

Activity 1

I Can Guess Your
Numbers

Teacher Notes

Concepts

- ◆ Number Theory
- ◆ Divisibility Rules
- ◆ Prime Factorization
- ◆ Exponents
- ◆ Problem-Solving

Calculator Skills

- ◆ Division
- ◆ Multiplication
- ◆ Algorithms to test for divisibility by 7, 11, and 13

Materials

- ◆ TI-30X IIS
- ◆ Student Activity pages (p. 8-9)
- ◆ Transparency

Objective

- ◆ Students will use the calculator as a tool to improve number sense, analysis, and reasoning.

Topics Covered

- ◆ Investigating mathematics content in problem-solving
- ◆ Integrating mathematics with problem-solving strategies
- ◆ Formulating mathematical generalizations discovered through investigations

Introduction

Many of us have encountered situations in which mathematics is “just a set of rules” that never seem to apply to anything outside the mathematics classroom. While rules are important, the context in which they are applied seems even more important. Exploratory activities, experiments, and projects may require descriptive accounts of what is observed and informal arguments based on evidence gathered. In this activity, students are given the product of six numbers between 10 and 20. They will use mathematics rules and experimentation to find the six numbers and justify their results.

Investigation

1. Announce to the students that you will be able to find any six numbers between 10 and 20, given the product of the six numbers. Explain to the students that you are going to step outside the classroom door while they work as a class to choose six numbers between 10 and 20, and determine the product. When you return, you will enter the product on the calculator and determine the six numbers.
2. Give the class time to decide upon six numbers between 10 and 20, and determine the product. (Note: The numbers do not have to be unique, but do not tell the students unless they ask!) One student should serve as the spokesperson for the class to tell you when to return to the classroom.
3. Have the spokesperson for the class write the product of the six numbers on the board.

Each student will enter the product in his or her calculator. You will enter the product on the overhead calculator.

4. Put the transparency on the overhead and review the divisibility rules. Present the algorithms to test for divisibility by 7, 11, and 13 from the student instructions page.
5. Ask the students to use the divisibility rules to tell you what number to divide by on the calculator. Suggest that they do the division on their calculators and record their results as you go.

Suppose the class poses 12,640,320 as the product of their six numbers.

Since the number is even, divide by 2 on the overhead calculator and record the factor '2' on the Student Activity page, out of the view of the students. The number now displayed on the calculator is 6,320,160.

Divide by 2 on the calculator again, and record another factor 2 on the Student Activity page. The number now displayed on the calculator is 3,160,080. Repeat the process above, and record another factor 2 on the page. The number now displayed is 1,580,040. Continue dividing by 2 until the number displayed is 197,505.

The one's place is now a 5, so you may now divide by 5. Record the factor 5 on the record sheet, and note that the number 39,501 is now displayed.

Since the sum of the digits is $3 + 9 + 5 + 0 + 1 = 18$, the number is divisible by 9. Record the factor 3 on the sheet two times, and divide the number by 9 on the calculator. The number now displayed is 4,389. Since the sum of the digits is $4 + 3 + 8 + 9 = 24$, the number is divisible by 3. Divide by 3 and record a factor 3 on the sheet. The number now displayed is 1,463.

Now start at the one's place and add alternate digits moving from right to left. That is, $3 + 4 = 7$. Add the remaining digits starting at the ten's place and moving from right to left. That is, $6 + 1 = 7$. Since the difference between the two sums is zero, the number is divisible by 11.

(See Student Instructions page.) Divide 1,463 by 11 on the calculator and record the 11 as a factor on the sheet. The resulting number is now displayed as 133.

Use the algorithm to show that 133 is divisible by 7. (See Student Instructions page.) Record the factor 7, and observe that the number displayed is 19. Since 19 is a prime number, record it as the final factor on the sheet.

6. Record the resulting factorization of the original product out of the view of the students:

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 3 \times 3 \times 3 \times 11 \times 7 \times 19$$

Mathematical Reasoning

Reason as outlined below and write the six numbers on the board as they are “discovered”.

1. Two of the numbers between 10 and 20 are 11 and 19.
2. Since $2 \times 7 = 14$ and 14 is the only number between 10 and 20 with a factor of 7, another number is 14.
3. Since 15 is the only number between 10 and 20 with a factor of 5 and $3 \times 5 = 15$, another number is 15.
4. This leaves $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$. Since $18 = 2 \times 3 \times 3$, another number is 18.
5. The remaining factors $2 \times 2 \times 2 \times 2 = 16$, and thus the final number is 16.

You can now show the check of the six numbers by multiplying $11 \times 19 \times 14 \times 15 \times 18 \times 16$ on the calculator to get 12,640,320.

Mathematical Connections

Have the students analyze how you are able to find the six numbers. When the students feel comfortable with this, give the students the Student Activity page for practice. Let them use the calculator to bypass laborious computation. Remember, the emphasis is upon analysis and reasoning.

Wrap-Up

Have the students work in groups of four to find a product of six numbers between 10 and 20. Groups then should exchange products to see if they can find the six numbers. Students should write justifications for their solutions.

Extensions

- ◆ Students can work in groups of four to find calculator short cuts for divisibility by 7, 11, and 13 and display them on the overhead calculator.
- ◆ Students may write proofs for the algorithms for divisibility by 7 and 13.

Solutions

1. The product of the six numbers is 7,759,752.

Possible factors of the number	Number of times each factor is used
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: 11, 12, 13, 14, 17, 19.

2. The product of the six numbers is 15,163,200.

Factors of the Number	Record each factor
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: 13, 15, 15, 16, 18, 18.

3. Find six numbers between 10 and 20 such that the product must contain *at least one* of each of the factors in the table.

Possible factors of the number	Number of times each factor is used
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: 11, 13, 14, 15, 17, 19.
The product is 9,699,690.

Student Instructions

Algorithms to Test for Divisibility by 7, 11, and 13

Test 39,494 for divisibility by 7 as follows:

1. Cross out the one's place digit (4) and double it to get 8. Subtract 8 from the remaining number.

$$\begin{array}{r} 3949 \\ - \quad 8 \\ \hline 3941 \end{array}$$

2. Cross out the one's place digit (1) in the result and double it to get 2. Subtract 2 from the remaining number.

$$\begin{array}{r} 394 \\ - \quad 2 \\ \hline 392 \end{array}$$

3. Cross out the one's place digit (2) in the result and double it to get 4. Subtract 4 from the remaining number.

$$\begin{array}{r} 39 \\ - \quad 4 \\ \hline 35 \end{array}$$

Since 35 is divisible by 7, the number 39,494 is divisible by 7.

Test 1,062,347 for divisibility by 11 as follows:

1. Start at the one's place and underline alternate digits from right to left.

$$\underline{1} \ 0 \ \underline{6} \ 2 \ \underline{3} \ 4$$

2. Find the sum of the underlined digits. $7 + 3 + 6 + 1 = 17$
3. Find the sum of the remaining digits. $4 + 2 + 0 = 6$

The difference between the two sums is 11; therefore the number 1,062,347 is divisible by 11. A number is divisible by 11 if the difference in sums is 0 or 11.

Test 79,495 for divisibility by 13 as follows:

1. Cross out the one's place digit (5) in the result and quadruple it to get 20.
Add 20 to the remaining number.

$$\begin{array}{r} 7949 \\ + 20 \\ \hline 7969 \end{array}$$

2. Cross out the one's place digit (9) in the result and quadruple it to get 36.
Add 36 to the remaining number.

$$\begin{array}{r} 796 \\ + 36 \\ \hline 832 \end{array}$$

3. Cross out the one's place digit (2) in the result and quadruple it to get 8.
Add 8 to the remaining number.

$$\begin{array}{r} 83 \\ + 8 \\ \hline 91 \end{array}$$

4. Cross out the one's place digit (1) in the result and quadruple it to get 4.
Add 4 to the remaining number.

$$\begin{array}{r} 9 \\ + 4 \\ \hline 13 \end{array}$$

Since 13 is divisible by 13, the number 79,495 is divisible by 13.

Student Activity 1

Name _____

Date _____

Making Sense of Data—I Can Guess Your Numbers

Objective: You will use the calculator as a tool to improve number sense, analysis, and reasoning.

1. The product of the six numbers is 7,759,752.

Possible factors of the number	Number of times each factor is used
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: _____, _____, _____, _____, _____, _____.

2. The product of the six numbers is 15,163,200.

Factors of the number	Record each factor
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: _____, _____, _____, _____, _____, _____.

3. Find six numbers between 10 and 20 such that the product must contain *at least one* of each of the factors in the table.

Possible factors of the number	Number of times each factor is used
2	
3	
5	
7	
11	
13	
17	
19	

The six numbers between 10 and 20 are: _____, _____, _____, _____, _____, _____

The product is _____.

TRANSPARENCY

Reviewing Rules for Divisibility

2	The number is even
3	Sum of the digits is divisible by 3
4	Last 2 digits are divisible by 4
5	The number has 0 or 5 in the one's place
6	The number is divisible by 2 and by 3
7	(See Student Instructions for algorithm)
8	Last 3 digits are divisible by 8
9	Sum of the digits is divisible by 9
10	The number has 0 in the one's place
11	(See Student Instructions for algorithm)
12	The number is divisible by 3 and by 4
13	(See Student Instructions for algorithm)