What Goes Up:
Position and Time for a Cart on a Ramp

When a cart is given a brief push up a ramp, it will roll back down again after reaching its highest point. Algebraically, the relationship between the position and elapsed time for the cart is quadratic in the general form

$$y = ax^2 + bx + c$$

where y represents the position of the cart on the ramp and x represents the elapsed time. The quantities a, b, and c are parameters which depend on such things as the inclination angle of the ramp and the cart's initial speed. Although the cart moves back and forth in a straight-line path, a plot of its position along the ramp graphed as a function of time is parabolic.

Parabolas have several important points including the vertex (the maximum or minimum point), the y-intercept (where the function crosses the y-axis), and the x-intercepts (where the function crosses the x-axis). The x- and y-intercepts are related to the parameters a, b, and c given in the equation above according to the following properties:

1. The y-intercept is equal to the parameter c.

2. The product of the x-intercepts is equal to the ratio $\dfrac{c}{a}$.

3. The sum of the x-intercepts is equal to $-\dfrac{b}{a}$.

These properties mean that if you know the x- and y-intercepts of a parabola, you can find its general equation.

In this activity, you will use a Motion Detector to measure how the position of a cart on a ramp changes with time. When the cart is freely rolling, the position versus time graph will be parabolic, so you can analyze this data in terms of the key locations on the parabolic curve.
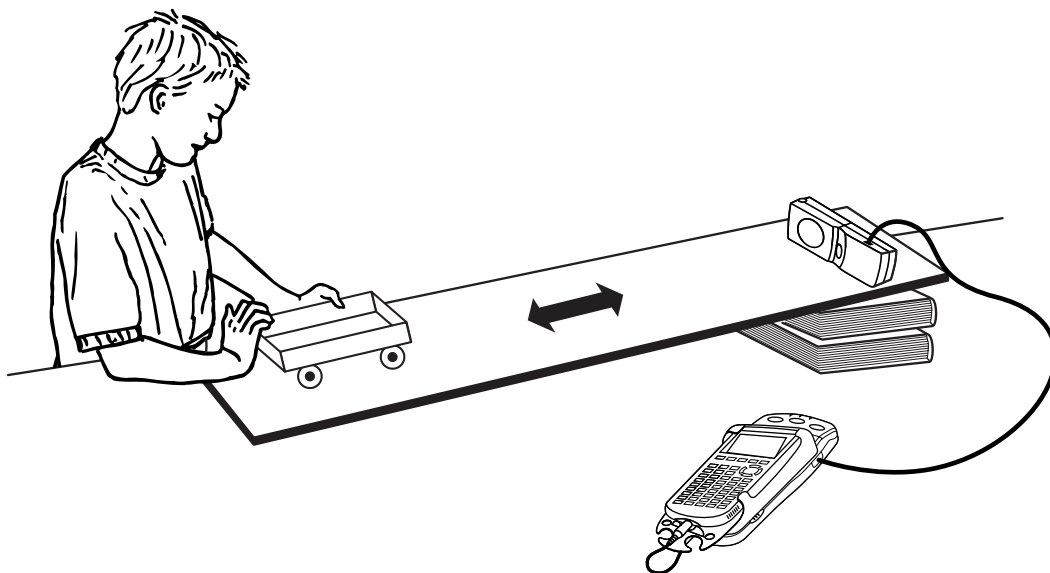
OBJECTIVES

- Record position versus time data for a cart rolling up and down a ramp.
- Determine an appropriate parabolic model for the position data using the *x-* and *y*-intercept information.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
CBR 2 or Go!Motion and direct calculator cable
  or Motion Detector and data-collection interface

4-wheeled cart
board or track at least 1.2 m
books to support ramp

PROCEDURE

1. Set up the Motion Detector and calculator.

   a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
   b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)

2. Place one or two books beneath one end of the board to make an inclined ramp. The inclination angle should only be a few degrees. Place the Motion Detector at the top of the ramp. Remember that the cart must never get closer than 0.4 m to the detector, so if you have a short ramp you may want to use another object to support the detector.

3. Set up EasyData for data collection.

   a. Start the EasyData application, if it is not already running.
   b. Select 〔File〕 from the Main screen, and then select **New** to reset the application.

4. So that the zero reference position of the Motion Detector will be about a quarter of the way up the ramp, you will zero the detector while the cart is in this position.

   a. Select 〔Setup〕 from the Main screen, and then select **Zero…**
   b. Hold the cart still, about a quarter of the way up the ramp. The exact position is not critical, but the cart must be freely rolling through this point in Step 6.
   c. Select 〔Zero〕 to zero the Motion Detector.

5. Practice rolling the cart up the ramp so that you release the cart below the point where you zeroed the detector, and so that the cart never gets closer than 0.4 m to the detector. Be sure to pull your hands away from the cart after it starts moving so the Motion Detector does not detect your hands.

6. Select 〔Start〕 to begin data collection. Wait for about a second, and then roll the cart as you practiced earlier.

7. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The graph should contain an area of smoothly changing distance. The smoothly changing portion must include two $y = 0$ crossings.

   Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select 〔Main〕 to return to the Main screen and repeat Step 6.

ANALYSIS

1. Since the cart may not have been rolling freely on the ramp the whole time data was collected, you need to remove the data that does not correspond to the free-rolling times. In other words, you only want the portion of the graph that appears parabolic. EasyData allows you to select the region you want using the following steps.

    a. From the distance graph, select ⌐Anlyz⌐ and then select **Select Region…** from the menu.
    b. If a warning is displayed on the screen; select ⌐OK⌐ to begin the region selection process.
    c. Use the Ⓓ and Ⓒ keys to move the cursor to the left edge of the parabolic region and select ⌐OK⌐ to mark the left bound.
    d. Use the Ⓓ and Ⓒ keys to move the cursor to the right edge of the parabolic region and select ⌐OK⌐ to select the region.
    e. Once the calculator finishes performing the selection, you will see the selected portion of the graph filling the width of the screen.

2. Since the cart was not rolling freely when data collection started, adjust the time origin for the graph so that it starts with zero. To do this, you will need to leave EasyData.

    a. Select ⌐Main⌐ to return to the Main screen.
    b. Exit EasyData by selecting ⌐Quit⌐ from the Main screen and then selecting ⌐OK⌐.

3. To adjust the time origin, subtract the minimum time in the time series from all the values in the series. That will start the time series from zero.

    a. Press ⌐2nd⌐ [L1].
    b. Press ⌐−⌐.
    c. To enter the min() function press ⌐MATH⌐, use Ⓓ to highlight the NUM menu, and press the number adjacent min(to paste the command to the home screen.
    d. Press ⌐2nd⌐ [L1] again and press ⌐)⌐ to close the minimum function.
    e. Press ⌐STO▶⌐, and press ⌐2nd⌐ [L1] a third time to complete the expression L1 – min(L1) ➔ L1. Press ⌐ENTER⌐ to perform the calculation.

4. You can find the two *x*-intercepts and the *y*-intercept by tracing across the parabola. Redisplay the graph with the individual points highlighted.

    a. Press ⌐2nd⌐ [STAT PLOT] and press ⌐ENTER⌐ to select Plot 1.

    b. Change the Plot1 settings to match the screen shown here. Press ⌐ENTER⌐ to select any of the settings you change.
    c. Press ⌐ZOOM⌐ and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.
    d. Press ⌐TRACE⌐ to determine the coordinates of a point on the graph using the cursor keys.
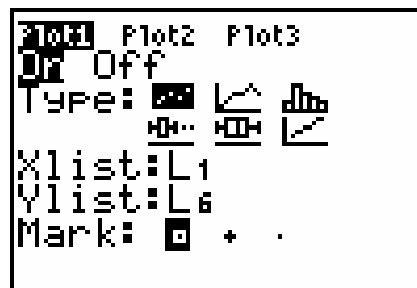
    Trace across the graph to determine the *y*-intercept along with the first and second *x*-intercepts. You will not be able to get to exact *x*-intercepts because of the discrete points, but choose the points closest to the zero crossing. Round these values to 0.01, and record them in the first Data Table on the *Data Collection and Analysis* sheet.

5. Determine the product and sum of the *x*-intercepts. Record these values in the second Data Table on the *Data Collection and Analysis* sheet.

6. Use the intercept values, along with the three intercept properties discussed in the introduction, to determine the values of *a*, *b*, and *c* for the general form parabolic expression $y = ax^2 + bx + c$. Record these values in the third Data Table.

   Hint: Write an equation for each of the three properties; then solve this system of equations for *a*, *b*, and *c*.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

7. Now that you have determined the equation for the parabola, plot it along with your data.

   a. Press ⬭Y=.
   b. Press ⬭CLEAR to remove any existing equation.
   c. Enter the equation for the parabola you determined in the Y₁ field. For example, if your equation is $y = 5x^2 + 4x + 3$, enter 5*x²+4*x+3 on the Y₁ line.
   d. Press ⬭◁ until the icon to the left of Y₁ is blinking. Press ⬭ENTER until a bold diagonal line is shown which will display your model with a thick line
   e. Press ⬭GRAPH to see the data with the model graph superimposed.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

8. You can also determine the parameters of the parabola using the calculator's quadratic regression function.

   a. Press ⬭STAT and use the cursor keys to highlight CALC.
   b. Press the number adjacent to QuadReg to copy the command to the home screen.
   c. Press ⬭2nd [L1] ⬭, ⬭2nd [L6] ⬭, to enter the lists containing the data.
   d. Press ⬭VARS and use the cursor keys to highlight Y-VARS.
   e. Select Function by pressing ⬭ENTER.
   f. Press ⬭ENTER to copy Y₁ to the expression.

      On the home screen, you will now see the entry QuadReg L1, L6, Y1. This command will perform a quadratic regression using the *x*-values in L1 and the *y*-values in L6. The resulting regression line will be stored in equation variable Y1.

   g. Press ⬭ENTER to perform the regression.

   ⇒ Record the regression equation with its parameters in Question 3 on the *Data Collection and Analysis* sheet.

   a. Press ⬭GRAPH to see the graph.

   ⇒ Answer Questions 4-6 on the *Data Collection and Analysis* sheet.

DATA COLLECTION AND ANALYSIS

Name   _____

Date   _____

DATA TABLES

y-intercept	First *x*-intercept	Second *x*-intercept

Product of *x*-intercepts	
Sum of *x*-intercepts	

a	
b	
c	

QUESTIONS

1. Substitute the values of *a*, *b*, and *c* you just found into the equation $y = ax^2 + bx + c$. Record the completed modeling equation here.

2. Is your parabola a good fit for the data?

3. Record the regression equation from Step 8 with its parameters.

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4. Are the values of a, b, and c in the quadratic regression equation above consistent with your results from your earlier calculation?

5. In the experiment you just conducted, the vertex on the parabolic distance versus time plot corresponds to a minimum on the graph even though this is the position at which the cart reaches its maximum distance from the starting point along the ramp. Explain why this is so.

6. Suppose that the experiment is repeated, but this time the Motion Detector is placed at the bottom of the ramp instead of at the top. Make a rough sketch of your predicted distance versus time plot for this situation. Discuss how the coefficient a would be affected, if at all.