



### About the Lesson

In this activity, students will explore distances in the coordinate plane. Students will substitute the coordinates of a segment's endpoints into the distance formula and compare the results to the measured length of the segment. Then students will find the distance between the endpoints using the Pythagorean Theorem. As a result, students will:

- Calculate the length of a line segment when given the coordinates of its ends.

### Vocabulary

- hypotenuse

### Teacher Preparation and Notes

- This activity is designed to be used in a high school or middle school geometry classroom.
- The distance formula for the distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ . The Pythagorean Theorem for a right triangle with legs  $a$  and  $b$  and hypotenuse  $c$  is  $a^2 + b^2 = c^2$ . Solving for the hypotenuse, the equation becomes  $c = \sqrt{a^2 + b^2}$ .
- Cabri Jr. does not allow the  $x$ -values and  $y$ -values of points to be used separately in calculations. The construction in Problem 1 uses a perpendicular line to project the coordinate on the appropriate axis, and then the distance from  $(0, 0)$  is used to find the  $x$ -values and  $y$ -values.
- **Note:** The coordinates can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To ensure that a point is actually at an integer value rather than a rounded decimal value, move the cursor over the coordinate value so it is highlighted and press  $\oplus$  to display additional decimal digits or  $\ominus$  to hide digits.

### Activity Materials

- Compatible TI Technologies:

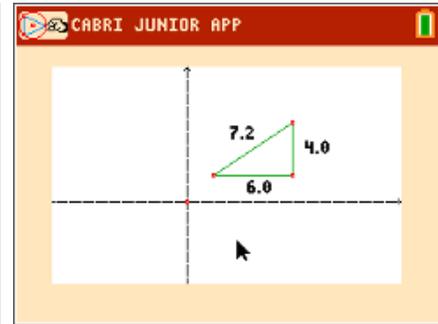
TI-84 Plus\*

TI-84 Plus Silver Edition\*

TI-84 Plus C Silver Edition

TI-84 Plus CE

\* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



### Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

### Lesson Files:

- Distances\_in\_the\_Coordinate\_Plane\_Student.doc
- Distances\_in\_the\_Coordinate\_Plane\_Student.pdf
- DISTNC1.8xv
- DISTNC2.8xv



**Tech Tip:** Before beginning the activity, the Cabri Jr. files: DISTNC1.8xv and DISTNC2.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE software.

## Problem 1 – The Distance Formula

**Note:** If the file *DISTNC1* is distributed to student calculators, skip Steps 1 and 2 and proceed to Step 3.

**Step 1:** Students should open a new Cabri Jr. file. If the axes are not currently showing, they should select **Hide/Show > Axes**.

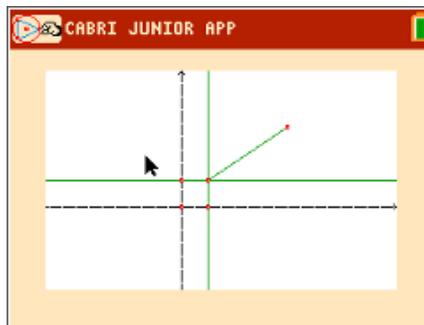
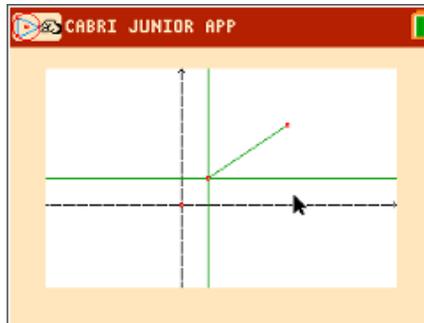
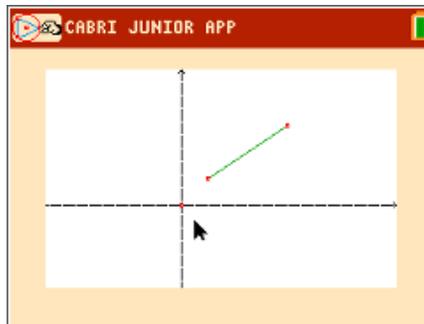
Students are to construct a segment in the first quadrant using the **Segment** tool.

**Step 2:** In order to use the x-values and y-values of the endpoints separately in calculations, students will use a perpendicular line to project the coordinates on the appropriate axis.

Direct students to select the **Perp.** tool to construct a line through one endpoint of the segment perpendicular to each axis.

Find the intersection of the perpendicular line with the axis using the **Point > Intersection** tool. Then, hide the perpendicular line with the **Hide/Show > Object** tool.

Repeat this process for the second endpoint.





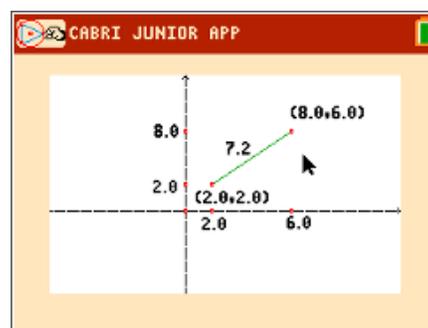
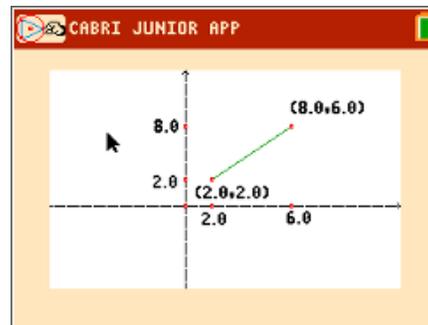
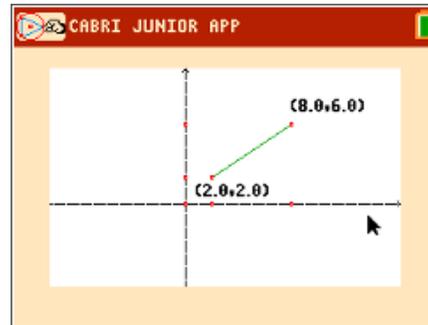
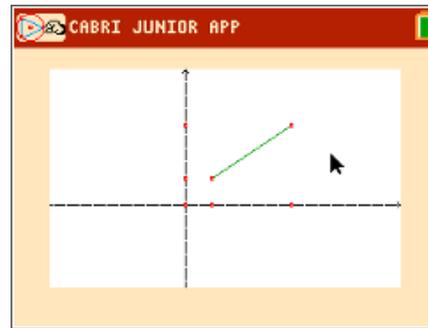
**Step 3:** Students will select **Coord. & Eq.** and show the coordinates for the endpoints of the segment.

If the coordinates of the endpoints are not integers, they need to use the **Hand** tool to drag the endpoints until the coordinates are integers.

**Step 4:** Students should measure the distance from each axis point to the origin using the **Measure > D. & Length** tool. These distances should match the x-values and y-values of the coordinates.

Students can now drag the endpoints and observe that the axis point distances change as the segment endpoints move, but still match the coordinates.

**Step 5:** Have students measure the length of the segment using the **Measure > D. & Length** tool.

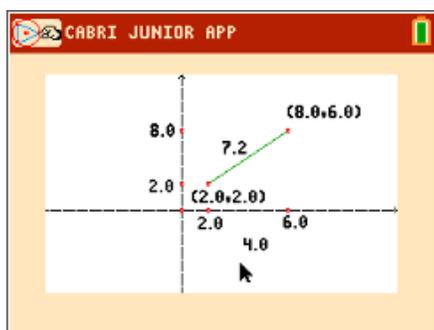




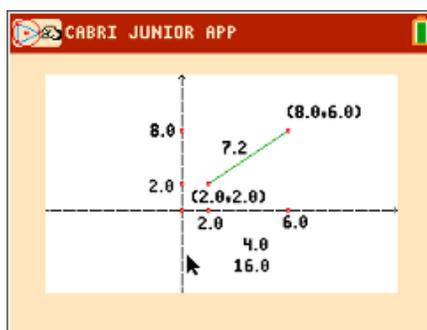
**Step 6:** The **Calculate** tool can perform calculations on pairs of numbers. The distance formula calculations will be broken down into individual steps. Be sure to select coordinates in the proper order! Students should complete the following steps in order:

1. **Subtract** the two x-values.
2. **Multiply** this result by itself.
3. **Subtract** the two y-values.
4. **Multiply** this result by itself.
5. **Add** the two products.
6. **Square root** this sum.

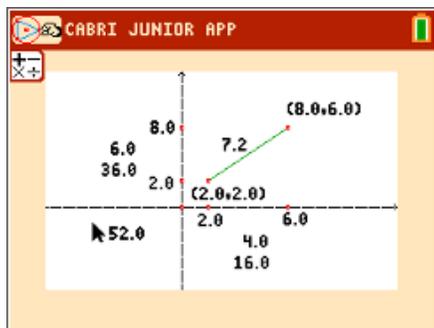
Subtract:



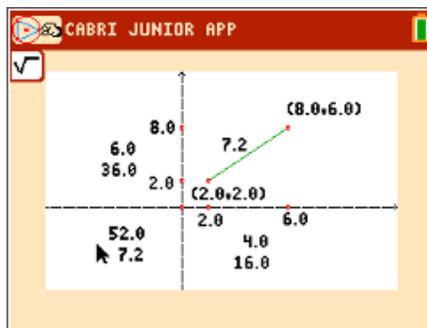
Multiply:



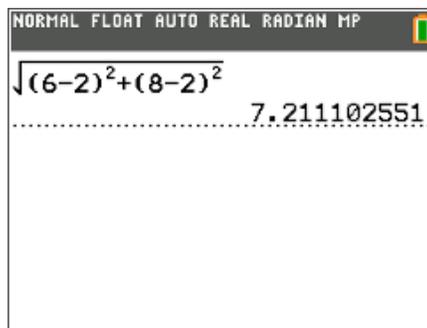
Add:



Square root:



**Note:** Students can do the calculations on the Home screen rather than within the Cabri Jr. file.





Now if students completed the calculations in the Cabri Jr. file, they can drag the segment endpoints and observe the calculation results as they update.

Discuss how these calculation results relate to the measured length of the segment.

1. Open up a new document in the Cabri Jr. app. Construct a segment. Find the coordinates of the endpoints and the measured length. Use the distance formula to calculate the length.

Length	Endpoints	Measured Length	Calculated
<u>Answers:</u>			
	(2, 2) and (8,6)	7.2	7.2111
	(5, 3) and (7, 5)	2.8	2.82843

2. What is important to remember when using the distance formula?

Answer: To not mix up the  $x$  and  $y$  coordinates.

3. What happens to the distance formula when your segment is horizontal or vertical? Give an example using endpoints.

Answer: For vertical lines  $(x_2 - x_1)^2 = 0$ , so the distance formula becomes  $|y_2 - y_1|$

For horizontal lines,  $(y_2 - y_1)^2 = 0$ , so the distance formula becomes  $|x_2 - x_1|$

(3, 6) and (3, 2)

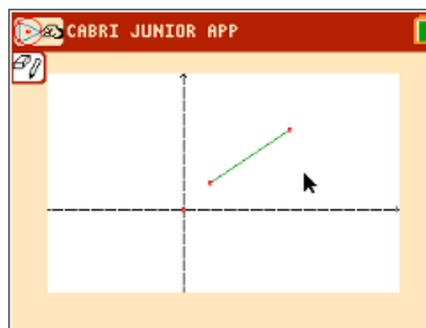
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 3)^2 + (2 - 6)^2} = \sqrt{0 + (-4)^2} = \sqrt{16} = 4$$

## Problem 2 – The Distance Formula and the Pythagorean Theorem

**Note:** If the file *DISTNC2* is distributed to student calculators, skip Steps 1, 2 and 3. Proceed with Step 4.

**Step 1:** Students should open a new Cabri Jr. file. If the axes are not currently showing, they should select **Hide/Show > Axes**.

Students are to construct a segment in the first quadrant using the **Segment** tool.

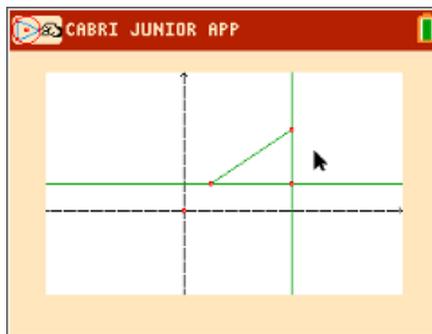




**Step 2:** Students will be constructing a small right triangle for the segment such that the segment is the hypotenuse of the right triangle.

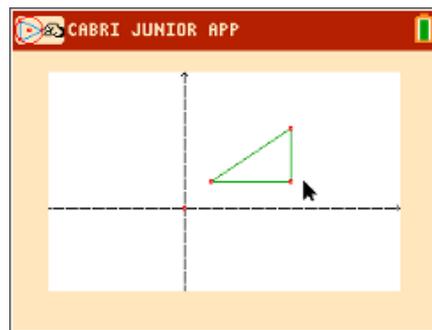
Select the **Perp.** tool and construct a line through the upper segment endpoint perpendicular to the  $x$ -axis. Construct a second line through the lower segment endpoint perpendicular to the  $y$ -axis.

Have students find the intersection of the perpendicular lines by selecting **Point > Intersection**.



**Step 3:** Direct students to hide the perpendicular lines with the **Hide/Show > Object** tool. Do not hide the intersection point.

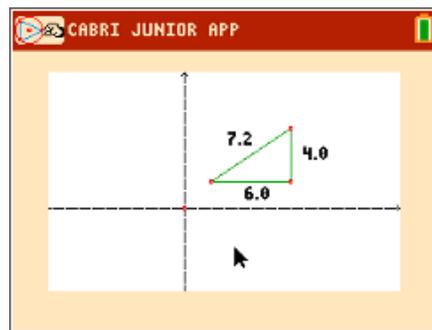
They should then construct segments for the legs of the right triangle with the **Segment** tool.



**Step 4:** Tell students to find the length of each side of the triangle.

Discuss the lengths of the three sides of the triangle and decide which is longest.

Identify the legs and the hypotenuse.



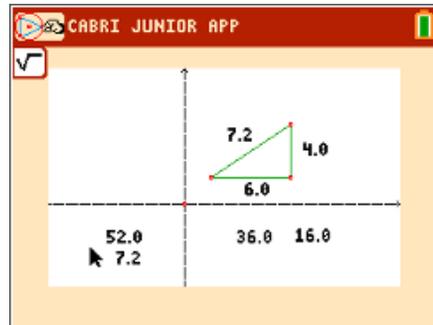
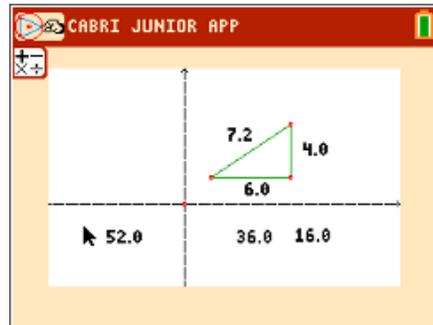
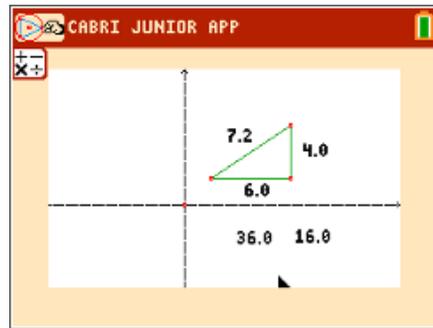


**Step 5:** Use the **Calculate** tool to perform the following calculations:

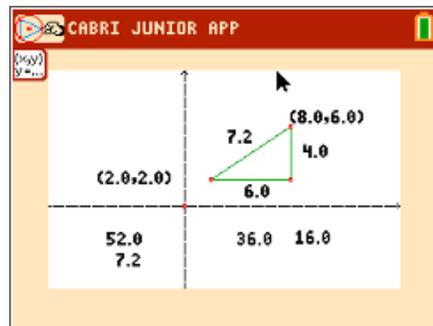
Students should complete the following steps in order:

1. **Multiply** the length of one leg by itself.
2. **Multiply** the length of the other leg by itself.
3. **Add** the two values.
4. **Square root** this sum.

**Note:** If desired, have students do the calculations directly on the worksheet rather than within the Cabri Jr. file.



**Step 6:** Students should show the coordinates of the segment endpoints and calculate the length using the distance formula on their worksheet.



Students should drag the endpoints of the segment and observe the relationship between the three values:

- Measured length
- Calculated length (distance formula) on worksheet
- Calculated Pythagorean distance

Discuss the connections between the distance formula and the Pythagorean Theorem. Challenge students to explain how  $\sqrt{a^2 + b^2}$  is derived from the Pythagorean Theorem.



4. Find the length of all three sides of your triangle. Which side is the longest? Can two of the sides be equal lengths? Which two?

**Answer:** 6, 4, 7.2. The longest side is the hypotenuse. The two legs can be equal lengths

5. Use the Pythagorean Theorem to calculate the length of your segment in another way.

Endpoints	Measured Length	Pythagorean Length
(5, 8) and (6, 2)	6.1	$\sqrt{37}$
(4, 6) and (3, 9)	2.2	$\sqrt{5}$

**Answers:**

6. What is the relationship between the Pythagorean Theorem and the distance formula?

**Answer:** The distance between the points is the length of the hypotenuse ( $c$ ).

The length of the legs ( $a$  and  $b$ ) of the right triangle are  $(x_2 - x_1)$  and  $(y_2 - y_1)$ .

$$\text{So } c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = \sqrt{(a)^2 + (b)^2}$$

$$c^2 = a^2 + b^2$$



## Exercises

7. What formula gives the distance between the points  $(x_1, y_1)$  and  $(x_2, y_2)$ ?

**Answer:**  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Determine the length of the segment with the following endpoints:

8.  $(1, 2)$  and  $(5, 10)$

**Answer:**  $\sqrt{80}$  or  $4\sqrt{5}$

9.  $(5, 8)$  and  $(9, 5)$

**Answer:** 5

10.  $(7, 4)$  and  $(4, 7)$

**Answer:**  $\sqrt{18}$  or  $3\sqrt{2}$

11.  $(-2, 3)$  and  $(3, 5)$

**Answer:**  $\sqrt{29}$

12.  $(1, -9)$  and  $(-2, -7)$

**Answer:**  $\sqrt{13}$

13.  $(3, 5)$  and  $(3, -11)$

**Answer:** 16

Given an endpoint and a length of a segment, find a possible other endpoint:

14. Endpoint:  $(3, 1)$ ; Length 5

**Sample Answer:**  $(6, 5)$