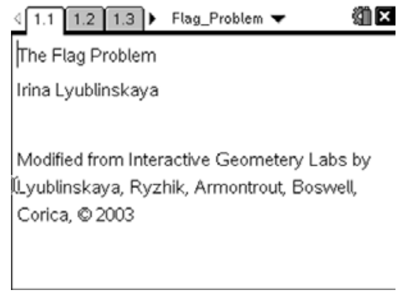
# The Flag Problem

by Dr. Irina Lyublinskaya

## Activity overview




In this activity students explore the area of a triangle with base being one of the legs of a right angled trapezoid and an opposite vertex being a point on the other leg of the trapezoid.
Statement of the problem: *The Mathematics Club at a high school decides to design a new flag for the club office. They want the shape of the flag to be a trapezoid with one leg at right angles to the bases. In the design of the flag, they create a white triangle using the perpendicular leg as a side of the triangle. The remaining triangle vertex is a point on the opposite leg of the trapezoid. The president of the club decides she wants the area of the white triangle to be half the area of the trapezoid. Where should she locate the point on the opposite leg of the trapezoid?*

Teacher is encouraged to have students work through the following stages of this problem.

- visualize the right angled trapezoid with an interior triangle with area ½ of the trapezoid
- describe or draw a diagram of what has been visualized
- investigate the problem with TI-Nspire technology; by this stage students should start to consider how the proof of their conjecture(s) will be approached
- give a proof of the solution to the problem

## Concepts

Key Property: the area of a trapezoid is equal to the height times the length of mid-segment.

## Teacher preparation

*Before carrying out this activity teacher should review with the students the following concepts: trapezoid, base, legs, area of triangle. The screenshots on pages 2-6 demonstrate expected student results. Refer to the screenshots on page 7 for a preview of the student TI-Nspire document (.tns file).*

## Classroom management tips

- *This activity is designed to be* ***teacher-led*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

## TI-Nspire Applications

*Graphs, Geometry, List & Spreadsheets, Notes*

TEXAS INSTRUMENTS

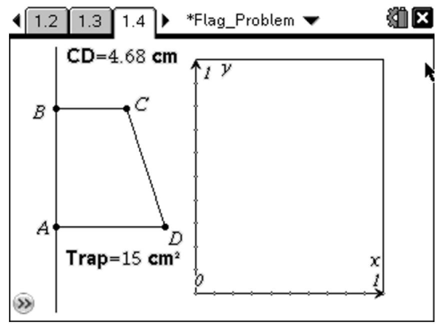
# The Flag Problem

by: Dr. Irina Lyublinskaya
Grade level: secondary
Subject: geometry
Time required: 45 to 90 minutes

Materials: TI-Nspire

Step-by-step directions

## Problem 1 – Right Angled Trapezoid




**Step 1.** Students open file Flag_Problem.tns, read problem statement on pages 1.2 and 1.3 and answer first question.

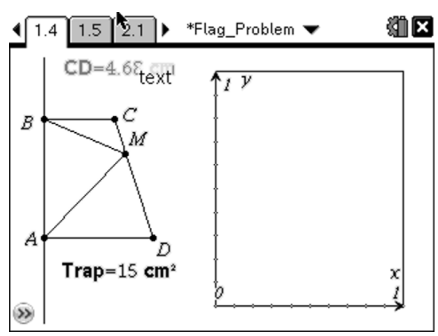
**Q1.** Close your eyes and imagine the trapezoid. Where would you locate the point so that the area of the triangle is ½ the area of the trapezoid?

**A.** The correct answer is: the vertex of the triangle should be placed at the midpoint of the opposite leg.

At this initial stage of the problem many students may offer incorrect answers. They should not be corrected, since they can correct themselves after the exploration using TI-Nspire.

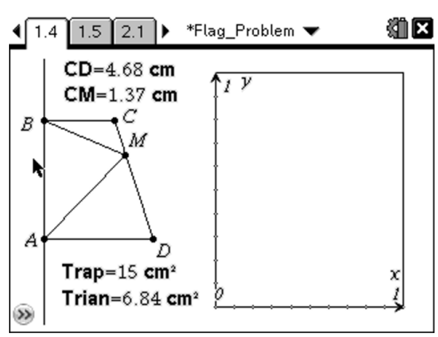
**Step 2.** Students should move to page 1.4 where they will complete constructions and explore the situation.

On this page a right angles trapezoid ABCD is constructed to left of the analytic window. The area of the trapezoid and the length of the leg CD are displayed. Students will construct a triangle with base AB and vertex on leg CD, measure position of the vertex on CD and area of triangle, capture data into data table on page 1.5 and analyze the data using the scatter plot on the page 1.4. This will help them to finalize their conjecture about position of the vertex of the triangle. For less experienced TI-Nspire users these constructions could be pre-made allowing students to do exploration part only. The following are the steps of construction:




**Step 3.** Click on [menu], 6: Points & Lines, 2: Point On. Move cursor over the segment CD and press [click]. The point will be constructed. Label this point M by clicking [menu], 1: Actions, 5: Text.

**Step 4.** Construct triangle ABM by clicking [menu], 8: Shapes, 2: Triangles. Then click on two vertices and press [enter]. The triangle will be constructed.



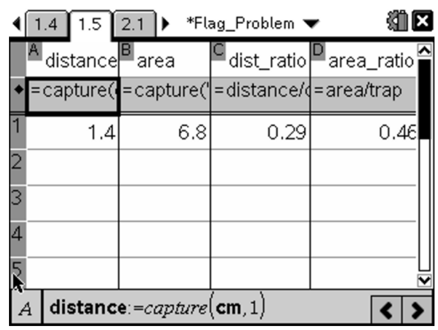

**Step 5.** Measure distance CM and store it as CM. In order to do that click [menu], 7: Measurement, 1: Length. Click on points C and M and move measurement above the trapezoid. Press [enter] to paste the measurement in selected location. Select the measurement so it is shaded and click [ctrl] [sto→ var]. In the open dialogue type CM in place of vars and press [enter].

**Step 6.** Measure area of the triangle ABM and store it as trian. In order to do that click [menu], 7: Measurement, 2: Area. Click on any side of the triangle and move measurement below the trapezoid. Press [enter] to paste the measurement in selected location. Select the measurement so it is shaded and click [ctrl] [sto→ var]. In the open dialogue type trian in place of vars and press [enter].

**Q2.** Drag point M along the segment CD. At what position do you think the area of the triangle is ½ of the area of the trapezoid? Is this different from what you predicted earlier?

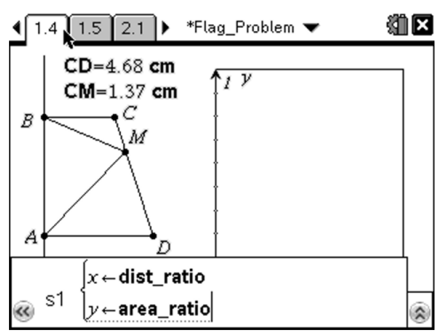
**A.** By moving the point M students should see that the area of triangle is ½ of the area of trapezoid when CM = ½ CD.

In the next part of the activity students will capture the distance CM and the area of the triangle ABM, find the ratios of CM/CD and areas of triangle and trapezoid and plot a scatter plot to confirm their conjecture. The following are the steps of construction:




**Step 7.** Move to Lists & Spreadsheets page 1.5. Move course to the diamond line of the column A. Click [menu], 3: Data, 2: Data Capture, 1: Automated Data Capture. Type CM for the variable and press [enter]. Move course to the diamond line of the column B. Click [menu], 3: Data, 2: Data Capture, 1: Automated Data Capture. Type "trian for the variable and press [enter].

**Step 8.** Move cursor to diamond line of the column C. Define dist_ratio as distance/CD; define area_ratio as area/trap. In order to do that type in the diamond row of the C column = distance/CD; type in the D column = area/trap.




**Step 9.** Plot ratio of areas vs. ratio of distances. In order to do that, move back to page 1.4. Press [ctrl] G to display function entry line. Then click [menu], 3: Graph Type, 4: Scatter Plot to change the type of graph. Press [click] to choose dist_ratio for the x. Choose area_ratio for y. Press [menu] and the point will be displayed on the graph.

**Step 10.** Grab and move point M along the segment CD. The scatter plot will be displayed simultaneously. Click [menu], 5: Trace, 1: Graph Trace. Use NavPad arrows to trace the scatter plot and determine what is the ratio of d/l when ratio of areas is 0.5.

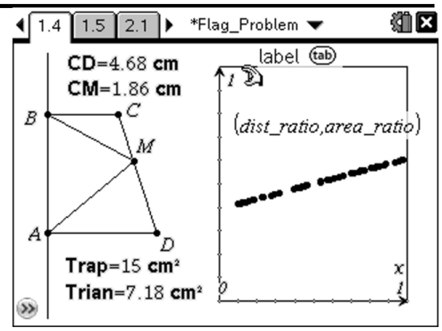
**Q3.** Formulate your final conjecture and prove it.

**A.** The area of the triangle is half of the area of the trapezoid when point M is a midpoint of the segment CD.

Proof: Let M be the midpoint of CD, N be the midpoint of AB.

Construct a line through M perpendicular to AD and BC and let E and F are points of intersections correspondingly. DM = MC, ∠DME = ∠FMC as vertical and ∠DEM = ∠MFC = 90° by construction ⇒ ΔDEM = ΔFMC (by hypotenuse and an angle).

Area of ABCD = area of ABFMD + area of FMC = area of ABFMD + area of DEM = area of ABFE. Since EF ⊥ AE ⇒ EF || AB ⇒ AEFB is a rectangle, so its area = AB·NM. Area of ABM = ½ AB·NM = ½ are of ABCD.




Activity extensions

## Problem 2. Non-Right Trapezoid

**Step 1.** Students read the problem statement on page 2.1, make a conjecture and then move to page 2.2 to explore this situation.

**Q4.** You are given a non-right trapezoid ABCD. Locate a point on the side CD so that the area of the triangle is ½ the area of the trapezoid.

**Step 2.** Measure the area of the trapezoid and store it as variable Trap. Measure the length of the segment CD and store it as variable CD.

**Step 3.** Measure the area of the traingle and store it as variable Trian. Measure the length of the segment CM and store it as variable CM.

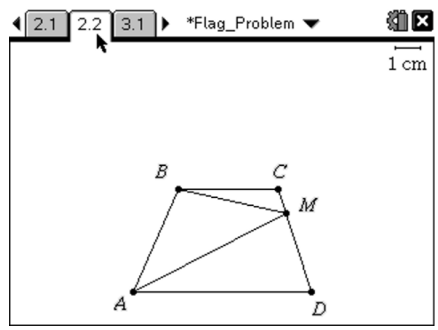
**Step 4.** Move point M along the segment CD to observe at which position of the point M the area of the triangle is equal half of the area of the trapezoid.

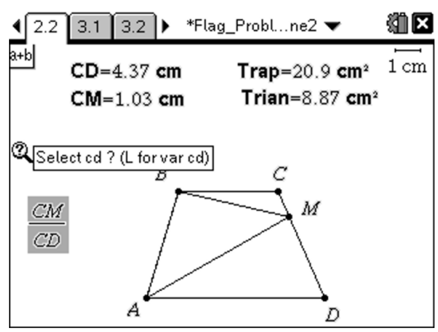
**A.** The point should be located in the middle of the segment CD.

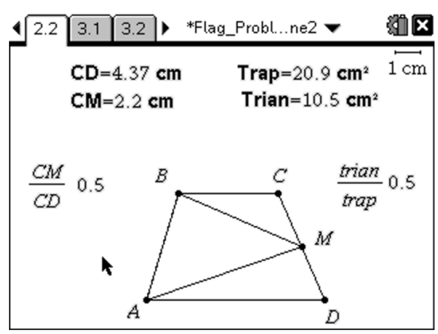
**Step 5.** Students could use calculations to confirm their prediction. In order to do that click (menu), 1: Actions, 5: Text and type in the open text box CM/CD. Click (menu), 1: Actions, 7: Calculate and click on the expression. When asked, move cursor of the variable value and press (enter).

**Step 6.** Repeat step 5 to find ratio of triangle area to trapezoid area. Adjust position of the point M so that the ratio of areas is ½ and observe the ratio of distances.

**Step 7.** Drag vertices of the trapezoid to confirm that this



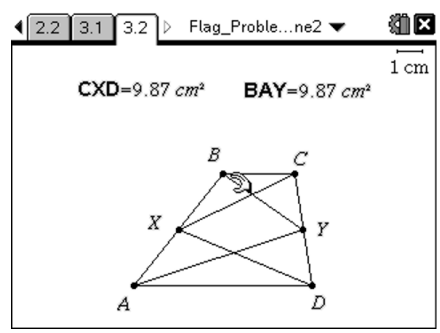



relationship holds true for different trapezoids.

Proof: a) prove that if M is midpoint of CD, area of ABM is ½ of the area of ABCD. Let M be the midpoint of CD. Extend lines AM and BC until they intersect at point L. Since ∠DAM = ∠MLC, ∠DMA = ∠CML and DM = MC ⇒ΔADM = ΔMCL, so their areas are equal. Then, area of ABCD = area of ABCM + area of ADM = area of ABCM + area of MCL = area of ABL. From congruence of triangles AM = ML, so BM is the median in ABL ⇒ area of ABM = area of BML = ½ ABL ⇒ area of ABM = ½ area of ABCD.

b) Prove that if area of a triangle ABN is ½ of the area of ABCD, then N is a midpoint of CD. Let N is a point on CD such that area ABN = ½ area ABCD. Let M is the midpoint of CD, then area AMB = ½ area ABCD as proved above. Then, area ANB = area AMB. Since both triangles have common base and equal areas, they should have equal altitudes to side AB. Then, distance from point N to AB is equal to the distance from M to AB ⇒ NM || AB, which is contradictory to the condition that ABCD is a trapezoid. Then, N = M.

## Problem 3. Area of Triangles




**Step 1.** Students read the problem statement on page 3.1, make a conjecture and then move to page 3.2 to explore this situation.

**Q5.** Let ABCD be a trapezoid with BC || AD. X and Y are the midpoints of AB and CD respectively. Compare the areas of triangles BAY and CDX.

**Step 2.** Construct midpoints on sides AB and CD by selecting (menu), 8: Construction, 5: Midpoint and clicking on each segment. Label the points X and Y.

**Step 3.** Construct triangles ABY and CDX and measure their areas.

**A.** The areas of these two triangles are equal.

**Step 4.** Drag vertices of the trapezoid to confirm that this relationship holds true for different trapezoids.

Proof: The proof follows immediately from the main problem. Since Y is the midpoint of CD, area of BAY = ½ area of ABCD. Since X is the midpoint of AB, area of CDX = ½ area of ABCD. Then, area of ABY = area of CDX.

# The Flag Problem

by: Dr. Irina Lyublinskaya

Grade level: 9-12
Subject: geometry
Time required: 45 to 90 minutes

Materials: TI-Nspire

## Student TI-Nspire Document

*Flag_problem.tns.*