EXPLORATIONS ™

Activity 6

Objectives

- To use the **Translation** tool on the Cabri® Jr. application
- To investigate the properties of a translation
- To extend the concepts of translations to the coordinate plane

Cabri® Jr. Tools

Translations in the Plane

Introduction

Transformational geometry is an important branch of geometry. The most basic transformation is a translation. In this activity, you will perform a translation using the Cabri Jr. application and explore the properties of translations. In the second part of this activity, you will extend the concept of translations to the coordinate plane.

This activity makes use of the following definitions:

Translation — a transformation that moves a figure a given number of units in a given direction

Pre-image — the original object that is to be transformed

Image — the new object created by applying the conditions of a transformation

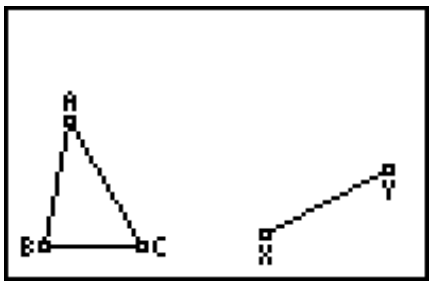
Part I: Properties of a Translation

Construction

I. Construct a triangle and a translation segment.

Draw scalene triangle ΔABC on the left side of the screen.

Draw $\overline{XY}$ on the right side of the screen.



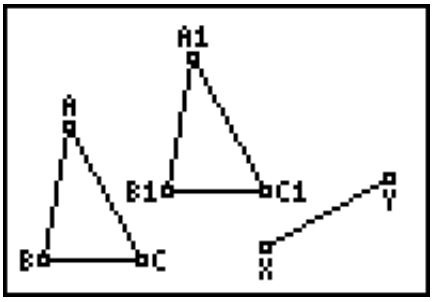
II. Translate the triangle.

To use the **Translation** tool you must do the following:

- Select the item to be translated.
- Select a segment or select two points representing a segment that will determine the length and direction the item will be translated.

Translate ΔABC the length of $\overline{XY}$. Label the vertices of the image triangle $A1$, $B1$ and $C1$ where $A1$ is the image of vertex A, $B1$ is the image of vertex B, and $C1$ is the image of vertex C.

Note: *If part or all of the image does not appear on the screen, try shortening* $\overline{XY}$ *, or making* ΔABC *smaller until the image appears on the screen.*

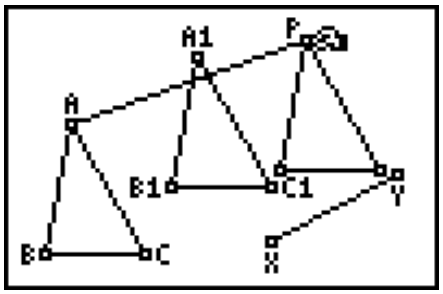


Exploration

Observe how the image (the new triangle) changes when you drag the vertices and sides of ΔABC, and ΔABC itself.

Observe how the image changes when you drag $\overline{XY}$ and when you drag one of the endpoints of $\overline{XY}$.

Construct a new segment, $\overline{AP}$, and translate ΔABC using $\overline{AP}$. Move point P until the image of the triangle created using $\overline{AP}$ aligns with $\Delta A1B1C1$. Use various measurement tools to investigate the relationship between $\overline{AP}$ and $\overline{XY}$. Be sure to investigate this relationship for different triangles ΔABC and different segments $\overline{XY}$.



Questions and Conjectures

1. Make a conjecture about how the **Translation** tool determines the direction of the translation. Explain how you would test your conjecture.
2. Is the translation segment unique? Explain.
3. Make a list of the properties of the pre-image that are retained in the image. Use various measurement tools (**Distance and Length**, **Angle** and **Slope**) to verify your answers.

Part II: Translations in the Coordinate Plane

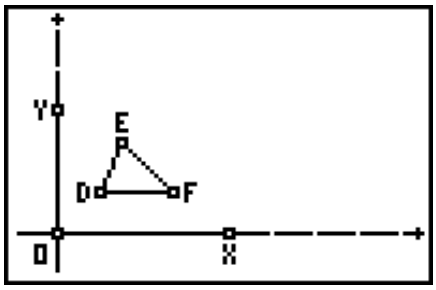
Construction

Draw a triangle to translate in the coordinate plane.

Clear the previous construction.

Show the axes on the screen. Drag the origin to the lower left part of the screen.

Draw scalene triangle ΔDEF in the first quadrant.



Construct $\overline{OX}$ having its first endpoint (O) on the origin and the second endpoint (X) on the positive x-axis.

Construct $\overline{OY}$ having its first endpoint (O) on the origin and the second endpoint (Y) on the positive y-axis.

Exploration

Translate ΔDEF using $\overline{OX}$ and observe the relationship between the coordinates of the vertices of ΔDEF, the corresponding vertices of its image, and the length of segment $\overline{OX}$. Be sure to drag a side and vertex of ΔDEF and point X to verify the relationships you believe exist.

Translate ΔDEF using $\overline{OY}$ and observe the relationship between the coordinates of the vertices of ΔDEF, the corresponding vertices of its image, and the length of segment $\overline{OY}$. Be sure to drag a side and vertex of ΔDEF and point Y to verify the relationships you believe exist.

Translate ΔDEF using $\overline{OX}$ and then translate the resulting image using $\overline{OY}$. Observe the relationship that exists between the coordinates of corresponding vertices of the three triangles and the lengths of the segments $\overline{OX}$ and $\overline{OY}$. Be sure to drag a side and vertex of ΔDEF and the points X and Y.

Questions and Conjectures

1. Make a conjecture about the relationship between the coordinates of corresponding vertices of a pre-image and its image when the triangle has been translated by a segment constructed from the origin to a point on the positive x-axis.
2. Describe the significance of the construction of $\overline{OX}$ to your conjecture. Is this the only segment that would have given the same results to the translation?
3. Make a conjecture about the relationship between the coordinates of corresponding vertices of a pre-image and its image when the triangle has been translated by a segment constructed from the origin to a point on the positive y-axis.

4. Describe the significance of the construction of $\overline{OY}$ to your conjecture. Is this the only segment that would have given the same results to the translation?
5. Make a conjecture about how a composition of translations works. Does the order of the two translations make a difference?
6. Can a composition of translations be accomplished using a single translation? Explain and be prepared to demonstrate.

Teacher Notes

Activity 6

Translations in the Plane

Objectives

- To use the **Translation** tool on the Cabri® Jr. application
- To investigate the properties of a translation
- To extend the concepts of translations to the coordinate plane

Cabri® Jr. Tools

Additional Information

A great way to summarize this activity would be to develop a working definition for translation as a whole class activity.

Challenge your students to show that given a triangle ΔGHI, translating a pre-image using side $\overline{GH}$ and then translating the image formed using $\overline{HI}$ is equivalent to translating the pre-image using side $\overline{GI}$. This is a very important idea in the study of vectors. Sides $\overline{GH}$ and $\overline{HI}$ are considered component vectors of vector $\overline{GI}$. In physics, vectors representing force, displacement, and velocity are often broken down into vertical and horizontal components.

In physics, vectors representing force, displacement, and velocity are often broken down into vertical and horizontal components represented by a right triangle having the original vector as the hypotenuse.

Part I: Properties of a Translation

Answers to Questions and Conjectures

1. Make a conjecture about how the Translation tool determines the direction of the translation. Explain how you would test your conjecture.

   The direction that the translation segment was constructed controls the direction of the translation. Imagine that the translation segment is a vector with direction and magnitude. The first point of the segment that is constructed is considered the tail of the vector and the second endpoint the head. This could be tested by performing the translation in the following way: select the individual endpoints of the segment instead of the segment itself when using the **Translation** tool.

2. Is the translation segment unique? Explain your reasoning.

   The students should discover that a given translation can be reproduced using different translation segments. This is demonstrated in this activity when $\overline{AP}$ is used to translate ΔABC onto $\Delta A1B1C1$ as well as when segment $\overline{XY}$ is moved around the screen.

3. Make a list of the properties of the pre-image that are retained in the image. Use various measurement tools (**Distance and Length**, **Angle** and **Slope**) to verify your answers.

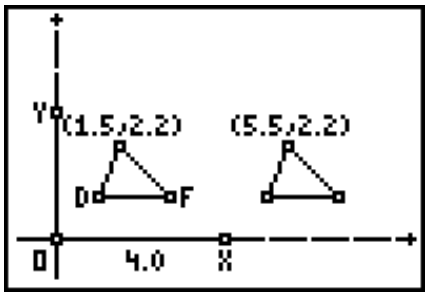
   The list of properties preserved by a translation includes side length, angle measure, slope of sides (rotational angle), area, perimeter, and orientation. The issue of orientation may not come up with students if they have not studied reflections before studying translations.

Part II: Translations in the Coordinate Plane

Answers to Questions and Conjectures

1. Make a conjecture about the relationship between the coordinates of corresponding vertices of a pre-image and its image when the triangle has been translated by a segment constructed from the origin to a point on the positive *x*-axis.

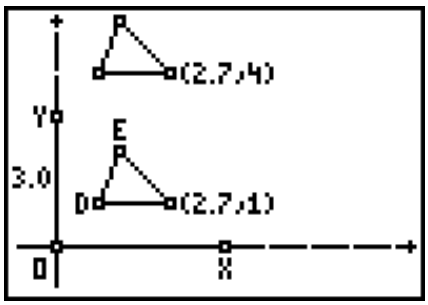
   In the Cabri® Jr. application, there is a one-to-one correspondence between length measure and the coordinate system. The figure shows a translation segment that is 4 units long on the *x*-axis. This segment translated the pre-image 4 units horizontally causing a change of 4 units in the *x*-coordinate of corresponding vertices. Also note that there is no change in the *y*-coordinate since the translation is only in the horizontal direction.



2. Describe the significance of the construction of $\overline{OX}$ to your conjecture. Is this the only segment that would have given the same results to the translation?

   Any horizontal segment constructed with its left endpoint drawn first and having the appropriate length will satisfy the above conjecture.

3. Make a conjecture about the relationship between the coordinates of corresponding vertices of a pre-image and its image when the triangle has been translated by a segment constructed from the origin to a point on the positive *y*-axis.

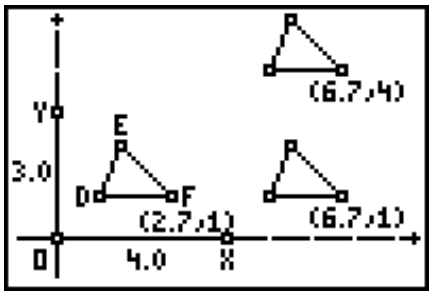


In the Cabri Jr. application, there is a one-to-one correspondence between length measure and the coordinate system. The figure shows a translation segment that is 3 units long on the *y*-axis. This segment translated the pre-image 3 units vertically, causing a change of 3 units in the *y*-coordinate of corresponding vertices. Also note that there is no change in the *x*-coordinate, since the translation is only in the vertical direction.

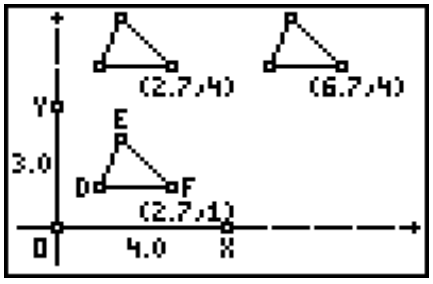
4. Describe the significance of the construction of $\overline{OY}$ to your conjecture. Is this the only segment that would have given the same results to the translation?

Any vertical segment constructed with its lowest endpoint drawn first and having the appropriate length will satisfy the above conjecture.

5. Make a conjecture about how a composition of translations works. Does the order of the two translations make a difference?



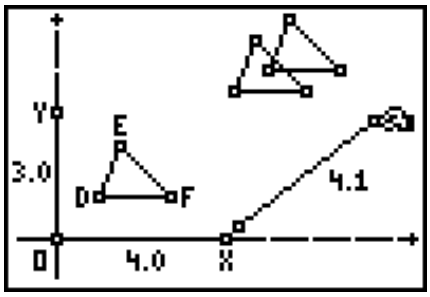
This composition of translations is the result of a translation of the pre-image by the segment on the *x*-axis followed by a translation of the first image by a segment on the *y*-axis. The combined double translation changes both the *x*- and *y*-coordinates by an amount equal to the length of the corresponding translation segments in the direction the segments were constructed.



Reversing the order of the translations produces the same final image with a different intermediate image.

6. Can a composition of translations be accomplished using a single translation? Explain and be prepared to demonstrate.

This composition of horizontal and vertical translations could be done with one translation. Draw an "unattached" translation vector on the screen and use it to translate the pre-image triangle. Drag the terminal end of the translation segment until the new image aligns with the previous final image.



Drag the segment by its body until the initial point is on the origin. In this position the new translational vector represents the diagonal of the original translational vectors. The new translational vector represents the sum or resultant of the original vectors.