Birthday Buddies!
By Stephen Arnold
Grade level: secondary (Years 7-12)
Subject: mathematics
Time required: 90 minutes

Activity overview

What is the chance of sharing a birthday with someone in your class? This simple question offers a rich context for mathematical modeling, which is potentially accessible to students from the early years of secondary school to seniors. Using *TI-Nspire*, students are offered the tools by which they can investigate the problem and build a meaningful model, which will deepen their understanding of the problem, and help them to further appreciate the applications of mathematics to their world.

Background

The Birthday Buddy problem is a well-known challenge which offers two key advantages as a context for mathematical modeling: it is personal (students of any age love tasks which are about *them*!) and the result is surprising! The counter-intuitive nature of the result reinforces for students the value of systematic and cross-representational modeling as a means of making sense of the real world.

Concepts

Interpreting graphs, probability (including complementary events), plotting data

Teacher preparation

This task requires no formal preparation. Teachers may, if they wish, gather student birthday data (days in column **A** and months in column **B**) and examine this in a scatter plot, having students locate their position on the screen. This will invite early consideration of the "birthday buddies" question.

Classroom management tips

This task is essentially a modeling exercise in which students collect data and attempt to build a graphical understanding of the problem. Initial discussion should focus on early perceptions of the problem – what do you think is the chance in a group the size of our class? What do you think happens as the group gets bigger?

As a class, discuss the use of **y = x** as a model: how well does this fit their expectations? Is it heading in the right direction, or would it need to be reversed? If the general shape of the graph is correct, then what will happen at the boundaries? For very small values of **x** (when **x = 0** and **x = 1**) and for very large values of **x** (> 365)? Using segments, it is possible to build early models of their understanding.

Students may then be paired with the *TI-Nspire* Birthday Buddies tool, allowing them to try different values and to build a scatter plot of the graph. Obviously, persistence and accuracy will be of benefit here. Working in pairs allows one student to generate the data and the other to enter it manually into lists, building a cooperative approach to problem solving, and encouraging discussion while they work. Alternatively, this stage could be run from a display unit, with students suggesting numbers to try, and then recording the results.

A later phase of the task involves working towards an algebraic solution. This should begin with physical involvement: have one student come to the front of the class, and ask all to imagine that this student is the only person in the room. Suggest that the task may be better approached by considering the chance of two people NOT sharing the same birthday. While this first student may choose from any of 365 days (with a probability of 365/365) the next student entering, not wishing to share the same birthday, may choose from only 364 out of 365 days! The chance of them sharing the same birthday, then, must be this product subtracted from 1. Continuing this line of thinking should assist most students to appreciate the product form of the function when it is introduced.

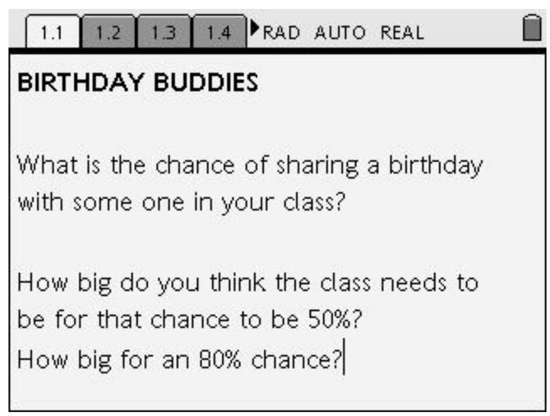
Technical prerequisites

Students should know how to:

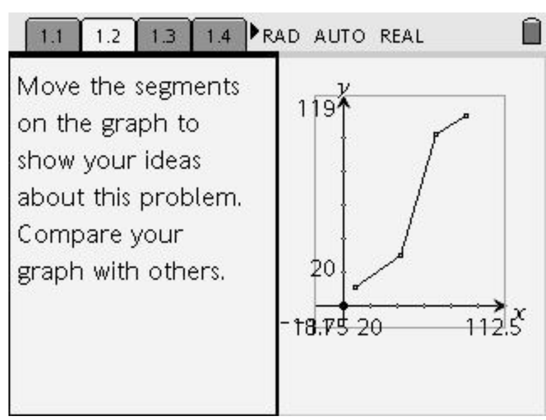
- Use function notation as a means of substituting values [**Calculator**];
- Enter data manually into lists and create a Scatter Plot [**Lists & Spreadsheet**];
- Create and graph functions [**Graphs & Geometry**].

Step-by-step directions

1. Begin with class discussion of the core question: What is the chance of two people in the same class sharing a birthday? (Variations may include: how many students do you think you would need for a 50% chance? An 80% chance? What do you think happens to the probability as the class size increases?)

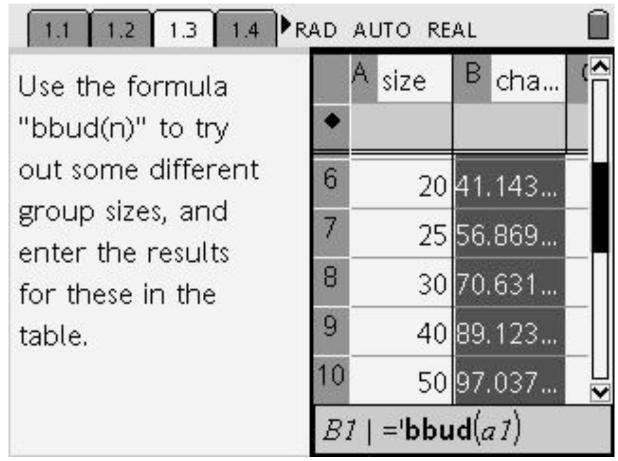


Students may work on their own devices and be brought back together to share, or this phase could be a group activity, led from the front. It is most important that students use this opportunity to build meaning and their own ideas concerning this problem. They are also being encouraged to move from numerical to graphical thinking in this context.



The use of line segments can be a powerful aid to student thinking and visualization. Initially, the focus should be on the general direction of the graph; later, it moves to the extremes: what is happening for very small and very large values. Segments can be used to build a graphical estimate of the function.

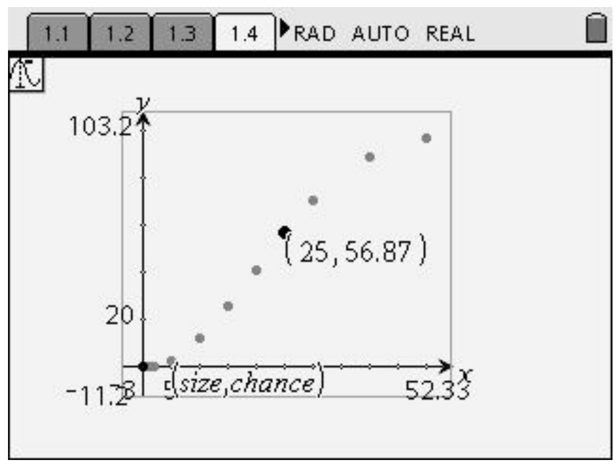
2. Students now use the tool provided to generate values, exploring the problem numerically at first, and then working towards a graphical representation.



3. For younger students, this could be the extent of the activity. Most, though, will want to know how it all works! This involves a careful building up of steps, ideally using physical involvement of students!

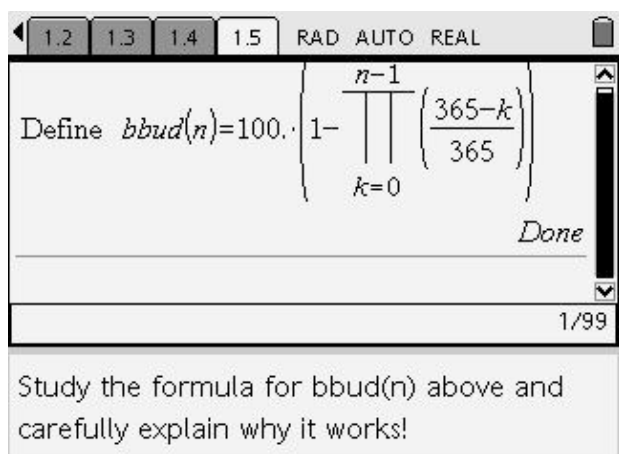
   Have a student come to the front of the room and inform the class that this is the only person in the room. This person is free to choose any of 365 days out of 365 for their birthday! (Note the introduction of the probability ratio concept in the wording!)

   Another person enters the room – in order NOT to share the same birthday, this person may choose only from 364 out of 365 days. The probability of these two people NOT sharing the same birthday, then, is given by the product of the two fractions. Invite students to discuss and continue this line of reasoning.



4. The final phase of the task, then, involves building an algebraic function which models this process. Students may develop this themselves (individually, in pairs or in small groups) or they may be given the function and be required to explain how and why it works! The function should be examined and verified both numerically (using *Lists & Spreadsheet* Tool) and graphically (using *Graphs & Geometry* Tool).

   This interpretation of multiple representations is an important learning outcome of this modeling activity.



(Possible CAS extension)

Assessment and evaluation

As for any modeling experience, students should be expected and required to document their process carefully, and to clearly explain both their assumptions and the implications of their results.

Many aspects of this activity should give students cause to stop and think. The number required for a 50% chance is only 23 students: by the time the class size reaches 30, the probability is already over 70% of two people sharing the same birthday. It is easy, especially in mathematics, to be so absorbed by the computations (and, in the case of technology, the button pushing!) that one forgets to stop and reflect – part of the teacher's responsibility lies in drawing attention to those aspects of the experience that are worthy of further discussion!

The quality of student responses will, of course, vary enormously. Some will simply use the tools provided to produce specific answers which show little evidence of deep understanding or thoughtful reflection. Such tasks are great examples of situations in which specific answers are not the goal – rather, it is the process that matters here, and the depth of observation and questioning which accompanies this learning journey.

The primary learning outcome desired from this activity is that students become more confident and capable modelers: that they have learned skills which can be applied to other questions which may be asked, both by themselves and others. The offering here is a carefully sequenced and scaffolded modeling experience which provides a framework for future investigations.

Activity extensions

As an extension of the initial suggested collection of birthday data into lists, sort the data in one list with respect to the other, and observe that the scatter plot remains unchanged except for the order of tracing – it is easier to identify shared birthdays now. For some extension work on correlation, both lists could then be sorted independently and the scatter plot assumes almost a linear form, which students may then try to manually fit a straight line to. In this way, the key concepts of correlation and line of best fit are introduced meaningfully.

The principal extensions activity arising from this task concerns the simple question, "Why?" Why does the probability rise so rapidly? This is a surprising result, and students should be encouraged to explore it further, even if there is no clear-cut answer available.