Math Objectives

- Students will understand the effect that each of the parameters *A, B, C*, and *D* has on the graph of a sinusoidal function of the form $y = \pm A \sin(B(x - C)) + D$ or $y = \pm A \cos(B(x - C)) + D$.
- Students will understand that sinusoidal functions can be expressed in a variety of equivalent forms.
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).
- Students will look closely to discern a pattern or structure (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- amplitude
- horizontal shift
- parameters
- period
- cycle
- vertical shift

About the Lesson

- This lesson involves examining graphs, or partial graphs, of sinusoidal functions to determine the values of their parameters and to express them in various ways involving sine and cosine functions
- As a result, students will:
  - Write the equation for each graph in the forms $y = \pm A \sin(B(x - C)) + D$ or $y = \pm A \cos(B(x - C)) + D$
  - Rewrite functions $f(x) = A \sin(Bx) + C \sin(Dx)$ to the form $f(x) = \pm\sqrt{M}\sin(K(x-L))$ where *M* is a positive integer
  - Analyze graphs and verify their rewritten functions using algebra and "trig identities."
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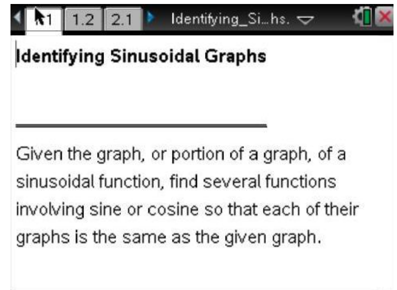
TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge and to monitor students' understanding.
- Use Quick Poll to assess students' understanding and to compare their answers.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity
Identifying_Sinusoidal_Graphs_Student.pdf
Identifying_Sinusoidal_Graphs_Student.doc

TI-Nspire document
Identifying_Sinusoidal_Graphs.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: Make sure student calculators are set to radian measure mode.

Teacher Tip: You might want students to complete the activity: **Basic Trigonometric Transformations** prior to this activity.
If they need a hint determining the parameters of the sinusoidal function whose graph is given, tell students that finding the maxima, minima, and zeroes of the graph using Graph Trace or the "Analyze Graph" tools can help.

Roughly speaking, sinusoidal functions are periodic functions that can be expressed using sine or cosine functions. Specifically, they can be written in the form,

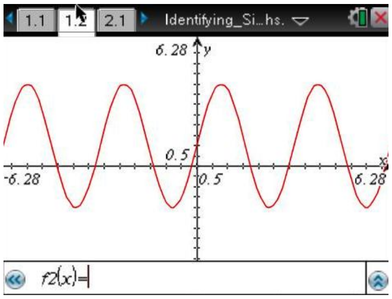
$$f(x) = \pm A \sin(B(x - C)) + D \text{ or}$$
$$f(x) = \pm A \cos(B(x - C)) + D$$

for appropriate values of A, B, C, and D where A and B are positive.
Recall that A is the amplitude, B = 2π/period, C is the horizontal shift, and D is the vertical shift.
You might have also used the forms

$$f(x) = \pm A \sin(Bx - C) + D \text{ or}$$
$$f(x) = \pm A \cos(Bx - C) + D.$$

The term 'sinusoid" was first used by Scotsman Stuart Kenny in 1789 while observing the growth patterns of soybeans.

Move to page 1.2.



1. Consider this graph of a sinusoidal function [in radian measure].
   a. Determine a function *f(x) = A sin (B(x – C)) + D* whose graph is the same as the one given on Page 1.2.

   Sample Answers: $f(x) = 3 \cdot \sin(2 \cdot x) + 1$

   b. Enter your function on the entry line below the graph to check that your graph coincides with the given graph. If not, change your function until the two graphs match.
   c. Determine a function *f(x) = -A sin (B(x – C)) + D,* whose graph is the same as the one given on Page 1.2.

   Sample Answers: $f(x) = -3 \cdot \sin(2 \cdot (x - \frac{\pi}{2})) + 1$

d. Enter your function on the entry line below the graph to check that your graph coincides with the given graph. If not, change your function until the two graphs match.

e. Determine a function *f(x) = A cos (B(x – C)) + D* whose graph is the same as the one given on Page 1.2.

Sample Answers: $f(x)=3\cdot\cos(2\cdot(x-\frac{\pi}{4}))+1$

f. Enter your function on the entry line below the graph to check that your graph coincides with the given graph. If not, change your function until the two graphs match.

g. Determine a function *f(x) = -A cos (B(x – C)) + D,* whose graph is the same as the one given on Page 1.2.

Sample Answers: $f(x)=-3\cdot\cos(2\cdot(x-\frac{3\pi}{4}))+1$

h. Enter your function on the entry line below the graph to check that your graph coincides with the given graph. If not, change your function until the two graphs match.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

2. a. Which of the values of A, B, C, or D remain unchanged in your four functions? Why?

Sample Answers: The values of A, B, and D are unchanged since the amplitude, period, and vertical shift are unchanged. Only the “reference point for the graph” – the horizontal shift – changes.

b. If the given graph is shifted *π* units to the right, write the function that will result in this graph.

Sample Answers: Since the period of this graph is π units, the graph is unchanged, and the functions remain the same.

c. If the given graph is shifted *π*/2 units to the left, write the function that will result in this graph.

Sample Answers: : $f(x)=-3\cdot\sin(2\cdot x)+1$. Substitute (*x + π/2*) for x in any of the answers to

1a and 1c to find other answers.

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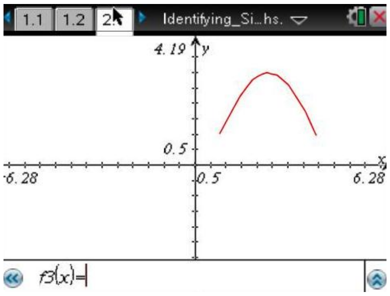

In the next several problems, you will be given only a portion of the graph of a sinusoidal function and asked to find several functions so that each of their graphs contains the portion of the graph that is given.

Teacher Tip: Suggest that students first make a rough paper-and-pencil sketch of the complete graph and estimate the *y*-coordinates of maxima and minima and the *x*-coordinates of the endpoints of one period.

3. Consider a half-cycle of the graph of a sinusoidal function.
   a. Determine a function of the form *f(x) = A sin (B(x – C)) + D* or *f(x) = -A sin (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = 2 \cdot \sin(x - \frac{\pi}{4}) + 1$ or $f(x) = -2 \cdot \sin(x + \frac{3\pi}{4}) + 1$

   b. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

   c. Determine a function of the form *f(x) = A cos (B(x – C)) + D* or *f(x) = -A cos (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = 2 \cdot \cos(x - \frac{3\pi}{4}) + 1$ or $f(x) = -2 \cdot \cos(x + \frac{\pi}{4}) + 1$

   d. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

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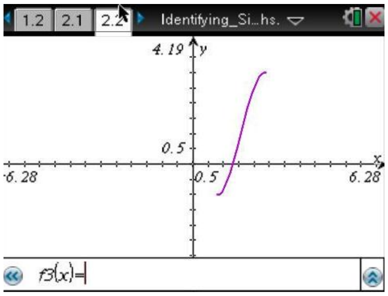

4. Consider a half-cycle of the graph of another sinusoidal function.
   a. Determine a function of the form *f(x) = A sin (B(x – C)) + D* or *f(x) = -A sin (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = 2 \cdot \sin(2 \cdot (x - \frac{\pi}{2})) + 1$ or
$f(x) = -2 \cdot \sin(2x) + 1$

b. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

c. Determine a function of the form *f(x) = A cos (B(x – C)) + D* or *f(x) = -A cos (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = 2 \cdot \cos(2 \cdot (x + \frac{\pi}{4})) + 1$ or $f(x) = -2 \cdot \cos(2 \cdot (x - \frac{\pi}{4})) + 1$

d. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

Teacher Tip: Suggest that students first make a rough paper-and-pencil sketch of the complete graph and estimate the *y*-coordinates of maxima and minima and the *x*-coordinates of the endpoints of one period.

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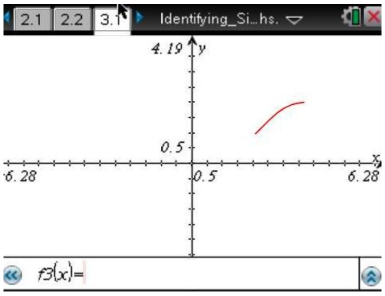


5. Consider a quarter-cycle of the graph of a sinusoidal function.
   a. Determine a function of the form *f(x) = A sin (B(x – C)) + D* or *f(x) = -A sin (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = \sin(x - \frac{2\pi}{3}) + 1$ or

$f(x) = -\sin(x + \frac{\pi}{3}) + 1$

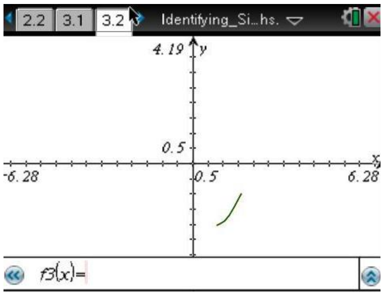
   b. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

   c. Determine a function of the form *f(x) = A cos (B(x – C)) + D* or *f(x) = -A cos (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = \cos(x - \frac{7\pi}{6}) + 1$ or $f(x) = -\cos(x - \frac{\pi}{6}) + 1$.

   d. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

Move to page 3.2.



6. Consider a quarter-cycle of the graph of another sinusoidal function.
   a. Determine a function of the form *f(x) = A sin (B(x – C)) + D* or *f(x) = -A sin (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x) = \sin(2 \cdot (x - \frac{\pi}{2})) - 1$ or

$f(x) = -\sin(2x) - 1$

   b. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

c. Determine a function of the form *f(x) = A cos (B(x – C)) + D* or *f(x) = -A cos (B(x – C)) + D* whose graph contains the given partial graph.

Sample Answers: $f(x)=\cos(2\cdot(x+\frac{\pi}{4}))-1$ or $f(x)=-\cos(2\cdot(x-\frac{\pi}{4}))-1$

d. Enter your function on the entry line below the graph to check that this graph contains the partial graph. If not, change your function until it does.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 3 at the end of this lesson.

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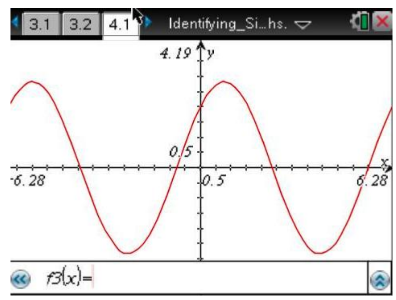


Some sinusoids can also be expressed as a sum or difference of sine and cosine functions.

7. a. Rewrite the function $f1=2\cdot\sin x+2\cdot\cos x$ in the form $f2=\pm\sqrt{M}\sin(K(x-L))$ where M is a positive integer by analyzing its graph. Enter your function on an entry line to check your answer.

Sample Answers: $f2=\sqrt{8}\sin(x+\frac{\pi}{4})=2\sqrt{2}\sin(x+\frac{\pi}{4})$

Teacher Tip: Students can determine the value of M by finding a local maximum of the graph and by using the fact that M is an integer.

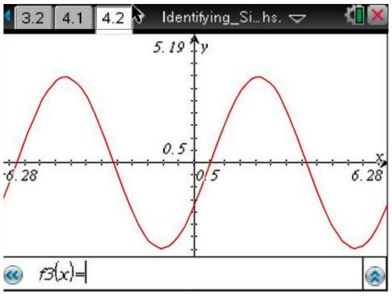
b. Algebraically verify that *f1 = f2* using a “trig identity”.

Sample Answers: $f2=2\sqrt{2}\sin(x+\frac{\pi}{4})=2\sqrt{2}(\sin x\cdot\cos(\frac{\pi}{4}))+2\sqrt{2}(\cos x\cdot\sin(\frac{\pi}{4}))$ =

$2\sqrt{2}(\sin x\cdot\frac{\sqrt{2}}{2}+\cos x\cdot\frac{\sqrt{2}}{2})=2\cdot\sin x+2\cdot\cos x=f1$

Teacher Tip: Ask one or more students to share their algebraic derivation for 7b so that other students receive some guidance for completing the derivation in 8b if needed.

Move to page 4.2.



8. a. Rewrite the function $f1 = 3 \cdot \sin x - \sqrt{3} \cdot \cos x$ in the form $f2 = \pm\sqrt{M}\sin(K(x-L))$ where M is a positive integer by analyzing its graph. Enter your function on an entry line to check your answer.

Sample Answers: $f2 = \sqrt{12}\sin(x - \frac{\pi}{6}) = 2\sqrt{3}\sin(x - \frac{\pi}{6})$

b. Algebraically verify that *f1* = *f2* using a "trig identity".

Sample Answers: $f2 = 2\sqrt{3}\sin(x - \frac{\pi}{6}) = 2\sqrt{3}(\sin x \cdot \cos(\frac{\pi}{6})) - 2\sqrt{3}(\cos x \cdot \sin(\frac{\pi}{6}))$ =

$2\sqrt{3}(\sin x \cdot \frac{\sqrt{3}}{2} - \cos x \cdot \frac{1}{2}) = 3 \cdot \sin x - \sqrt{3} \cdot \cos x = f1$

Extension

1. Ask a student to create a partial graph of a sinusoidal function and share it with the class to see if other students can determine one or more functional representations.
2. Ask students what relationships they see between the values of the parameters in the two ways of writing a sinusoidal function like those in Questions 7 and 8.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The effect that each of the parameters has on the graph of a sine or cosine function.
- How to express sinusoidal functions using equations involving sine and/or cosine functions by examining their graphs and the parameters in the equations of the graphs.

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Note 1

Question 1, Name of Feature: Quick Poll

You might want to send a Quick Poll to check understanding of the values of A, B, C, and D in the various functional representations of the given sinusoidal graph.

Note 2

Name of Feature: Screen Capture

You might want to use Screen Capture to compare choices for student answers to both 2.1 and 2.2 parts a and c.

Note 3

Name of Feature: Screen Capture

You might want to use Screen Capture to compare choices for student answers to both 3.1 and 3.2 parts a and c.