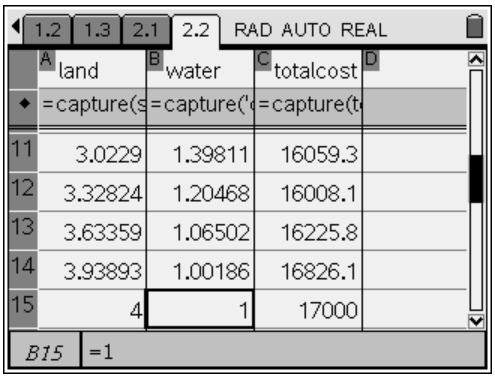

Figure 20

Note: ctrl G will allow you to move to any row in the spreadsheet. Press ctrl G 1 (See Figure 21.) and enter to move to Row 1 in the spreadsheet.

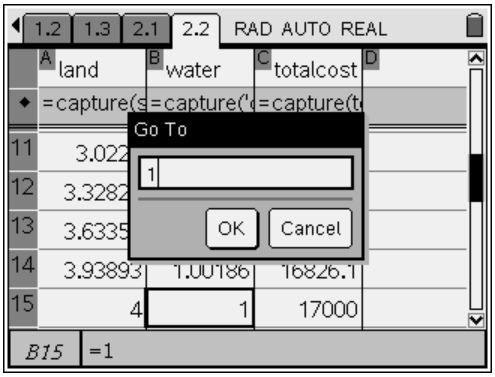

Figure 21

19. Add a Graphs and Geometry page.
Press (home) (2).

Set the graph type to Scatter Plot.
Press (menu) (3) (4) (enter). Use the ▼ to select "water" as the independent variable and (enter).
Press (tab) (enter) and ▼ to select "totalcost" as the dependent variable (See Figure 22.) and (enter).
Press (ctrl) (G) to hide the entry line.

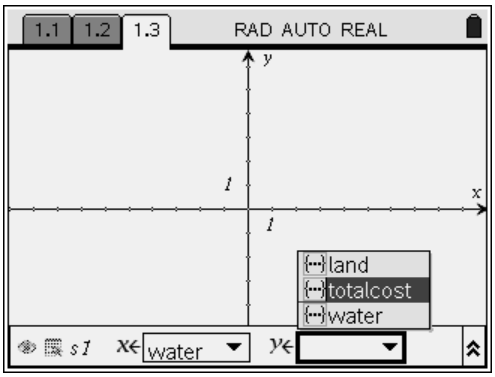

Figure 22

20. Set the window so we can view the data.

Press (menu) (2) (8) to show axes end values.
Place the cursor on the y-min and press (enter) (enter).
Press (ans) (2) (tab) which carries you to the y-max. Type (2)(2)(0)(0)(0) and (tab) to x-max. Type (6) and (tab) to y-min and type (1)(0)(0)(0)(0) and (tab) and (esc) (esc).
See Figure 23.

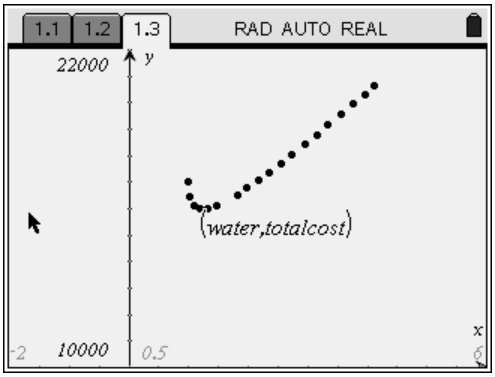

Figure 23

21. Before proceeding copy page 1.3 to 2.1.

Press (ctrl) (home) (4) to insert a new problem.
Press (2) to open a Graph page. To copy press (ctrl) ▲ to go to the Page Sorter. See Figure 24.
With the border around page 1.3 press (ctrl) (C).
Press ▼ ▼ so the border is around page 2.1.
Press (ctrl) (V) to paste page into problem 2.
Place border around 2.1 and press (clear). Follow this procedure to copy 1.3 to 3.1.

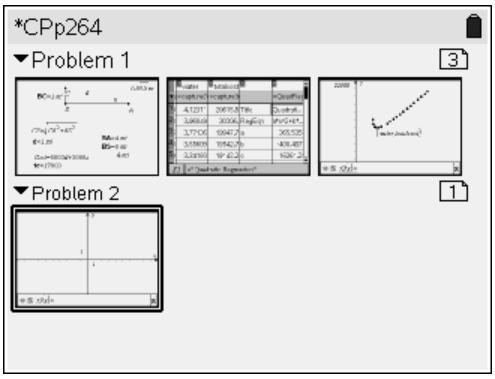

Figure 24

22. Find a quadratic function that best fits this data.

To change the Graph Type from Scatter Plot to Function press (menu) (3) (1). Press (ctrl) ◀ to return to the Lists and Spreadsheet page.

Place your in column D. Press (menu) (4) (1) (6) to get a Quadratic Regression. See Figure 25. Press ▼ to "water" and (enter). Press (tab) ▼ to "totalcost" and (enter). Press (enter) to get your Regression Model. See Figure 26 on the next page. Press (ctrl) ▶ to return to graph.

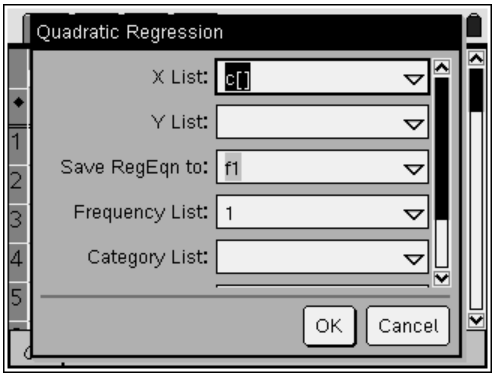

Figure 25

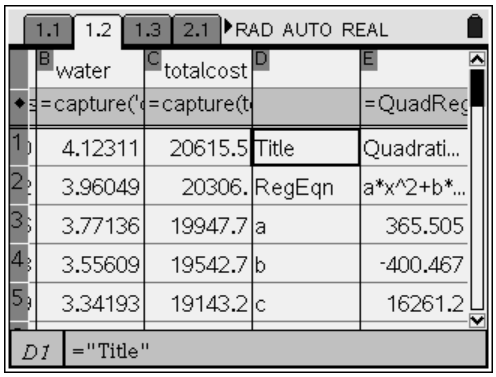

Figure 26

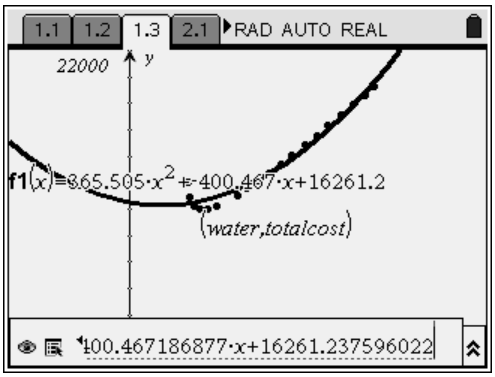

Figure 27

Press ▲ and enter to see the graph of the quadratic model. See Figure 27.

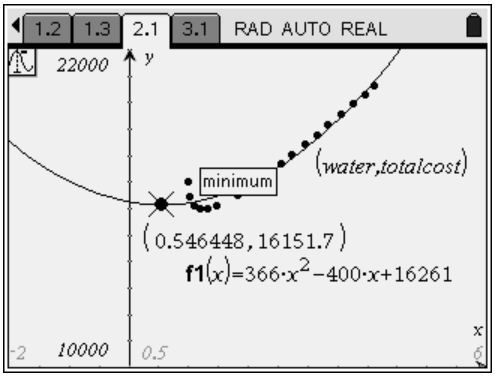

Figure 28

23. Since I cannot trace on a Regression Model type 3 6 6 X x² ÷ 4 0 0 X +/- 1 6 2 6 1 into problem 2.1 as f1(x) and enter. Press menu 5 1 to trace and approximate the minimum value. See Figure 28. This is an unreasonable answer since it is 1 mile to shore. Therefore we will grab our graph near the vertex and move the parabola until the vertex is near the lowest point of the stat graph. Also grab the parabola near the top of the screen and work for a better fit. Find the minimum point now. See Figure 29.
Note: The x value of the ordered pair represents the distance traveled by water. When solved, the value for x is 3.43 which is over by 0.18 mi while the minimum value is under by $0.80.

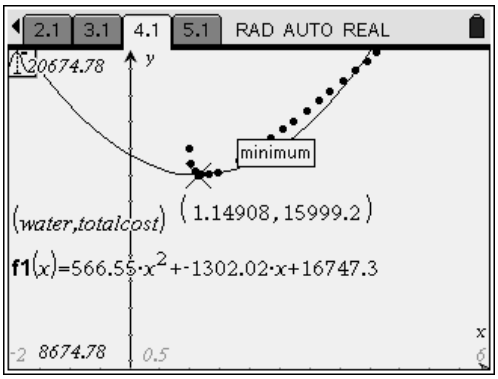

Figure 29

24. Write an equation for total cost in terms of x to find the actual minimum value.

The equation is $tc = 5000\sqrt{1+(4-x)^2} + 3000x$.
Insert a new problem (Problem 3).
On page 3.1 press menu 2 to get a Graphs and Geometry Page. Use the same window as in Step 20 and type the equation into f1(x) and enter. Find the minimum value of this function by pressing menu 5 1 and trace until the minimum appears. See Figure 30.

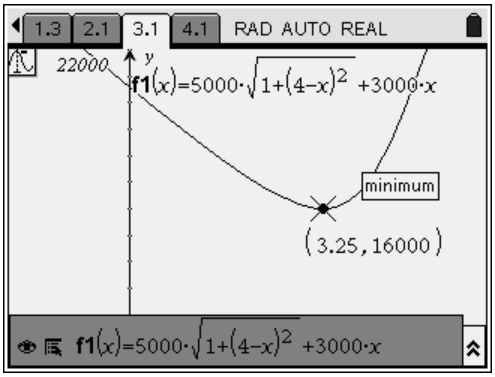

Figure 30