# *NUMB3RS* Activity: The Escape Game
## Episode: “Assassin”

**Topic:** Game Theory **Grade Level**: 7-12
**Objective:** Introduce game theory
**Time:** about 15 minutes

## Introduction

John von Neumann (1903-1957), a Hungarian mathematician whose contributions spread over many different mathematical areas, is probably best known for his pioneering work on game theory in the 1920s. Von Neumann’s work has been applied in fields such as economics and politics. For example, competing businesses must decide the best price for a product based on the prices of their competitors. The situations can be thought of as games in which decisions made by two or more players affect everyone involved. Game theory focuses on finding the best strategies to resolve conflict using mathematical models.

You and your students may be familiar with the work of John Nash, popularized in the 2001 movie *A Beautiful Mind*. Nash shared a Nobel Prize in Economics in 1994 for his work applying game theory to economics.

## Discuss with Students

In **“Assassin**,**”** Charlie is trying to figure out where a killer might strike given that he knows who the victim will be. By analyzing the movements of the would-be victim, Charlie models the decisions that the assassin will make. This is an example of behavioral game theory, where the motives of two players (assassin and victim) are contradictory and the behavior of one player affects the behavior of the other.

***NUMB3RS* Example** Agent Don Eppes is chasing a fugitive. Don and the fugitive are on separate subway trains traveling in the same direction; there are two stops left, **A** and **B**. If Don and the fugitive get off at the same stop, then Don will catch the fugitive. If they get off at different stops, then the fugitive will get away. This scenario can be modeled with the following *payoff matrix*:

$$\begin{array}{cc} & \textbf{Stop chosen by fugitive} \\ & \begin{array}{cc} \quad\textbf{A} \quad & \quad \textbf{B} \end{array} \\ \textbf{Stop chosen by Don} \begin{array}{c} \textbf{A} \\ \textbf{B} \end{array} & \begin{bmatrix} \textit{caught} & \textit{free} \\ \textit{free} & \textit{caught} \end{bmatrix} \end{array}$$

***Student page answers**: **1**. Results will vary depending on the experiment. **2**. 50%; **3.** 100%; **4**. B; Don would capture the fugitive 80% of the time; **5**. **a**. No; randomly choose the stop; **b**. randomly choose his stop.*

Name: ______________________________ Date: _______________

## *NUMB3RS* Activity: The Escape Game

Play the following game 10 times with a partner. Take turns being Don and the fugitive. Randomly choose which stop you will take and record whether the fugitive will escape or be captured. You can use the function **`randInt(0,1)`** on your TI calculator to randomly generate a 0 or 1 (find this function by pressing **`Math`** and then **`PRB`**). Alternately, you could flip a coin to decide the stop to take.

$$\begin{array}{cc} & \textbf{Stop chosen by fugitive} \\ & \begin{array}{cc} \quad\textbf{A} & \qquad\textbf{B} \end{array} \\ \textbf{Stop chosen by Don} & \begin{array}{c} \textbf{A} \\ \textbf{B} \end{array} \begin{bmatrix} \textit{caught} & \textit{free} \\ \textit{free} & \textit{caught} \end{bmatrix} \end{array}$$

For example, if you are Agent Don Eppes and the random number you generate is 0 (or you flip heads) then you have chosen stop **A**. If the random number you generate is 1 (or you flip tails) then you have chosen stop **B**. Your partner then generates a random number to represent the fugitive's choice. If the fugitive's choice is the same as Don's, then the fugitive is caught. If the choices are different, then the fugitive escapes.

Play the game 10 times. Record whether the fugitive escapes or is captured.

Results: ________ ________ ________ ________ ________

________ ________ ________ ________ ________

**1**. Based on your results, what are the chances that the fugitive will be caught each time? ______________________________

**2**. If you continued the game for several hundred more times, what would you expect the chances are that the fugitive will be caught each time? __________

**3.** If Don knew in advance where the fugitive would get off, what would the chances of capture be? ______________________________

**4.** If Don knows that the fugitive gets off at Stop **B** 80% of the time, what stop should Don always choose? Explain your answer. ______________________

______________________________________________________________

**5. a.** Is it smart for the fugitive to get off at one stop more often than the other? Explain how the fugitive should decide which stop to choose. ___________

______________________________________________________________

**b.** Based on your answer to part **a**, how should Don decide which stop to get off at? ______________________________

# Extensions

## Activity 1: Prisoner’s Dilemma

### Introduction

Pretend you are a criminal. Agent Don Eppes has captured you and someone you have worked with, but don’t know very well. Don has evidence linking one or both of you to a crime. You are being kept in separate rooms and cannot communicate with each other. Don outlines the following options for you and you must decide without knowing what your partner has decided:

- If you confess and testify against your partner, you will go free and your partner will spend 10 years in jail.
- If your partner confesses and you deny, he will testify against you. You will spend 10 years in jail and your partner will go free.
- If you and your partner both confess, you will each spend 6 years in jail.
- If you and your partner both deny, you will each spend 3 years in jail.

This situation can be represented with the following payoff matrix, where the 1st number is the number of years that you will spend in jail, and the 2nd number is the number of years that your partner will spend in jail:

$$\begin{array}{cc} & \textbf{Your Partner} \\ & \begin{array}{cc}\textbf{Confess} & \textbf{Deny}\end{array} \\ \textbf{You} \begin{array}{r}\textbf{Confess} \\ \textbf{Deny}\end{array} & \begin{bmatrix} 6,6 & 0,10 \\ 10,0 & 3,3 \end{bmatrix} \end{array}$$

### For the Student

- A zero-sum game is a game in which one player’s payoff is equal to another player’s loss. Is this a zero-sum game? Explain.
- Use the payoff matrix to help you decide what to do. Look at each of your options and the number of years you will spend in jail. Which choice should you make to spend the least amount of time in jail *regardless* of what your partner chooses?
- If you and your partner were allowed to speak to each other, do you think this would change your strategy?

## Activity 2: Rock, Paper, Scissors

### For the Student

- Create a payoff matrix to model the game Rock, Paper, Scissors.
- How big should the matrix be?
- What are the possible outcomes of this game?
- Is this a zero-sum game?

## Additional Resources

**http://www.egwald.com/operationsresearch/gameintroduction.php**

This introduction to game theory, with many examples, is more appropriate for higher classes.

 