

Math TODAY™

Teacher Edition



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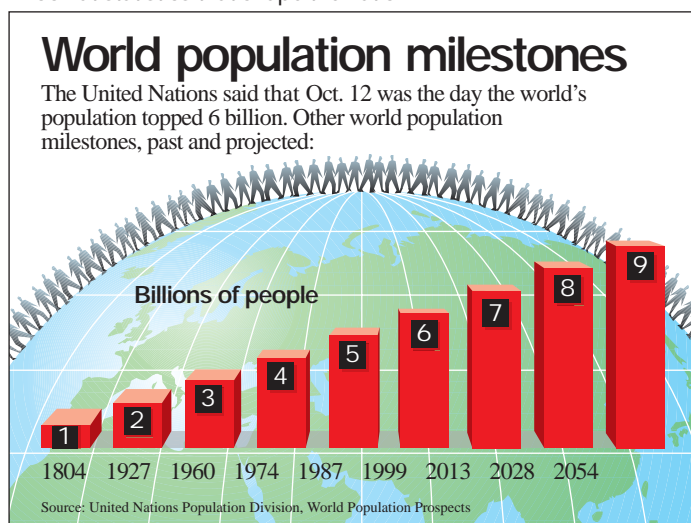


World population milestones

By: Bob Tower

USA SNAPSHOTS®

A look at statistics that shape the nation



By James Abundis and Bob Laird, USA TODAY

Activity Overview:

Students will create a scatter plot of the world population for the years 1950 through 2050. The United Nations Population Division data will allow students to explore real-life data that will be modeled by a logistic function. Students will have the opportunity to explore the end behavior of this model and explain what this means about the world population estimates. The derivative will be used to explore when the world population increased most rapidly during this time period.

Concepts:

- Evaluating, synthesizing and analyzing real-world data with a logistic model
- Limiting behavior (end behavior) of a function/graph
- Instantaneous rate of change/derivative

Objectives:

Students will:

- model real-life data with a logistic function.
- determine and interpret the limiting behavior of the logistic function.
- interpret the instantaneous rate of change at the inflection point of a graph.

Activity at a Glance:

- Grade level: 9-12
- Subject: Calculus
- Estimated time required: 15-30 minutes

Materials:

- TI-83 Plus family or TI-84 Plus family
- Overhead view screen calculator for instruction/demonstration
- Student handout
- Transparency
- USA TODAY newspapers (recommended)

Prerequisites:

Students should know:

- how to create a scatter plot with the calculator.
- how to use the regression capabilities of the calculator.
- the nDeriv feature of the calculator and how to create a graph using this feature.
- how to use the table and trace features of the calculator.



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Background:

This activity presents the students with the opportunity to explore the logistic function with real-life data. The students will use their knowledge about derivatives to determine when the most rapid increase in the world population occurred. This activity will give the students practice with instantaneous rate of change rather than average rate of change. Many population models start exponentially but eventually will slow down and approach a limit to growth called the carrying capacity.

Preparation:

- Provide one graphing calculator for each student.
- Each student should have a copy of the corresponding student activity sheet.

Classroom Management Tips:

- Students will have a better understanding of how to read the graphic and retrieve data if you use the transparency for a class discussion before the students start working.
- Students can work individually or in small groups on this activity. Working in groups is especially helpful as they learn the various features of the calculator.
- Allow students to talk about the "how" and "why" approach they used to find the solutions.
- Students may need to be reminded about how to use nDeriv with the calculator to create a graph of the derivative.
- Remind students to expand their window on their calculator if they are going to trace their graph to determine the limiting behavior.
- Use the years shown in the Snapshot "World population milestones" and determine the years when the logistic model did not reflect the population shown. Discuss with the class why this may happen with a regression model.
- This would be a good time to have the students determine the limit of the logistic regression model analytically and compare this answer with the limit they found graphically or numerically.

Data Source:

United Nations Population Division,
World Population Prospects

National Council of Teachers of Mathematics (NCTM) Standards*:

Algebra Standard

- Understand patterns, relations, and functions.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Problem-Solving Standard

- Solve problems that arise in mathematics and in other contexts.

Connections Standard

- Recognize and apply mathematics in contexts outside of mathematics.

*Standards are listed with the permission of the National Council of Teachers of Mathematics (NCTM), www.nctm.org. NCTM does not endorse the content or validity of these alignments.

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Activity Extension:

- Visit the United Nations Population Division site at www.un.org/esa/population/unpop.htm and look at the population of the United States or some other country or region to determine if the population data would be modeled by a logistic function.
- Population growth starts exponentially which shows unlimited growth. Then other factors may influence the growth so that it begins to slow down. Factors that may influence this growth are the environment and economics. Look in USA TODAY and find articles that address growth of a population or factors that may limit this growth and bring these articles to class for discussion.

Curriculum Connections:

- Biology
- Chemistry

Additional Resources:

- Student handout
- Transparency
- TI Technology Guide, for information on the following: TI-83 Plus family, TI-84 Plus family and List Editor
- TI-Navigator™ Basic Skills Guide for information on using the TI-Navigator Classroom Learning System

Teacher Notes:

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Assessment and Evaluation:

Activity 1:

Determine the logistic model for the world population data from 1950 through 2050. Use this model to determine the population in 1987 and compare this to the population shown in the Snapshot "World population milestones."

A. The following screen shots show the logistic regression model and the estimate for the world population in 1987 using the calculator. The population in 1987 would be 5 (one significant digit) or 5.0 (rounded to tenths).

```

Logistic
y=c/(1+ae^(-bx))
a=6.1689158E26
b=.0310222571
c=10.32879715
    
```

```

Y1(1987)
5.046901411
    
```

The world population found by the calculator does show that the population exceeded 5 billion in 1987 and this confirms the information shown in the Snapshot.

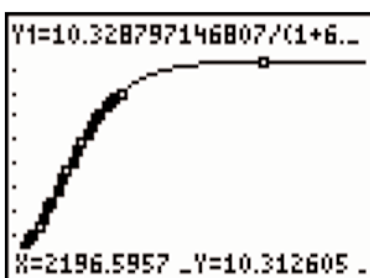
Activity 2:

Explain the limiting behavior of the logistic model and what this means.

A. The window settings below will allow the students to graphically see the limiting behavior of the logistic function. Students can trace and observe what happens to the population based on this model as shown in the second screen shot. The graph approaches 10 (two significant digits) or 10.3 when rounding to tenths. If students were to find the limit analytically of the logistic model the value is 10.328797146807.

```

WINDOW
Xmin=1940
Xmax=2300
Xscl=1
Ymin=1.412
Ymax=12
Yscl=1
Xres=1
    
```



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Assessment and Evaluation:

Q. Explain what this limiting behavior means.

A. The relative growth rate is positive but is slowing down due to other factors such as the environment.

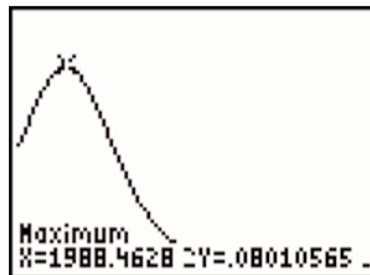
Activity 3:

Determine during which year the world population increased most rapidly.

The graph of the derivative has a maximum at 1988 which confirms that the population increased most rapidly during that year.

```

Plot1 Plot2 Plot3
\Y1=10.328797146
807/(1+6.1689157
890196E26e^(-.03
102225712932X))
\Y2=Deriv(Y1,X,
X)
\Y3=
    
```



If you are using the TI-Navigator Classroom Learning System, send the provided LearningCheck assessment to your class to gauge student understanding of the concepts presented in the activity. See the TI-Navigator Basic Skills Guide for additional information on how this classroom learning system may be integrated into the activity.