

Activity 5

Exploring the
Exponential
Function

When the value of a variable increases or decreases by a constant percent, the variable is said to change *exponentially*. When you have completed these activities, you will be able to identify exponential growth or decay from equations. In these activities, you will practice developing "eyeball" models for situations involving exponential growth.

If you invest money in a bank savings account, it grows by a constant percentage. For example, you might invest money at a simple 6% annual rate. If you invested this way, each year the bank would give you interest that amounted to 6% of the money in your account. Your money would grow exponentially.

If you drop a ball and let it bounce, like you did in Activity 4, with each bounce it rebounds less and less and the ball gets lower and lower. However, each bounce rebounds at a fixed percentage of the previous height. This is an example of *exponential decay*.

If a cup of hot chocolate was removed from the stove and allowed to cool, the temperature of the chocolate over time would illustrate exponential decay.

Exponential growth and decay are all around us. In this activity, you will explore exponential functions of the form $y = ab^x$. In this form, the constant b is called the growth factor.

Investigating the Effect of a and b on the Graph of $y = ab^x$

1. Press **[APPS]** and select Transformation Graphing by pressing the number at the left of **Transfrm**.



```
APPS
1: Finance
2: ALGEBRA1
3: CBL/CBR
4: Transfrm
```

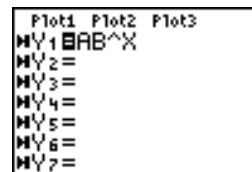
2. Press any key (except $\boxed{2\text{nd}}$ or $\boxed{\text{ALPHA}}$) to start Transformation Graphing.

Note: If you do not see the screen illustrated at the right, select 2: Continue.



3. In Func mode, press $\boxed{Y=}$ to display the Y= editor. Clear any functions that are listed and turn off any plots.
4. Enter $Y = AB^x$. Press $\boxed{\text{ALPHA}}$ \boxed{A} $\boxed{\text{ALPHA}}$ \boxed{B} $\boxed{\wedge}$ $\boxed{X,T,\theta,n}$.

If the Play-Pause Mode is not selected at the left of Y1 ($\boxed{>||}$), press $\boxed{\downarrow}$ until the cursor is over the symbol; then press $\boxed{\text{ENTER}}$ until the correct symbol is selected.

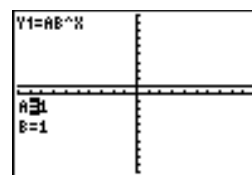


5. Press $\boxed{\text{WINDOW}}$ $\boxed{\uparrow}$ to display the settings screen for Transformation Graphing.

Set the settings as shown. To make these selections press $\boxed{\downarrow}$ $\boxed{1}$ $\boxed{\downarrow}$ $\boxed{1}$ $\boxed{\downarrow}$ $\boxed{1}$. This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.



6. Press $\boxed{\text{ZOOM}}$ $\boxed{6}$:ZStandard to display the graph.



Questions for Discussion

- The graph appears to be a line. Why? Explain your answer.
- If B remains 1 and A changes, what will happen to the graph? Make a hypothesis and then press the cursor control keys (either $\boxed{\leftarrow}$ or $\boxed{\rightarrow}$) a number of times to check.
- Changing the coefficient A does not seem to affect the rate of change of the exponential function? What does A affect?

Investigating $B > 1$

1. Press $\boxed{\text{WINDOW}}$ to display the settings screen for Transformation Graphing.

Set the **SETTINGS** as shown. To make these selections, press $\boxed{\uparrow}$ $\boxed{\downarrow}$ $\boxed{1}$ $\boxed{\downarrow}$ $\boxed{1}$ $\boxed{\downarrow}$ $\boxed{.25}$. This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.



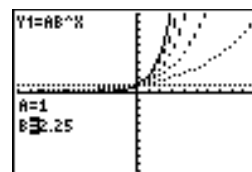
2. Press 2nd [FORMAT] \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow to highlight **TrailOn**. Press [ENTER]. This activates the Trail capability. With **TrailOn**, as the value of **B** is changed and the graph redrawn, a dotted line will remain in the location of the last curve.



Note: TrailOn is a feature that exists only with Transformation Graphing.

You can use 2nd [DRAW] 1:ClrDraw to clear the trails or repeat the steps above and highlight **TrailsOff**.

3. Press [GRAPH] to return to the graph screen.
4. Highlight the **B=**. Press \rightarrow a few times to increase the value of **B**. Pause after each increase to notice the change in the graph. You only need to change the value about 5 times.



What happens to the graph as **B** increases?

The value of **B** controls the rate at which the function grows and is called the growth factor. As **B** increases the function bends upward more steeply.

So far we have only investigated values of $B > 1$. We will save our exploration of $B < 1$ for later in these activities.

Investigating the Effect of A

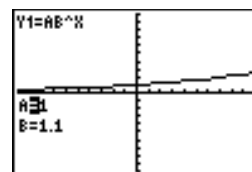
Before starting this investigation turn **TrailOn** to **TrailOff**. Press 2nd [FORMAT] \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow [ENTER].



1. Press [WINDOW] \uparrow to display the settings screen for Transformation Graphing. Set the **SETTINGS** as shown.

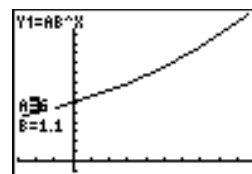


2. Press [GRAPH] to return to the graph screen. Highlight **A=**.



3. Press \rightarrow a few times to see the effect on the graph as **A** increases. Pause each time the curve is re-graphed and take note of the change. Pay special attention to the value of the y-intercept with each move.

- Press **WINDOW** and change the graph window to match the graph window at the right.
- Press **GRAPH** to re-display the graph with the new window.
- Enter specific values for A by typing the number and then pressing **ENTER**. Pay special attention to the y-intercept. Enter the values 2, 6, 10.



What effect does A have on the graph of $Y = AB^X$?

The value A is the y-intercept of the curve. The y-intercept is the value when $X = 0$, $Y = A \cdot B^0 = A \cdot 1 = A$. The y-intercept of an exponential function is frequently called the starting value of the dependent variable, the variable that is growing exponentially.

As an example, if you invested \$100 in a bank, at time $t=0$ you would have \$100. To set up an exponential model for the growth of your money, you would let $A=100$, the starting value. The value of B would reflect the interest rate.

Now look at cooling as a second example. If the temperature of a cup of hot chocolate was 60°C when it was taken from a stove and then placed outside the house on a table where the temperature was 0°C , the starting point for our equation would be time=0, temperature = 60. Expressed as $y=$ for the calculator, the value of A would be 150, the starting temperature. In this case, B would be affected by the rate at which the chocolate was cooling. The hot chocolate would be losing heat, an example of decay. The next activity will look at decay.

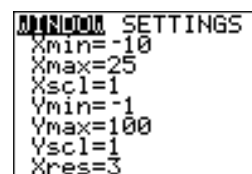
Revisiting B

You have considered exponential decay but have not viewed its graph. Before you proceed any further, look at an example of decay and see if you can sketch its graph.

The maximum height of each bounce of a bouncing ball illustrates exponential decay. Assume the ball is dropped from a height of 6 feet and rebounds to half that height, or 3 feet. On the next bounce, the same thing happens, it rebounds halfway again, or 1.5 feet. This pattern continues until the ball no longer bounces. Draw a rough sketch of the maximum height of the ball with each bounce. Let Y be the height and X be the number of the bounce.

Now that you have an idea of how an exponential decay curve would look, let's continue our investigation.

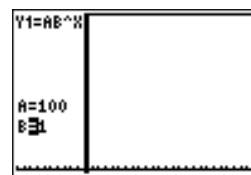
- Press **WINDOW** and set the window settings as shown at the right.



2. Press the cursor up key (\uparrow) until you see the settings menu for Transformation Graphing. Set the settings as shown at the right.



3. Press \square to return to the graph window.
4. Press \square until the **B=** is highlighted.



5. Press \square several times to lower the value of **B**. Observe values of **B** between 1 and 0. Pause after each graph change and notice the effect on the graph. You might want to enable **TrailOn**. If you do, be sure to turn that feature off when you finish this particular part of the activity.

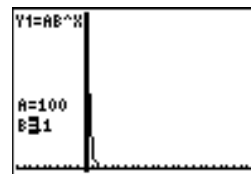
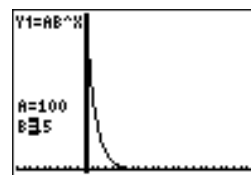
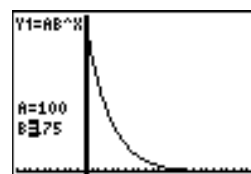
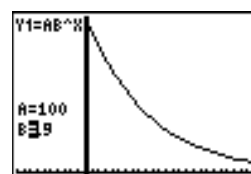
What effect does lowering the value of **B**, in the range $0 < B < 1$, have on the graph of the exponential function $Y = AB^X$?

When $0 < B < 1$ the values of the function, **Y**, decrease as **X** grows. The closer to $B = 0$, the faster the **Y** values decrease and thus the faster the rate of decay of the graph.

*Note: **B** can not be negative. Allow **B** to be negative and investigate the function values with the table feature of your calculator.*

Assuming $A > 0$ and $0 < B < 1$, can **Y** ever be negative for $Y = AB^X$? Explain your answer.

Since $0 < B < 1$, the more you multiply **B** by itself, the smaller the term becomes, but it can never be less than 0. As an example take $.9^1, .9^2, .9^3, \dots, .9^{10}$, when you evaluate them you get $.9, .81, .729, \dots, .349$. The value of each successive term is getting smaller but will never reach 0. Thus as long as **A** is not negative there is no way for AB^X to ever be negative, only very small.

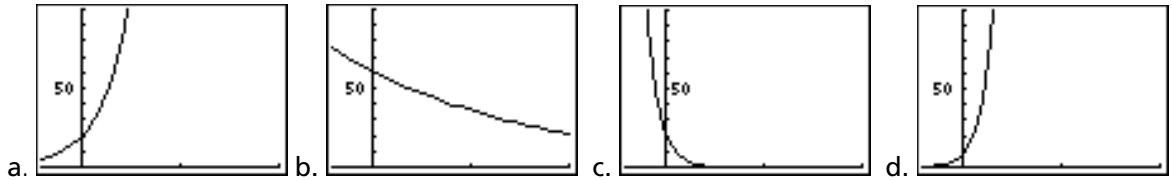


Curves that approach a value but never reach it are said to be *asymptotic*. The exponential equation $Y = AB^X$ is asymptotic to 0 because it approaches 0 as **X** gets large, but never reaches 0.

How Are You Doing?

Each graph is an example of an exponential function. Tell:

1. If the graph is an example of growth or decay.
2. The value of A .
3. Is $B < 1$ or $B > 1$?



4. Which graph has the larger value for B , b or c ? How about a or d ? Explain your answers.

Homework Page

Name _____

Date _____

The table shows the population of the world, in billions, since 1940. Use Transformation Graphing to develop an exponential model for the data.

Internet Search: Look for world population on the Internet. Use the data you find if it is different from that shown in the table below.

Year	Population (in billions)
1940	2.30
1950	2.52
1960	3.02
1970	3.70
1980	4.44
1990	5.27

Source: *The World at Six Billion*. Population Division, Department of Economics and Social Affairs, United Nations Secretariat. Working Paper ESP/P/W.154, New York, 1999. Reprinted with permission.

Before you enter the data, think about the parameters of the exponential function $Y = AB^x$. The value A is considered the starting value of the dependent variable. In this problem, the starting value is 2.3 billions when the year = 1940. But you would like the starting value to be when your independent variable, the year in this case, is 0. Subtract 1940 from each year.

Year	Translated Year (x)	Population (in billions) (y)
1940	0	2.30
1950	10	2.52
1960	20	3.02
1970	30	3.70
1980	40	4.44
1990	50	5.27

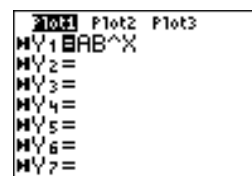
1. Enter the data in the Stat List of your TI-83 Plus. Press **[STAT]** and select **1:Edit**. Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty.

Note: To clear a list, move up to highlight the list name, press **[CLEAR]** **[ENTER]**.

L1	L2	L3	2
0			
10			
20			
30			
40			
50			

L2(?) =			

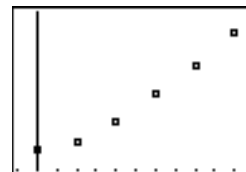
2. Press $\boxed{Y=}$ and clear all functions. Enter the general form of the exponential function.



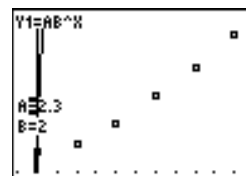
3. To display the plot, press $\boxed{2nd}$ [Stat Plot]. Select **1:Plot1**. Turn the plot on and set the plot menu as shown. Press \boxed{ZOOM} **9:ZoomStat** to display the plot.



It appears that an exponential model might be appropriate.



4. Be sure Transformation Graphing is installed and in Play-Pause Mode. Press \boxed{GRAPH} to re-draw the plot with the graph.
5. Select starting values for **A** and **B**. As an example let **A = 2.3** and **B = 2**. Use the up/down cursor control keys to first highlight **A=** and enter **2.3**, and then highlight **B=** and enter **2**. This gives you a starting point in your investigation.



Should the value of **B** be larger or smaller than shown in the graph? Explain.

The graph seems to be growing too fast—it is too steep. The value of **B** should be less than 2.

6. Revise your values for **A** and **B** until you have a reasonable model. Revise **B** first and then revise **A**. Express **B** to the thousandth place to get a good model.

What equation did you decide is a good model for the data?

7. Once you have a reasonable model, use the table feature of your calculator to estimate the world population in the year 2000. Hint: If 1940 was entered as 0 in our table, the year 2000 should be 60.

To use the table feature press $\boxed{2nd}$ [TBLSET] and set the **TABLE SETUP** as shown.



To read the table, press $\boxed{2nd}$ [TABLE].

Note: Your table might have different values depending on your model.

X	Y1
0	2.2
10	2.6142
20	3.1064
30	3.6913
40	4.3862
50	5.2121
60	6.1934

X=60

8. What does your model predict the population will be in 2010?
9. What does your model predict the population was in 1930?

Modeling Practice

The population of the mystical city of Transform is shown in the table.

Year	Population
1970	125,000
1975	201,300
1980	324,000
1985	522,000
1990	841,000
1995	1,354,000

10. Find a model for the population of Transform.
11. What will the population be in 2000?
12. At what rate is the population growing?

Note: Start by representing the year 1970 as 0.

Notes for Teachers

The purpose of this activity is to start a study of the exponential function. Once students have completed the activities they should be able to:

- ◆ recognize exponential growth or decay from an equation.
- ◆ tell which coefficient controls the rate of growth/decay.
- ◆ tell the effect of changes in the value of A .
- ◆ compare two graphs by looking at their equations.
- ◆ eyeball model a situation involving exponential growth.

There is no strong attempt to discuss the magnitude of B , just that it affects the rate at which the function grows. There is no work done with the actual rate; it is not meant to be part of the activity.

Optional Presentation Method

Activities 5 and 6 could be divided into three sections: growth, decay, and non-zero asymptotes. If you decide to use this division the sections would be as follows:

- ◆ *Growth*: Activity 5 from start to the section *Revisiting B*. The homework from Activity 5 would be homework for this section.
- ◆ *Decay*: Start where the new unit on Growth ends. Continue into Activity 6. End in Activity 6 with the section *Modeling the Experiment: Casting Out Sixes with Special Number Cubes*. The homework on page 61 would be homework for this section.
- ◆ *Non-zero Asymptotes*: This section would start where the previous section ended and continue for the rest of the activity. The homework for this section would be the CBL Practice Activity listed on page 63. This activity, *Experiment M5: Coffee To Go* can be found in the *CBL™ System Experiment Workbook*. This activity can also be downloaded for free from the Texas Instruments web site at education.ti.com. Any “cooling” experiment could also be used for this homework.

Warning!!!! This activity mentions the cooling of a cup of chocolate but *does not develop* the idea of asymptotes. On page 45, the statement about the hot chocolate has the chocolate starting at 60°C and cooling to 0°C . In this way the asymptote is at $y = 0$. It is felt that investigating decay completely, with non-zero asymptotes, would be too much for one set of activities. Also, most Algebra I courses do not deal with non-zero asymptotes. The discussion of asymptotes occurs in Activity 6. With this activity, you are trying to start the study of the exponential function and only want the students to realize there is decay and growth, and how to tell which is which.

Answers

Investigating the Effect of a and b on the Graph of $y=ab^x$: Questions for Discussion

1. When $A=B=1$, you have the equation $y = 1(1)^x$. No matter what value x takes, 1^x will remain 1. Thus the equation becomes $y = 1(1)$ or the line $y = 1$. In this case there is no growth or decay.
2. Since $B=1$, you will continue to have the line $y = A$. The general form of the equation becomes $Y = AB^x = A*1 = A$. B is the growth factor; as long it remains 1, there is no change.
3. A is the Y-intercept.

Investigating $B>1$

4. As B increases the curve gets steeper, the function grows at a faster rate.

Investigating the Effect of A

6. A is the y-intercept of the curve.

How Are You Doing?

1-3 Growth, 20, $b>1$

Decay, 60, $b<1$

Decay, 20, $b<1$

Growth, 10, $b>1$

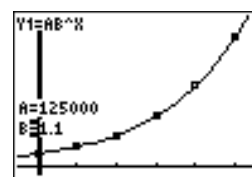
4. The value of B is greater in b because the curve doesn't bend as much, it decays slower. The value d is greater because the curve bends faster, it grows faster.

Homework Page

5. The value for B should be smaller. The graph is growing too quickly.
6. Answers will vary. $Y = 2.1(1.019)^x$ could be a reasonable model. Any reasonable answer should be accepted. There is no one correct answer. Answers in the neighborhood of $y = 2.2(1.0174)^x$ are exceptionally good. With exponential growth, the number of decimal places is very important.
7. The prediction of the population in 2000 should be in the 6 billion range. Be sure students label the answer as billions.
8. Approximately 7.4 billion. Be sure students label the answer as billions.
9. Approximately 1.85 billion. Be sure students label the answer as billions.

Modeling Practice

10. A reasonable model is $y = 125000(1.1)^x$.
11. The population in 2000 will be approximately 2,181,000.
12. The population is growing at approximately 10% per year.



X	Y1
0	125000
5	201314
10	324218
15	522156
20	840937
25	13556
30	2181175

$Y1 = 2181175.28361$