Closure Tables

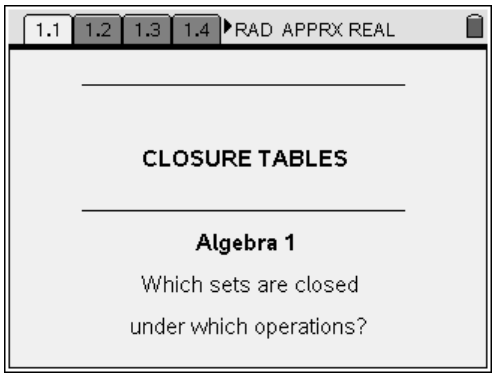
ID: 8639

Name _____

Class _____

In this activity, you will explore:

- *the properties of some sets under the operations of addition, subtraction, multiplication, and division*
- *the concept of closure and closure tables*



Open the file *Alg1Act30_ClosureTables_EN.tns* on your handheld. Listen as your teacher explains what it means for a set to be closed under an operation. Record your answers on the worksheet or as directed by your teacher.

Things to Remember…

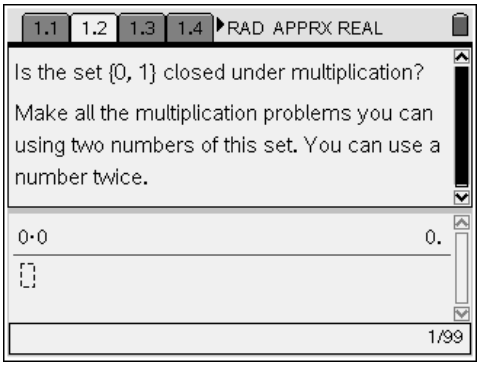
A set is ***closed*** under an operation if for any two numbers in the set, the result of the operation is also in the set. This definition will take on more meaning as you explore examples.

You can think of a set as a bag of numbered tiles. The set is closed under an operation if you can draw any two tiles from the bag, perform the operation with them, and put the answer back in the bag.

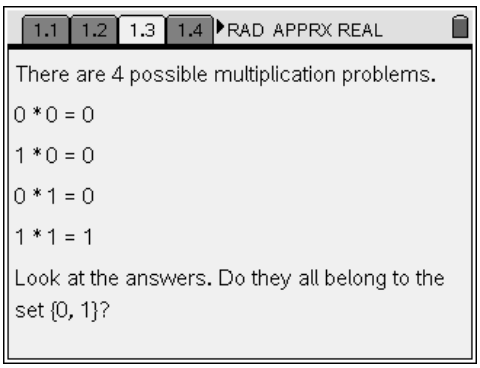
Problem 1 – The set {0, 1}

Let's begin with the set {0, 1}. This set has only two elements. Is it closed under multiplication?

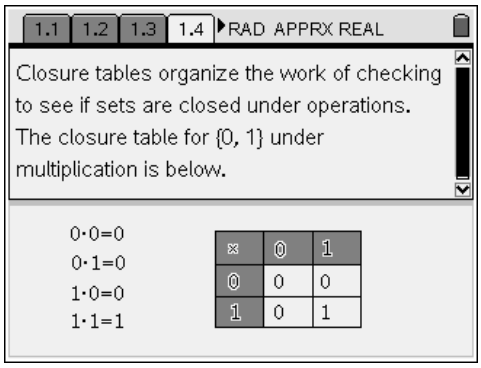
Make all the multiplication problems you can using two numbers from this set. You can use a number twice. Try the problems in the *Calculator* application.



There are 4 possible multiplication problems. Look at their answers. Do the answers belong to the set {0, 1}?



Closure tables are a way to organize your work when checking to see if sets are closed under operations. The closure table for {0, 1} under multiplication is shown.



Closure tables are similar to multiplication tables, but they can show any operation. To read them, find a number in the first column and a number in the first row. The intersection of these gives the result of the two numbers under that operation.

To use the closure table to determine if this set is closed under multiplication, look at each of the products in the table. All of the products belong to the set {0, 1} so this set is closed under multiplication.

Use closure tables to answer each of the following questions.

1. Is the set {0, 1} closed under addition?

2. Is the set {0, 1} closed under subtraction?

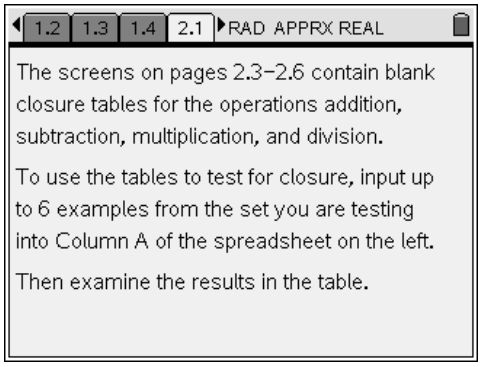
3. Is the set {0, 1} closed under division?

Note that closure under an operation depends on both the operation and the set.

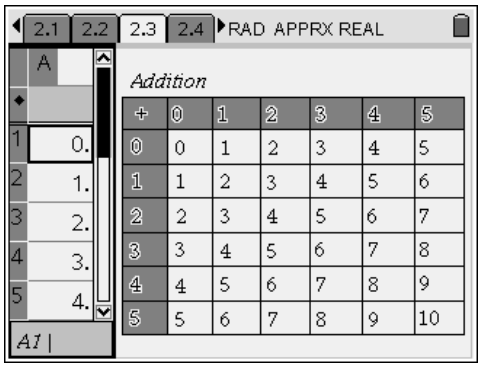
Problem 2 – The whole numbers

In this problem, you will examine the set of whole numbers to determine if they are closed under addition, subtraction, multiplication, and division.

A closure table with all of the whole numbers would be infinitely large! For our purposes, it is enough to use just a few examples of the whole numbers.



To use your handheld to complete the closure table, use ctrl + ◄ or ctrl + ► to move to the page with the operation you are testing. Then input up to 6 examples of whole numbers into the spreadsheet on the left.



+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

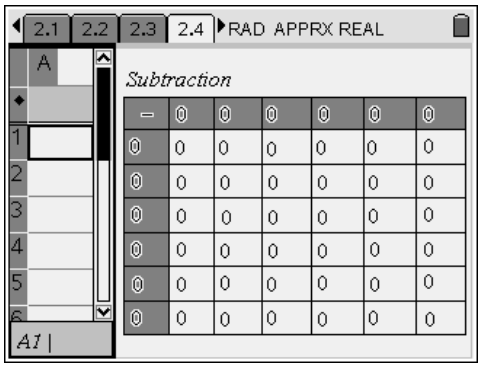
1. Examine the closure table for the whole numbers under addition. Do you see any patterns in the table? Do any of the sums repeat?

2. What property of addition causes this pattern?

3. Are all of the sums in the table whole numbers?

4. Are the whole numbers closed under addition?

Addition is commutative, so it doesn't matter what order you add two numbers in. Subtraction is **not** commutative. It **does** matter in what order you subtract.



−	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

5. With this in mind, use your handheld to create a closure table for the whole numbers under subtraction. Copy your table here.

−							

6. Do you see any patterns in the table?

7. Are all of the differences whole numbers?

8. Are the whole numbers closed under subtraction?

9. Create closure tables for the whole numbers under multiplication and division. Copy your tables here. Round your answers to the nearest tenth if necessary.

×							

÷							

10. Do you see a pattern in the table?

11. What property causes this pattern?

12. Are the whole numbers closed under multiplication?

13. Are the whole numbers closed under division?

Problem 3 – The integers

In this problem, you will determine if the set of integers is closed under addition, subtraction, multiplication, and division.

The set of integers is even larger than the set of whole numbers!

1. What types of numbers belong to the set of integers that do not belong to the set of whole numbers? List a few examples.

2. What examples of the integers will you use in the closure tables? Be sure to choose some negative and some positive integers.

Complete each closure table with the help of your calculator and the patterns you have observed.

3.

+							

4.

–							

5.

×							

6.

÷							

7. What pattern do you see in the multiplication and division tables? Where are the products and quotients positive? Where are they negative?

8. Under which operations are the integers closed?

Problem 4 – The even and odd numbers

In this problem, you will determine if the sets of even and odd integers are closed under addition, subtraction, multiplication, and division.

Choose some even and odd numbers to use in the closure tables.

Create each closure table with the help of your handheld.

1. Under which operations is the set of even numbers closed?

2. Under which operations is the set of odd numbers closed?

3. Complete the table to summarize your findings.

set	closed under +	closed under –	closed under ×	closed under ÷
{0,1}	no		yes	
whole numbers				
integers				
even numbers				
odd numbers				