Grade level: Secondary
Subject: Calculus
Time required: 30 to 45 minutes

Introduction to the Integral Test

by – Sean Turkington

Activity overview

This activity introduces the Integral Test for series convergence by utilizing the built-in CAS capabilities of the TI-Nspire CAS. Students are first told the conditions under which the Integral Test is applicable and the conclusions that can be drawn from it. Students read through an example that details the steps they might follow in using the Integral Test. Students then work through two examples of p-series one of which converges and one of which diverges. A third problem involves a series whose function is cumbersome to integrate by hand. The final problem introduces the concept behind the generalization of convergent and divergent p-series.

Concepts

Investigating convergence and divergence of positive term series through the Integral Test.

Teacher preparation

This activity is designed to be used in any Calculus II classroom or any calculus course in which infinite series are considered.

- *Students should be familiar with convergence and divergence of series prior to beginning this activity.*
- *The teacher should already have developed the idea of the Integral Test for convergence.*

Classroom management tips

- *This activity has three sections. The first is a walk-through of what the Integral Test is and how to implement it in a CAS environment. The second section is three problems for students to work through on their own. The final section prompts students to investigate the values of p for which a p-series converges.*

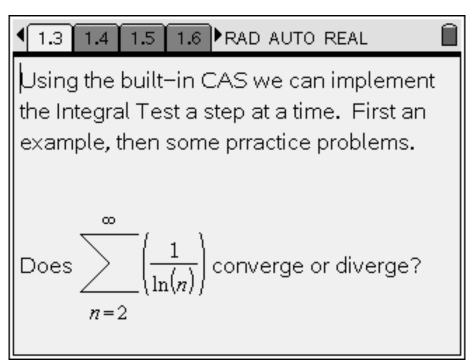
TI-Nspire Applications

Calculator, Notes, Lists & Spreadsheets

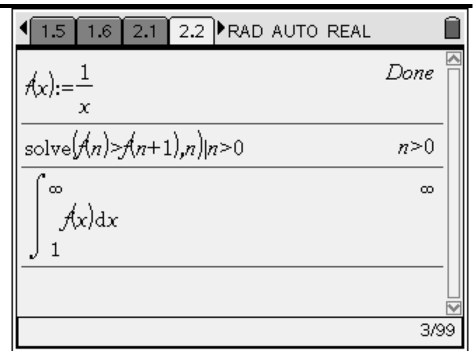
Step-by-step directions

Step-by-step activity directions with screenshots, sample data, etc. as needed. Screenshots should be created using the TI-Nspire handheld and resized to 70% for best visibility.

The first problem is just a walk-through with which the students should read and follow along. They may need to refer back to this as they complete the second and third questions. Students should advance through pages 1.1 to 1.6.

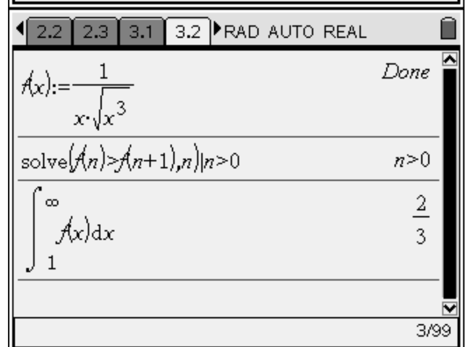


Students may need to refer back to the example problem in pages 1.3 through 1.6 as they work through problems 2 and 3. Sample student input for pages 2.2 and 3.2 are shown to the right. Responses to 2.3 and 3.3 are shown as well.

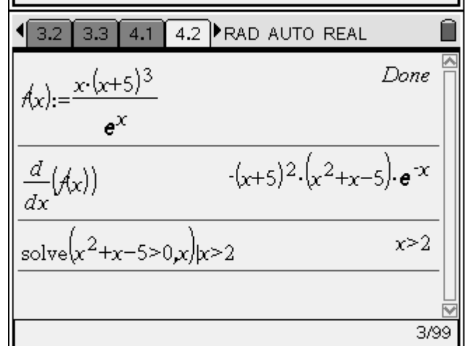


Students may choose to define the function in one of two ways. Either by selecting 1: Define from the Tools menu and typing the function or by defining the function as f(x):= the function. It is important that they realize these both accomplish the same job but have slightly different forms.

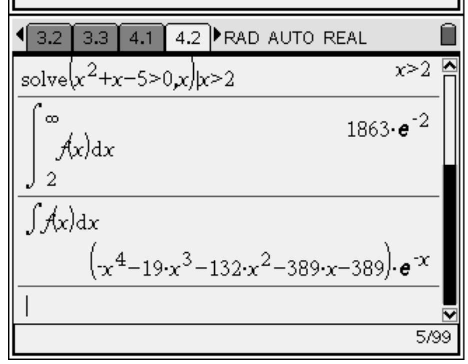
The first series diverges by the Integral Test. The second series converges by the Integral Test.



The series in the fourth problem is very complicated. In fact, the CAS does not easily show that the function is decreasing. Instead students will need to take the derivative and examine it. The derivative has three parts. The derivative has a coefficient of -1 and two of the three factors are always positive. Students need to determine if the third factor is always positive on the necessary interval.



This is exactly the sort of problem that CAS is most useful for solving. Out of interest, students might wish to find the indefinite integral just to appreciate the work they did not have to do to see if that integral converges. This series converges by the Integral Test.



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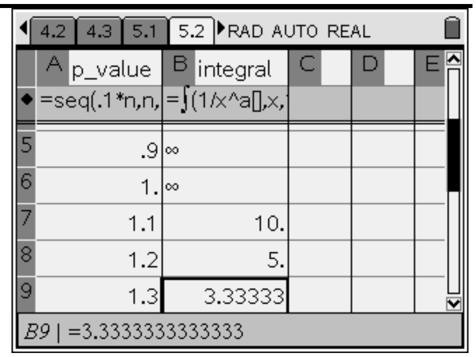
The third section of the activity asks students go guess for what values of p the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. They make their guesses based on the information contained in the spreadsheet on page 4.2. They should put their guess in the space provided on page 4.3. The correct answer is that a p-series will converge for p>1 and diverge otherwise.

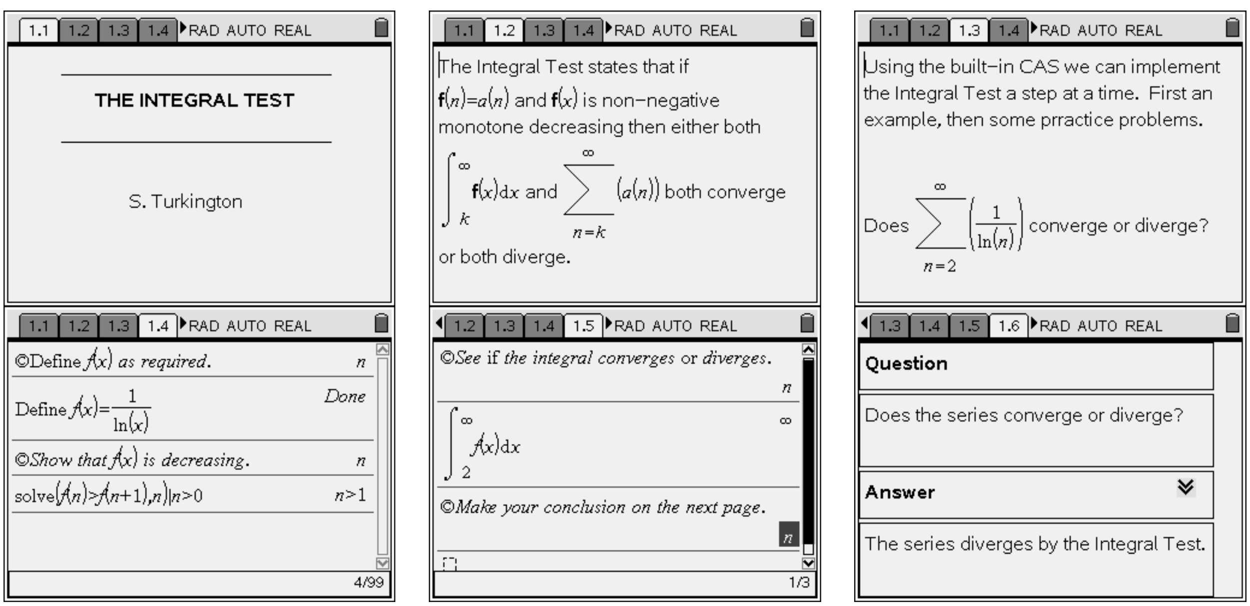


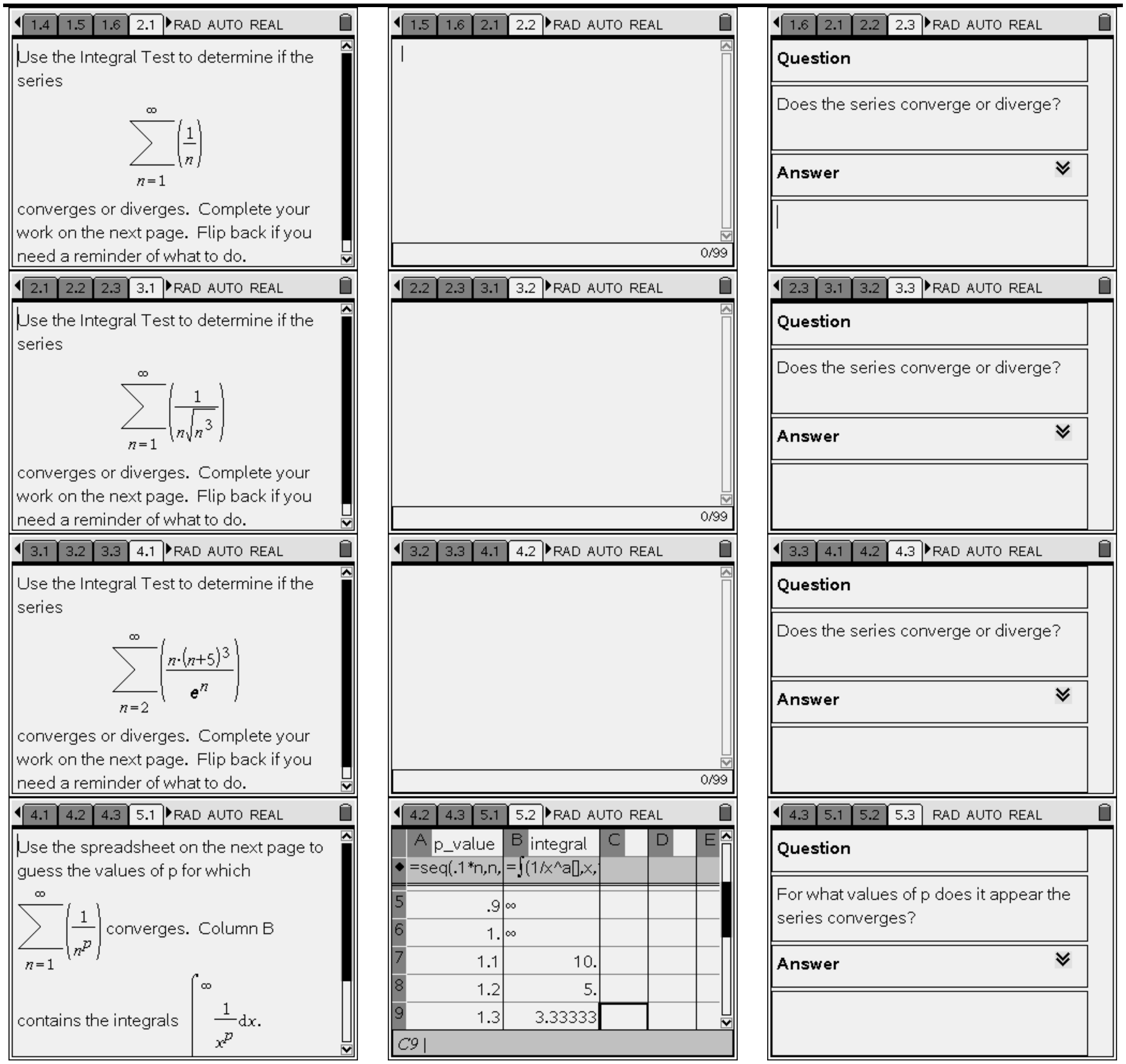
Assessment and evaluation

- The teacher should check student responses to pages 2.3, 3.3, 4.3, and 5.3.
- The teacher may also want to check the students' work on pages 2.2, 3.2, and 4.2.
- The teacher should check for student understanding. In particular the step in which you are solving for the values of n>0 for which a(n)>a(n+1).

Student TI-Nspire Document

Filename goes here.




1.4 1.5 1.6 2.1 RAD AUTO REAL
Use the Integral Test to determine if the series
$\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$
converges or diverges. Complete your work on the next page. Flip back if you need a reminder of what to do.
1.5 1.6 2.1 2.2 RAD AUTO REAL
0/99
1.6 2.1 2.2 2.3 RAD AUTO REAL
Question
Does the series converge or diverge?
Answer
2.1 2.2 2.3 3.1 RAD AUTO REAL
Use the Integral Test to determine if the series
$\sum_{n=1}^{\infty}\left(\frac{1}{n\sqrt{n^3}}\right)$
converges or diverges. Complete your work on the next page. Flip back if you need a reminder of what to do.
2.2 2.3 3.1 3.2 RAD AUTO REAL
0/99
2.3 3.1 3.2 3.3 RAD AUTO REAL
Question
Does the series converge or diverge?
Answer
3.1 3.2 3.3 4.1 RAD AUTO REAL
Use the Integral Test to determine if the series
$\sum_{n=2}^{\infty}\left(\frac{n\cdot(n+5)^3}{e^n}\right)$
converges or diverges. Complete your work on the next page. Flip back if you need a reminder of what to do.
3.2 3.3 4.1 4.2 RAD AUTO REAL
0/99
3.3 4.1 4.2 4.3 RAD AUTO REAL
Question
Does the series converge or diverge?
Answer
4.1 4.2 4.3 5.1 RAD AUTO REAL
Use the spreadsheet on the next page to guess the values of p for which
$\sum_{n=1}^{\infty}\left(\frac{1}{n^p}\right)$ converges. Column B
contains the integrals $\int_1^{\infty}\frac{1}{x^p}dx$.
4.2 4.3 5.1 5.2 RAD AUTO REAL
A p_value B integral C D E
=seq(.1*n,n, =∫(1/x^a[],x,
5 .9 ∞
6 1. ∞
7 1.1 10.
8 1.2 5.
9 1.3 3.33333
C9
4.3 5.1 5.2 5.3 RAD AUTO REAL
Question
For what values of p does it appear the series converges?
Answer