Advanced Algebra

with the

TI-89

Sample Activity: Exploration 2

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EXPLORATION 2 FROM TABLES TO GRAPHS

Annual income twenty pounds, annual expenditure nineteen nineteen six, result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery.
— Charles Dickens 1851

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Broom Hilda has discovered too late the power of compound interest. One dollar invested at an annual interest rate of 3% grows according to the table shown on the right, that is,

After one year, the dollar has accumulated 3¢ interest, so the investment has grown to a value of $1.03.

In the second year, the entire $1.03 earns interest (not just the original $1.00 invested) and so the investment has grown to $1.03 plus the interest on $1.03. So the total value is $1.03 + (.03)($1.03). Applying the distributive law, we express this as $$(1.03)^2$.

Each year the investment grows to 1.03 times its value at the end of the previous year, so the value at the end of three years is $$(1.03)^3$.

In general, the value at the end of the n^{th} year is $$(1.03)^n$, so the value at the end of the 1500^{th} year is $$(1.03)^{1500}$.

Number of Years, n	Value at the End of n^{th} Year
1	1.03
2	$(1.03)^2$
3	$(1.03)^3$
• Then follows 1500 years of compounding •	
1500	$(1.03)^{1500}$

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WORKED EXAMPLE

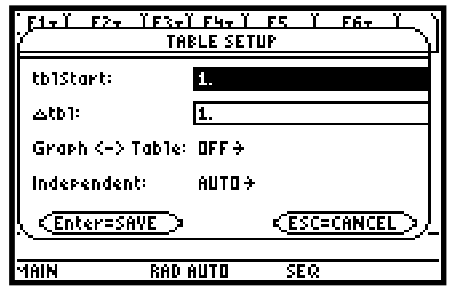
a) Create a table showing the value of Broom Hilda's $10 investment at the end of the n^{th} year for $n = 1$ to 100, if the investment accumulated interest at 3% per year. Use your table to determine how many years it took for the investment to double.

b) Graph the value of the investment during the first 100 years. Trace along the curve to determine how many years it took for the investment to grow to 5 times its original value. What would be the value of the investment after 1500 years?

SOLUTION

a) The value of the investment at the end of the n^{th} year is $10(1.03)^n$. We must make a table of the function $u1(n) = 10(1.03)^n$, for positive integral values of n. To do this, we must first select SEQUENCE mode by pressing MODE . Then adjust the MODE settings so they match the display at the top of page 9.

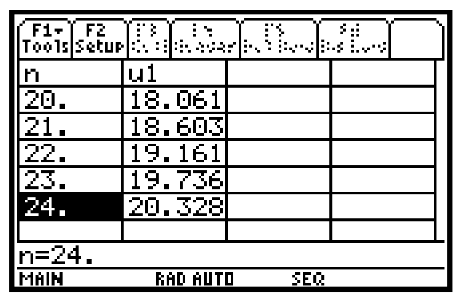
To set the initial value of n (i.e. $n = 1$) and the incrementing value for n (i.e., $\Delta tbl = 1$), press: ◆ [TblSet]. Then use your cursor keys to move through the fields in the TABLE SETUP and enter the values as in the display shown here.



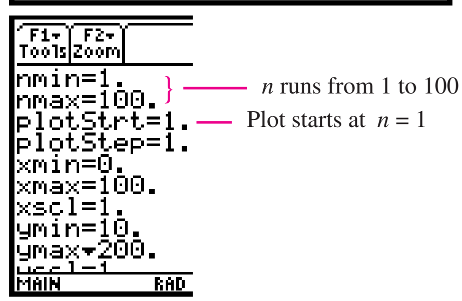
To define $u1=10(1.03)^n$, we press these keys: ◆ [Y=].

We then enter: 1 0 (1 . 0 3) ^ alpha N ENTER

To generate the table we enter ◆ [TABLE]. At first, the table displays the values of u1(n) for $n = 1$ to 5. We use the cursor key to move down through the values of n until the u1(n) column entry exceeds $20, (i.e., the $10 investment is doubled). We obtain the display shown on the right. The value 20.328 opposite $n = 24$ indicates that the investment doubled to $20.33 in about 24 years.

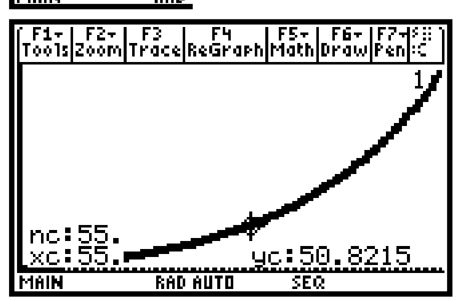


b) To graph the value of the investment over its first 100 years, we press ◆ [WINDOW] and set the window variables to the values shown in the display.



Then we press: ◆ [GRAPH].

To trace along the graph, we press: F3 ▶ and we continue to press the cursor key until the cursor reaches a point with y-coordinate (i.e., yc) greater than 50. The display shows that this occurs when $n = 55$. That is, the investment of $10 grows to 5 times its original size in about 55 years.



The accumulated value of $10 @ 3% per annum over 1500 years is $10(1.03)^{1500}$ or about $180 000 000 000 000 000 000. This is more than all the money in the world today!
Smoking is very expensive.

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Exercises & Investigations

❶. Graph u1(n) for $1 \le n \le 10$ where u1(n) is given by:
a) 3n b) 5n – 20 c) $n^2 \div 3$ d) $\sqrt{80n}$

❷. a) Explain why an investment of $1 compounded at a rate of *i*% per annum for *n* years yields $(1+\frac{i}{100})^n$ dollars.
b) Write an expression for the value of $*P* after *n* years of compounding at a rate of *i*% per annum.

❸. a) Access the command **fPart(** by pressing CATALOG and scrolling. Create a table of values for u1(n) = **fPart**(n/7) for n = 1 to 28. Explain any pattern you see.
b) What does the command **fPart(** do?
c) Graph u1(n) for n = 1 to 28. What pattern do you see?
d) Use your pattern to predict the value of u1(40).

❹. a) Create a table of values for 7^n for $n = 1$ to 8.
b) Graph 7^n for $n = 1$ to 8. Trace along your graph to find the value of 7^6.

❺. a) An investment of $10 000 which earns *simple interest* at the rate of 8% per annum earns $800 interest every year. Create a table which shows the value u1(n) of the investment at the end of the nth year for n =1 to 20.

b) To the table you created in part a) add another column showing the value u2(n) of the investment at the end of the nth year for n =1 to 20 when interest is compounded annually.

c) How much greater is the value of the investment at the end of 20 years when the interest is compounded?

d) Graph u1(n) and u2(n) for n = 1 to 20. Trace along the graph to find the values of u1(n) and u2(n) when n = 10. Compare the graphs of u1(n) and u2(n) as n increases.

❻. Set the window variables to: $0 \le x \le 10$; $0 \le y \le 70$. Graph u1(n) where u1(n) is given by:
a) u1(n) = u1(n–1) + 5 ← This means that the nth term is
u(1) = 1 5 more than the (n–1)th term.

b) u1(n) = 2u1(n–1) ← This means that the nth term
u(1) = 2 is double the (n–1)th term.

Trace along each graph to find the value of u1(6) in each case.

c) Graph u1(n) where $u1(n) = \frac{1}{u1(n-1)} + 1$ and u1(1) =1, Graph in the range $0 \le x \le 20$; $0 \le y \le 2$, for n = 1 to 20. Trace along the graph to evaluate u1(20). Compare your value of u1(20) with the value of $\frac{1+\sqrt{5}}{2}$. Describe what you discover.

d) Graph u2(n) where u2(n) = u2(n–1) + u2(n–2) and u2(-1)=1 and u2(0)= 1.
Enter ui2 = {1,1} to indicate the two initial values. Trace along the graph to find the value of u2(10). Explain how you can calculate any term of this sequence if you know the two previous terms. Do you know the name of this famous sequence?

> To graph u2(n) and not u1(n), highlight u1(n) and press key F4 to de-select u1(n) .

e) Graph u3(n) where $u3(n) = \frac{u2(n-1)}{u2(n-2)}$ and ui3 = {1,1}. Graph in the range $0 \le x \le 20$; $0 \le y \le 2$, for n = 1 to 20. Trace along your graph to evaluate u3(10). Compare with your answer in part c). Explain what you discover.

The Rule of 72

Proceed as in the *Worked Example* to create a table of values for u1(n) = $(1.12)^n$ for n = 1, 2, 3, … 18. Use your table to determine how long it takes an investment to double if it increases in value at 12% per annum.

Is the woman in the cartoon correct?

Repeat the procedure above to find the time it takes for the investment to double for compounding rates of 3%, 6%, 8%, 12%, 18% and 24%. Describe a simple rule for estimating the doubling time for any compounding rate.

Press MODE and select FUNCTION mode.

Press: ◆ [Y=] and enter the function $Y_1(x) = (1.12)^x$. Graph $Y_1(x)$ in the domain $0 \le x \le 18$ for $0 \le y \le 4$. Graph $Y_2(x) = 2$. Trace to the point of intersection to check the doubling time you found above.

Solutions to the Selected Exercises

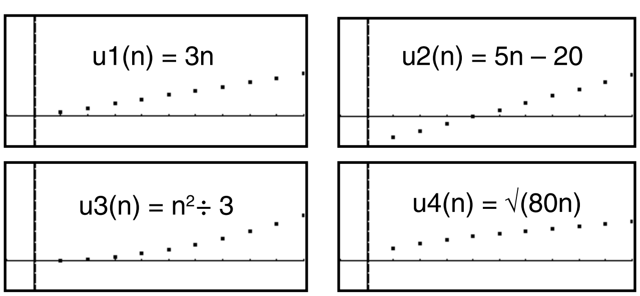
Exploration 2

1. We set the window variables as shown in the left display, and define u1, u2, u3 and u4 as shown in the display on the right.

```
nmin=1.
nmax=10.
plotstrt=1.
plotstep=1.
xmin=-1.
xmax=10.
xscl=1.
ymin=-20.
ymax=70.
yscl=1.
```

```
PLOTS
 u1=3·n
 ui1=
 u2=5·n-20
 ui2=
 u3=n²/3
 ui3=
✓u4=√(80·n)
 ui4=
u4(n)=√(80*n)
```

When we select each of u1 through u4 in turn (by pressing F4) and graph them, we obtain these graphs.



2 a) The value of an investment of $1 growing at a rate of i% per annum increases by a factor of $(1 + \frac{i}{100})$ each year. Therefore to find its value after n years, we multiply it by $(1 + \frac{i}{100})$ n times.

b) $ $P\,(1 + \frac{i}{100})^n$

3. a) We define u1(n) = fPart(0.142857*n) and obtain this tabl

n	u1
3.	.42857
4.	.57143
5.	.71429
6.	.85714
7.	0.

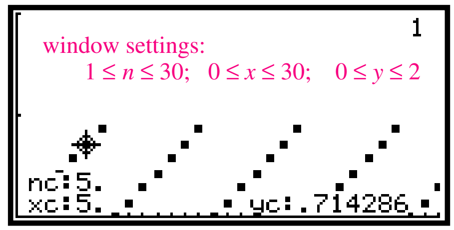
n=7.

As we scroll down the table, we discover that the numbers run throug a periodic cycle; i.e. they repeat ever seven numbers.

b) fPart(x) returns the fractional par of the number x.

c) The periodic pattern described i part a) is evident in the graph belov

d) u1(40) = u1(33) = …=u1(5). Fro the graph, we see u1(5) = 0.71428



5. a &b) The value of the investment after n years of simple intere is u1(n) = 10000 + 800n. The value of the investment after n years of compound interest is given by u2(n) = 10000(1.08)ⁿ. The values of u1(20) and u2(20) are shown in the bottom row of the table.

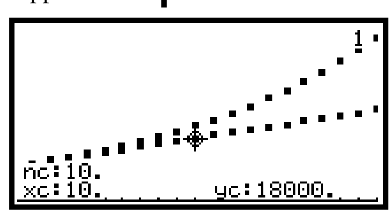
n	u1	u2
16.	22800.	34259.
17.	23600.	37000.
18.	24400.	39960.
19.	25200.	43157.
20.	26000.	46610.

n=20.

c) We see that 8% simple interest yields $26,000 after 20 years, while 8% compounded interest yields $46,610 showing a difference of about $20, 610! That's why the only person who is likely to give you a simple interest loan is a close relative.

d) When we set the window variables as shown here, and then graph u1(n) and u2(n), we obtain the graph shown in the display below. When we move the cursor along the lower curve to xc: 10, we see that the value of the investment under simple interest is $18,000 after 10 years. Moving the cursor upward onto the upper curve reveals that the value under compounding is $21,589.25 after 10 years. We observe that the gap increases as n increases.

```
nmin=1.
nmax=20.
plotstrt=1.
plotstep=1.
xmin=0.
xmax=20.
xscl=1.
ymin=0.
ymax=50000.
yscl=1.
```



6. a) u1(6) = 26 b) u1(6) = 64 c) From the table or grap u1(20) =1.618033998…The golden ratio ≈ 1.6180339

d) u2(n) is the famous Fibonacci sequence, 1, 1, 2, 5, 8, … i which each term is the sum of the two previous terms. u2(10) = 5

e) u3(10) = 1.61905. The ratios of successive terms of the Fibonac sequence converge to the golden ratio.