EXPLORATIONS

Activity 8

How Fast for Whiplash— Going with the Flow

Teacher Notes

Concepts

- Using ratios to model change
- Direct variation
- Average rates of change

Calculator Skills

- Using decimals in modeling
- Building tables with ratios

Materials

- TI-30X IIS
- Student Activity pages (p. 77-78)
- Transparency

Objective

- In this activity, students will learn to recognize direct variation as a rate of change and to apply rates of change in problem situations. They also will learn to use average rates of change to make decisions in problem situations.

Topics Covered

- Applying the process of mathematical modeling to real-world problem situations
- Making connections between mathematics and other disciplines
- Representing situations that involve variable quantities with algebraic expressions

Introduction

Skaters sometimes play a game called Pop the Whip, in which they form a revolving line by joining hands, and the person on the inside grabs a steel pole as the center of rotation. Each individual in the revolving line will skate at a speed that is dependent upon their distance from the center of rotation. How are speed and distance related in this situation?

Investigation

1. Display the transparency. The data in the table represents speeds and distances collected from one Pop the Whip game. Use the overhead calculator to find the ratio of $\frac{\text{speed}}{\text{distance}}$ and record the results in the third column. (The first two are done for you, and the remaining answers are shown below.)

Speed (ft/sec): s	Distance from pole (ft): d	Speed/distance ratio
1.70	2	0.8500000000
3.75	4.5	0.8333333333
5.92	7	0.8457
7.95	9.5	0.8368
10.1	12	0.8417

2. Point out that there is a common ratio of about 0.84 feet/second of speed for each one foot distance from the pole (center of rotation).
3. Explain that when the ratio of two variables is constant, the rate of change is called a *direct variation*. This means that if $\frac{y}{x} = m$, for some constant value m, then y = mx.
4. Work with the students to help them find that the direct variation equation for the example would be $\frac{s}{d} = 0.84$, so s = 0.84d.
5. Use this equation to predict the speed of a skater who is 18 feet from the pole.

   Speed = 0.84 X 18 = 15.12 feet per second!

Wrap-Up

Use graphs to show how direct variation and average rates of change are different.

Extension

A child's shoe size is an example of a variable showing a positive rate of change over time. Ask students to give an example of a variable that will show a negative rate of change over time. For this variable, have them estimate the rate of change and explain the estimate.

Solutions

For each of the following tables in the Student Activity pages, find the common ratio (if one exists). If there is a common ratio, write an equation relating the two variables.

1. A bungee jumping apparatus is used to model the amount of stretch by using an elastic band and fishing weights. The different amounts of stretch, in centimeters, are given in the table for the amount of weight, in ounces, on the end of the elastic band.

Amount of Stretch (centimeters): s	Weight (ounces): w	Ratio of Stretch/Weight
13	2	*6.5*
26	4	*6.5*
32.5	5	*6.5*
45.5	7	*6.5*
65	10	*6.5*

Common Ratio? *6.5* Equation: *s = 6.5w*

a. What would be the amount of stretch (in centimeters) for 3 ¼ ounces What of weight?

21.125 cm

b. How much weight (in ounces) would stretch the elastic band 50 centimeters?

7.69 oz.

2. Wind resistance encountered by a cyclist in a race is measured for different speeds and given in the table.

Wind resistance (pounds): r	Speed (miles/hour): s	Ratio of resistance/speed
0.41	12	0.034166667
0.72	16	0.045
0.91	18	0.050555556
1.12	20	0.056
1.36	22	0.061818182

Common Ratio? *No* Equation: *None*

3. The table below shows sales of LPs and CDs in the United States from 1984 to 1988 (in millions).

Year	1984	1985	1986	1987	1988
LPs	208.8	164.9	125.6	99.8	74.1
CDs	5.1	25.2	54.7	100.3	151.2

For each time period in the table below, use the data above to find the actual change in LP sales and the actual change in CD sales.

Time Period	LP Sales	CD Sales
1984-1985	-43.9	*20.1*
1985-1986	*-39.3*	*29.5*
1986-1987	*-25.8*	*45.6*
1987-1988	*-25.7*	*50.9*

4. If you were a music producer in the year 1988, how many CDs would you produce in 1989?

   More than 151.2 million! Probably about 187.7 million.

5. How many LPs would you have produced in 1989?

   Fewer than 74.1 million. Probably about 40.4 million.

6. In what year (if any) would you cease to produce LPs? Justify your answer.

   In 1990 or 1991

Student Activity 8

Name _____

Date _____

Finding Rates of Change—How Fast for Whiplash

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Common Ratio? Equation:

a. What would be the amount of stretch (in centimeters) for 3 ¼ ounces of weight?

b. How much weight (in ounces) would stretch the elastic band 50 centimeters?

2. Wind resistance encountered by a cyclist in a race is measured for different speeds and given in the table.

Speed (miles/hour)	Wind resistance (pounds)	Ratio of resistance/speed
12	0.41	
16	0.72	
18	0.91	
20	1.12	
22	1.36	

Common Ratio? Equation:

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5. How many LPs would you have produced in 1989?

6. In what year (if any) would you cease to produce LPs? Justify your answer.

TRANSPARENCY
Pop the Whip Game

Speed (ft/sec) *s*	Distance from pole (ft): *d*	Speed/distance ratio
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3.75	4.5	0.833333333
5.92	7	
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10.1	12	