John Unson, Gates Chili HS, Rochester, NY Time required: 30 minutes

Activity Overview

This lesson is intended to allow students to investigate the angle & arc relationships when 2 secants in a circle are drawn from a common external point. Pages include a statement of the theorem, a dynamic geometry demonstration, several problems that apply the theorem, and a 2-column geometric proof of the theorem.

Teacher Preparation

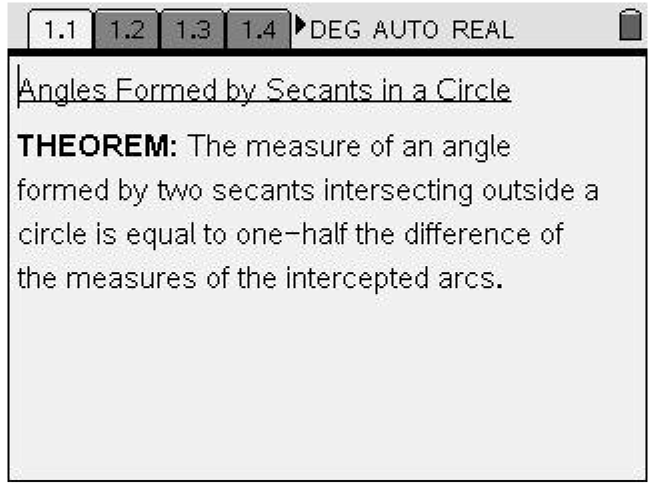
This lesson is created for use in a middle school or high school geometry class.

- Inscribed angles have a vertex on the circle and a measure equal to one-half the measure of the intercepted arc.
- Angles formed by secants drawn from a common external point have a measure equal to one-half the difference of the measures of the intercepted arcs. This is the premise of this lesson.
- Since the geometry application does not have an “arc measure” tool, the measure of an arc is equated to the measure of the central angle that intercepts the arc.

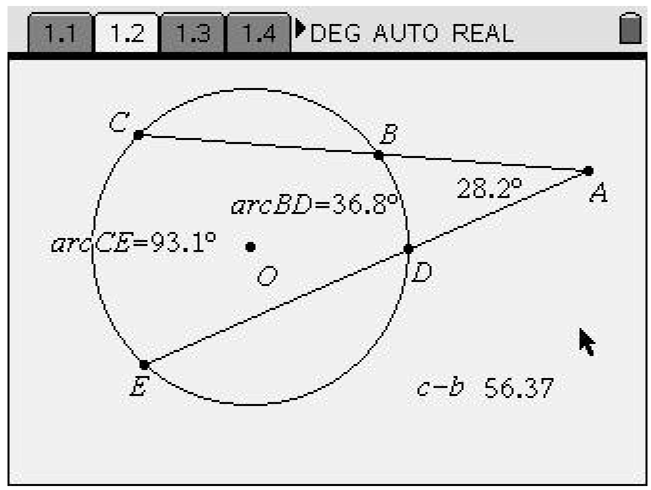
Classroom Management

- This lesson is intended to allow students to investigate the angle-arc relationships using the TI-Nspire Geometry Application.
- The student worksheet contains additional diagrams that allow the student to work on each of the problems, as well as the geometric proof.

On page 1.1, the theorem is stated for the students. Most students will experience greater success working with the variety of angles formed in circles if the location of the vertex (inside, outside, or on the circle) is emphasized. In this case, the secants drawn from a common external point causes the vertex to lie *outside* the circle.



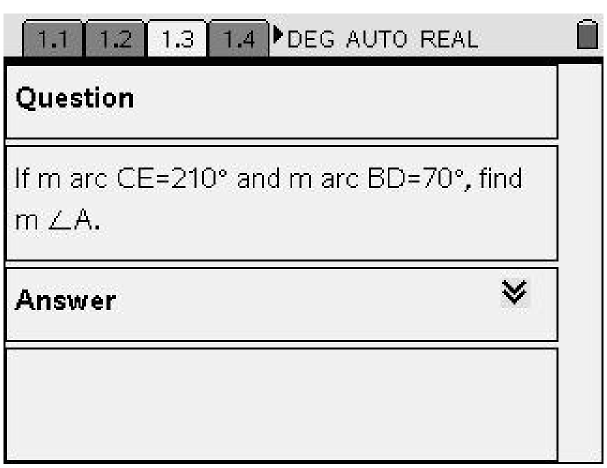
On page 1.2, the Geometry Application allows the student to manipulate the measures of the arcs by dragging the endpoints of the secants, or the location of angle A, to different locations. As the measures of the arcs change, so does the difference, and the student is expected to be able to confirm that angle formed by the secants is in fact, equal to one-half of this difference.



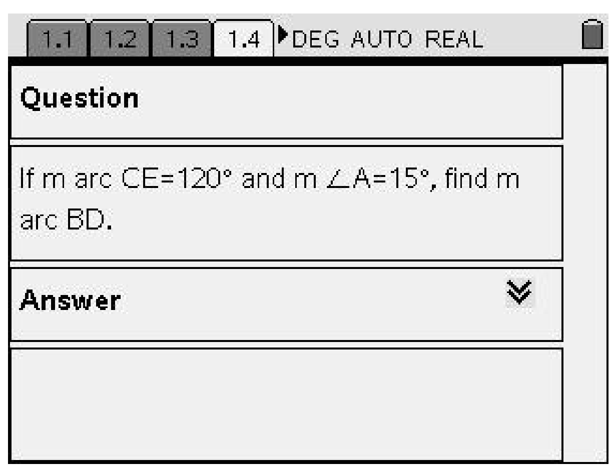
The questions on pages 1.3-1.6 each require use of the theorem and correspond to the diagrams provided on the student worksheet.

Answers:

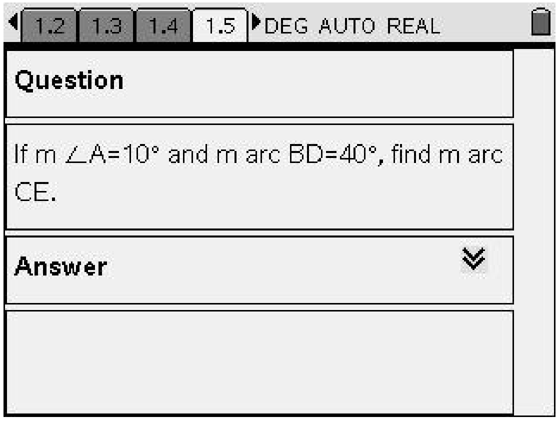
1.3 – m ∠A = 70˚



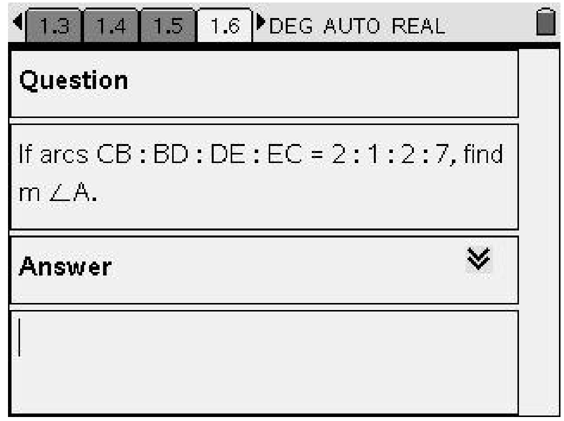
1.4 – m arc BD = 90˚



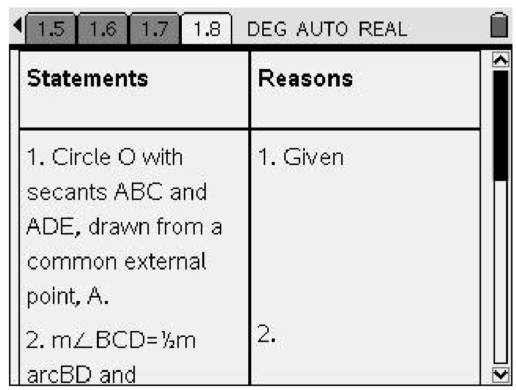
1.5 – m arc CE = 60˚



1.6 – m ∠A = 90˚



Page 1.7 instructs the student to complete the geometric proof that follows.

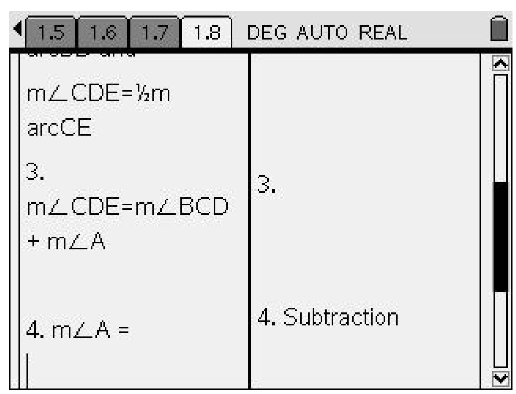


Page 1.8 illustrates a 6 step geometric proof of the theorem.

It is necessary to draw an additional chord (CD) in order to clearly identify the inscribed angles referred to in the proof.

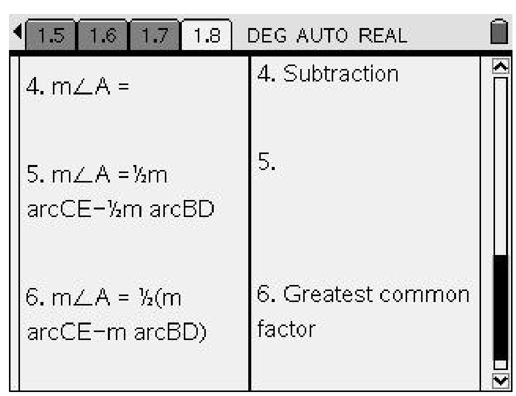
The missing items are:

Reason #2 – The measure of an inscribed angle is equal to one-half the measure of the intercepted arc.



Reason #3 – An exterior angle of a triangle is equal to the sum of the 2 remote interior angles.

Statement #4 – m ∠A = m ∠CDE – m ∠BCD



Reason #5 – Substitution.

The inclusion of the 6th step is only necessitated in order to state the theorem's conclusion in its factored form.