EXPLORATIONS™

Activity 7

Reflections in the Plane

Objectives

- To use the **Reflection** tool on the Cabri® Jr. application
- To investigate the properties of a reflection
- To extend the concepts of reflection to the coordinate plane

Cabri® Jr. Tools

Introduction

Reflections across a line are useful for studying symmetry, proving theorems, and constructing figures. In the first part of this activity, you will learn how to perform a reflection using the Cabri Jr. application and will explore the properties of reflections. In the second part of this activity, you will extend the concept of reflections to the coordinate plane.

This activity makes use of the following definition:

Reflection — a transformation that displays a mirror image of a figure across a certain line.

Pre-image — the original object that is to be transformed.

Image — the new object created by applying the conditions of a transformation.

Orientation — refers to the rotation order of corresponding vertices. If the pre-image is represented by the vertices A, B, and C rotated in counter-clockwise order, then orientation will be preserved in the image when the vertices $A1$, $B1$, and $C1$ are in counter-clockwise order.

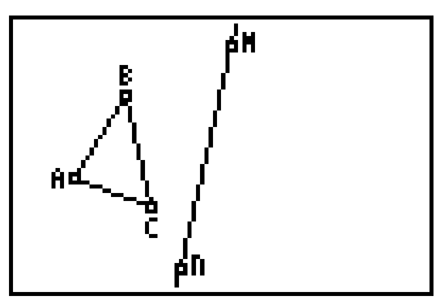
Part I: Properties of a Reflection

Construction

I. **Draw a triangle and a reflection line.**

Draw a scalene $\triangle ABC$ on the left side of the screen.

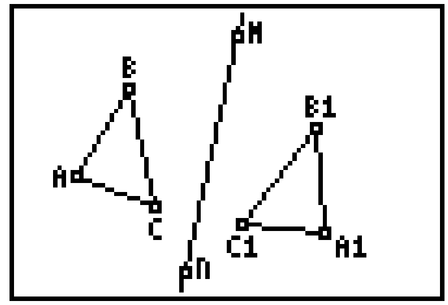
Draw $\overleftrightarrow{MN}$ near the center of the screen.



II. Reflect the triangle.

To use the **Reflection** tool you must do the following:

- Select the item to be reflected.
- Select the line of reflection.



Reflect $\triangle ABC$ over $\overleftrightarrow{MN}$. Label the corresponding vertices of $\triangle ABC$, $A1$, $B1$, $C1$.

Note: *If part or all of the image does not appear on the screen, try moving* $\overleftrightarrow{MN}$, *moving* $\triangle ABC$, *or making* $\triangle ABC$ *smaller until all of the image appears on the screen.*

Exploration

Observe the relationship between the pre-image and the image by dragging the vertices and sides of $\triangle ABC$, $\triangle ABC$ itself, points M and N, and $\overleftrightarrow{MN}$.

Connect one pair of corresponding vertices of the pre-image and the image using a line segment. Use various measurement tools to investigate the relationship between $\overleftrightarrow{MN}$ and the segment connecting corresponding vertices. Drag a vertex or side of the triangle to verify that any relationship observed is true for any triangle reflected over a line.

Questions and Conjectures

1. Make a conjecture as to how the **Reflection** tool determines the location of the reflection. Explain how you would test your conjecture.
2. Make a list of properties of the pre-image triangle that are preserved in its image. Use various measurement tools (**Distance and Length**, **Angle**, and **Slope**) to verify your answers.
3. Can the **Translation** tool be used to create the same image created using the **Reflection** tool? Explain and be prepared to demonstrate.

Part II: Reflections in the Coordinate Plane

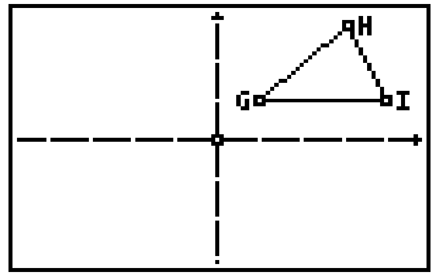
Construction

Draw a triangle to reflect in the coordinate plane.

Clear the previous construction.

Show the axes on the screen. Drag the origin to the center of the screen.

Draw scalene triangle ΔGHI in the first quadrant.



Exploration

Reflect ΔGHI across the **x-axis** and observe the relationships between the coordinates of the vertices of ΔGHI and the corresponding vertices of its image. Be sure to drag the sides and vertices of ΔGHI and ΔGHI itself to verify the relationships you believe exist.

Reflect ΔGHI across the **y-axis** and observe the relationships between the coordinates of the vertices of ΔGHI and the corresponding vertices of its image. Be sure to drag a side and vertices of ΔGHI and ΔGHI itself to verify the relationships you believe exist.

Reflect ΔGHI across the **x-axis** and then reflect the resulting image across the **y-axis**. Observe the relationships that exist among the coordinates of the corresponding vertices of the three triangles.

Questions and Conjectures

1. Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and those of its image when the triangle has been reflected across the **x-axis**.
2. Is there a relationship between slopes of the sides of the pre-image triangle and those of the image triangle when the pre-image is reflected across the **x-axis**? Explain your reasoning.
3. Make a conjecture about the relationship between the coordinates of the corresponding vertices of a triangle and those of its image when the triangle has been reflected across the **y-axis**.
4. Is there a relationship between slopes of the sides of the pre-image triangle and those of the image triangle when the pre-image is reflected across the **y-axis**? Explain your reasoning.
5. Make a conjecture about how a composition of reflections works. Does the order of the two reflections make a difference?
6. Can a composition of reflections be done using a single reflection? Explain and be prepared to demonstrate.

Extension

Investigate reflections of a triangle across the lines $y = x$ and $y = -x$. (You can construct these lines by bisecting the angles formed by the intersection of the coordinate axes.) Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and its image. Additionally, make a conjecture about the relationship between the slopes of the corresponding sides in the pre-image and image.

Teacher Notes

Activity 7

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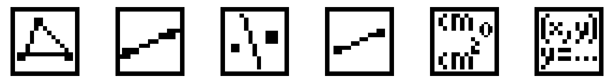
Cabri® Jr. Tools



Additional Information

It is recommended that you complete Activity 6, Translations in the Plane, prior to doing this activity.

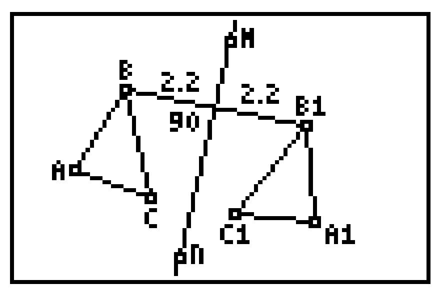
A great way to summarize this activity would be to develop a working definition for reflection as a whole class activity.

Part I: Properties of a Reflection

Answers to Questions and Conjectures

1. Make a conjecture as to how the **Reflection** tool determines the location of the reflection. Explain how you would test your conjecture.

   The image point of a vertex is a point on the line perpendicular to the reflection line passing through the given vertex. The point is on the opposite side of the reflection line at a distance that is equal to the distance from the vertex to the reflection line. The same relationship is true for a pair of corresponding points on the image and pre-image.

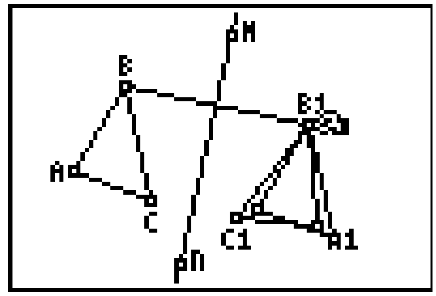


2. Make a list of properties of the pre-image triangle that are preserved in its image. Use various measurement tools (**Distance and Length**, **Angle**, and **Slope**) to verify your answers.

   The properties that are preserved in an image are as follows: length, area, angle measure, and perimeter. Unlike translations, orientation is not preserved. If the pre-image is represented by the vertices *A*, *B*, and *C* rotated in counter-clockwise order then the image will be represented by the vertices *A*1, *B*1, *C*1 in clockwise order. It can also be argued that rotational orientation is changed since the slopes of corresponding sides are not equal.

3. Can the **Translation** tool be used to create the same image created using the **Reflection** tool? Explain and be prepared to demonstrate.

In general, ΔABC will not translate onto $\Delta A1B1C1$ because a reflection changes the orientation of the image compared to the pre-image. Special cases, such as equilateral triangles, may seem to work, but this is not true for all shapes.

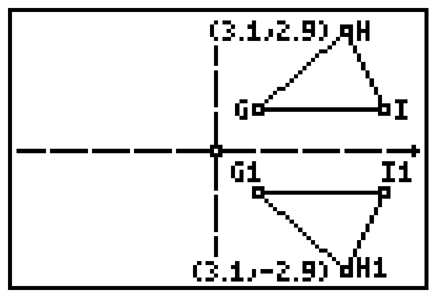


Part II: Reflections in the Coordinate Plane

Answers to Questions and Conjectures

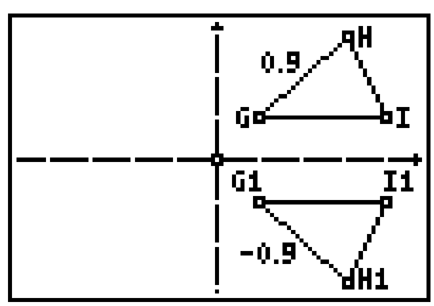
1. Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and those of its image when the triangle has been reflected across the **x-axis**.

When ΔGHI is reflected over the x-axis to form its image $\Delta G1H1I1$, each pair of corresponding points of the pre-image and image have opposite y-coordinates and the same x-coordinates. The coordinates of corresponding points are represented by the general mapping $(x, y) \rightarrow (x, \text{-}y)$. Since reflections across a line are perpendicular to the line, when the reflection line is the x-axis, the x-coordinates remain unchanged. Since line reflections cause corresponding points to be equidistant from the reflection line, when the reflection line is the x-axis, the y-coordinates reverse signs, but maintain their absolute values.



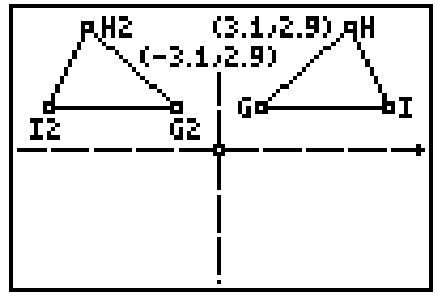
2. Is there a relationship between the slopes of the sides of the pre-image triangle and those of the image triangle when the pre-image is reflected across the **x-axis**? Explain your reasoning.

The sign of the slope changes when a segment is reflected over the x-axis. This is true except for the special cases of horizontal and vertical sides of a triangle. This can be shown to be true by considering the slope formula and the relationship between $(y_2 - y_1)$ and $((\text{-}y_2) - (\text{-}y_1))$.



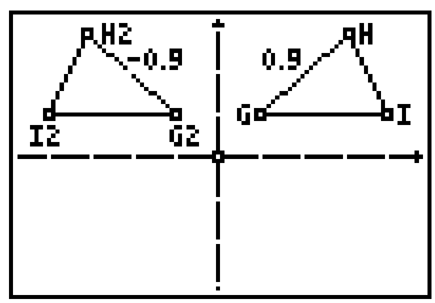
3. Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and those of its image when the triangle has been reflected across the **y-axis**.

   When ΔGHI is reflected over the y-axis to form its image, each pair of corresponding points of the pre-image and image have opposite x-coordinates and the same y-coordinates. The coordinates of corresponding points are represented by the general mapping $(x, y) \rightarrow (^{-}x, y)$. Since reflections across a line are perpendicular to the line, when the reflection line is the y-axis, the y-coordinates remain unchanged. Since line reflections cause corresponding points to be equidistant from the reflection line, when the reflection line is the y-axis, the x-coordinates reverse signs, but maintain their absolute values.



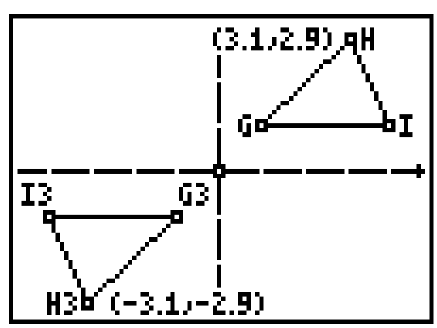
4. Is there a relationship between the slopes of the sides of the pre-image triangle and those of the image triangle when the pre-image is reflected across the **y-axis**? Explain your reasoning.

   The sign of the slope changes when a segment is reflected over the y-axis. This is true except for the special cases of horizontal and vertical sides of a triangle. This can be shown to be true by considering the slope formula and the relationship between $(x_2 - x_1)$ and $((^{-}x_2) - (^{-}x_1))$.



5. Make a conjecture about how a composition of reflections works. Does the order of the two reflections make a difference?

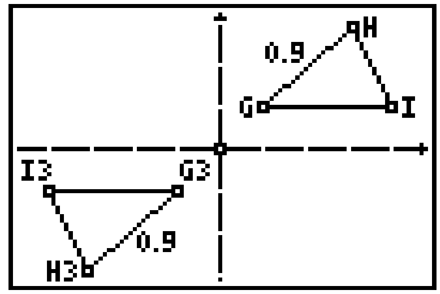
   When ΔGHI is reflected over both axes (a composition of reflections), each pair of corresponding points of the pre-image and image have opposite coordinates. The coordinates of corresponding points are represented by the general mapping $(x, y) \rightarrow (^{-}x, ^{-}y)$. A composition of reflections over both the x- and y-axis is also referred to as a *reflection through the origin*.



   The **Symmetry** tool on the Transformation Menu will reflect an object through a point in the same way an object is transformed by a composition of reflections over perpendicular lines.

6. Can a composition of reflections be done using a single reflection? Explain your reasoning and be prepared to demonstrate.

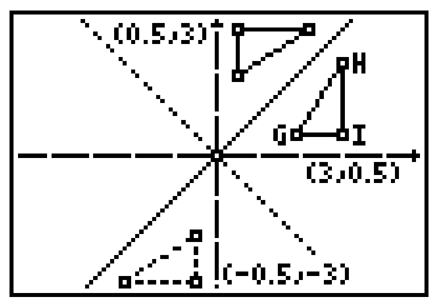
These two reflections create an image for which the slopes of the corresponding sides equal the slopes of the original triangle. For a general triangle, the reflection across a single line would not produce matching images because the orientation is reversed. In general, it takes an even number of reflections to produce an image having the same orientation as the pre-image. There are special cases (appropriately aligned isosceles or equilateral triangles) that will work.



Answers to the Extension

Investigate reflections of a triangle across the lines $y = x$ and $y = {}^-x$. (You can construct these lines by bisecting the angles formed by the intersection of the coordinate axes.) Make a conjecture about the relationship between the coordinates of corresponding vertices of a triangle and its image. Additionally, make a conjecture about the relationship between the slopes of the corresponding sides in the pre-image and image.

Reflections over the line $y = x$ are represented by the mapping $(x, y) \rightarrow (y, x)$ where the coordinates of the point are reversed. Reflections over the line $y = {}^-x$ are represented by the mapping $(x, y) \rightarrow ({}^-y, {}^-x)$.



For a reflection over the lines $y = x$ and $y = {}^-x$, the slopes of the reflected segments are reciprocals, hence the product of the slopes is 1. This will be true except for the special cases when the side of the triangle is horizontal or vertical.