Chords and Circles

ID: 9423

Time required
30 minutes

Activity Overview

Students will begin this activity by exploring how the chord in a circle is related to its perpendicular bisector. Investigation will include measuring lengths and distances from the center of the circle. These measurements will then be transferred to a graph to see the locus of the intersection point of the measurements as the endpoint of a chord is moved around the circle. In the extension, students will be asked to find an equation for the ellipse that models the relationship.

Topic: Circles

- *Deduce from the Perpendicular Bisector Theorem the following corollaries:*
  - *a) The perpendicular from the center of a circle to a chord bisects the chord.*
  - *b) The line joining the center of a circle to the midpoint of a chord is perpendicular to the chord.*
  - *c) The center of a circle is at the intersection of the perpendicular bisector of two non-parallel chords.*

Teacher Preparation

This activity is designed to be used in a high school geometry classroom.

- *Students should already be familiar with circles, chords of circles, and perpendicular bisectors.*
- *This activity assumes a basic working knowledge of the TI-Nspire handheld device, such as drawing shapes and figures and finding lengths of segments other measurements.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9423" in the keyword search box.***

Associated Materials

- *ChordsandCircles_Student.doc*
- *ChordsandCircles.tns*
- *ChordsandCircles_Soln.tns*

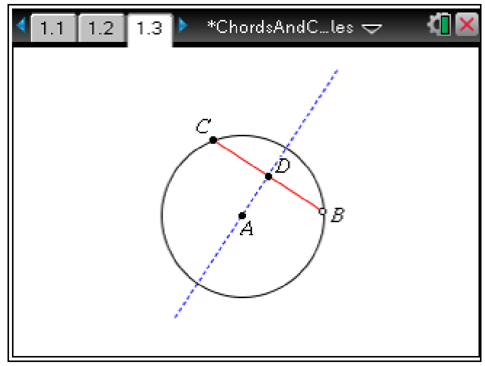
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Chords of a Circle (TI-Nspire technology) — 13178*
- *Circle Geometry: Property of the Segments of Two Chords Intersecting within a Circle (TI-Nspire technology) — 8340*

Problem 1 – Relationship between a chord and its perpendicular bisector

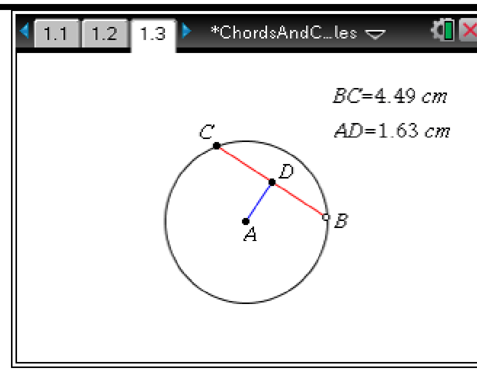
On page 1.3, students are given a circle *A* where point *D* is the midpoint of a chord *BC*. As they drag point *B* around the circle, they should observe that the perpendicular bisector of $\overline{BC}$ always passes through the center of the circle.



TI-Nspire Navigator Opportunity: *Screen Capture*

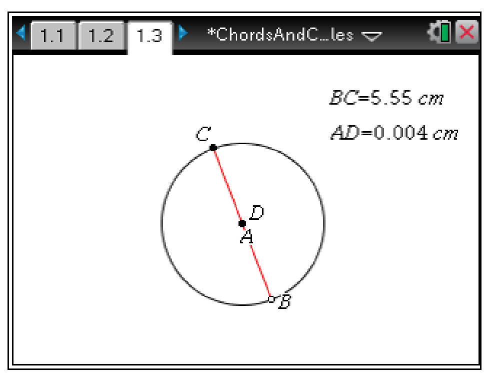
See Note 1 at the end of this lesson.

Next, students should use the **Hide/Show** tool from the Actions menu to hide $\overleftrightarrow{AD}$, followed by the **Segment** tool from the Points & Lines menu to draw $\overline{AD}$ (replacing the line with a segment).



Then they should use the **Length** tool (**MENU > Measurement > Length**) to display the lengths of $\overline{AD}$ and $\overline{BC}$. To label each measurement as shown in the screenshot to the right, you can double click in the text box, arrow to the beginning of the text using the NavPad, and type the appropriate label and equals sign.

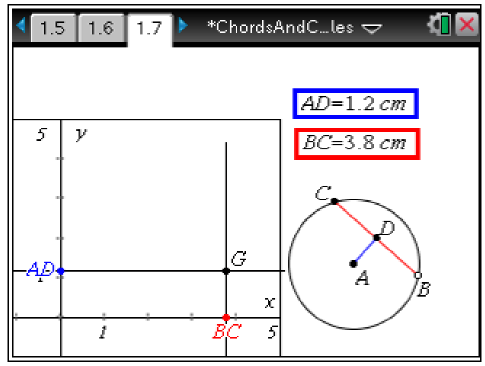
Direct students to drag *B* around the circle and ask how the lengths of $\overline{AD}$ and $\overline{BC}$ are related. They should conjecture that the lengths have an inverse relationship: As the length of chord $\overline{BC}$ increases, the distance from the chord to the center (represented by the length of $\overline{AD}$) decreases, and vice versa. They should also observe that when points *A* and *D* coincide (i.e., the length of $\overline{AD}$ is zero), the chord $\overline{BC}$ is a *diameter* of the circle.



TI-Nspire Navigator Opportunity: *Screen Capture*

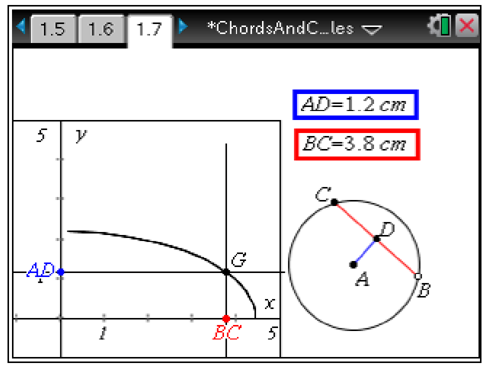
See Note 2 at the end of this lesson.

On page 1.7, the length of $\overline{BC}$ has been transferred to the *x*-axis and the length of $\overline{AD}$ has been transferred to the *y*-axis. Using the **Perpendicular** tool (**MENU > Construction > Perpendicular**), students are to construct a line perpendicular to the *x*- and *y*-axes through their respective points, Next, students should use the **Attributes** tool (Actions menu) to change the appearance of the lines and the **Intersection Point(s)** tool (Points & Lines menu) to mark their intersection, point *G*.



Dragging point *B* around the circle, students can watch the path of point *G*. Then they can use the **Locus** tool from the Construction menu to show the path, clicking first on point *G* and then on point *B*.

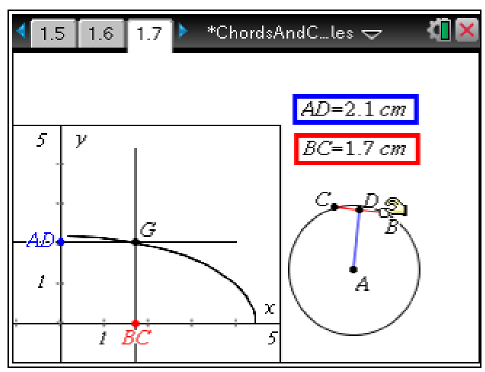
To help answer the questions on the worksheet, students should label point *G* with its coordinates using the **Coordinates and Equations** tool from the Actions menu.



TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 3 at the end of this lesson.

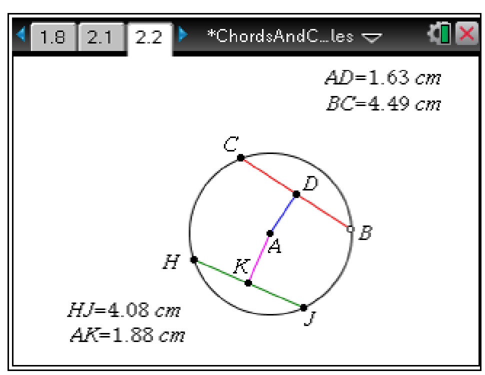
Ask students to describe the graph as they move point *B* around the circle. They should find that $\overline{AD}$ is a radius of the circle when point *G* coincides with the *y*-intercept and that $\overline{BC}$ is a diameter of the circle when point *G* coincides with the *x*-intercept. As point *G* moves from left to right, meaning that $\overline{BC}$ is getting longer, the *y*-coordinate of point *G* decreases.



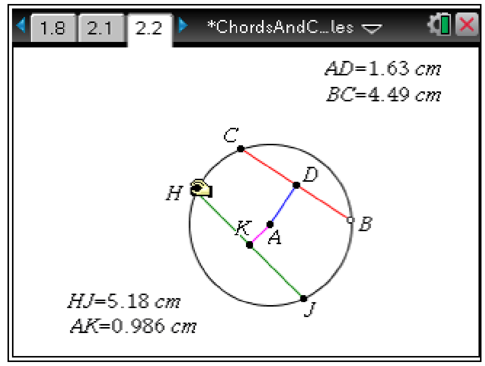
Problem 2 – Investigating congruent chords

On page 2.2, students will first draw a chord ($\overline{HJ}$) of the circle using the **Segment** tool. Then they can construct the midpoint (point *K*) of the chord using the **Midpoint** tool from the Construction menu, and use the **Segment** tool once again to connect the midpoint to the center of the circle ($\overline{AK}$).

Next, students should measure and label the lengths of $\overline{HJ}$ and $\overline{AK}$.



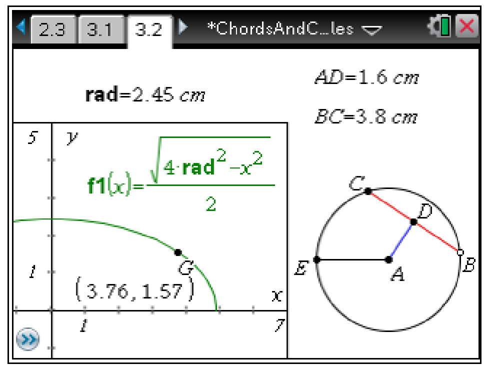
As students drag *H* or *J* around the circle, ask students to observe what happens as the lengths of the two chords approach the same measure. They should determine in Question 7, that the (perpendicular) distance from the center of a circle to congruent chords is always equal.



Note: It may be less frustrating on the part of the students in matching these segment lengths if they change the **Attributes** of their measurements to one decimal place.

Problem 3 – Extension

On page 3.2, students will find a diagram similar to the one they encountered on page 1.7. Have them display the radius of circle *A* using the **Length** tool to find the distance from point *A* to point *C* (A segment does not have to be drawn to measure the distance between two points.).



Next, they should store the radius as the variable **rad**. This can be accomplished by pressing ctrl + menu, select **STORE,** clicking once on the measurement, pressing ctrl + var, and typing the name for the variable.

Challenge students to find and graph the equation, in terms of the radius, of (the "top half" of) an ellipse that matches the locus.

Solutions

1. What is true about the perpendicular bisector of $\overline{BC}$?

   It always passes through the center of the circle.

2. How does the length of $\overline{BC}$ relate to that of $\overline{AD}$?

   They are inversely proportional. That is, as $\overline{BC}$ gets bigger, $\overline{AD}$ gets smaller and vice versa.

3. What happens to $\overline{BC}$ when point *D* coincides with point *A* (i.e., when the length of $\overline{AD}$ is zero)?

   It becomes the diameter of the circle.

4. What is true about $\overline{BC}$ and $\overline{AD}$ when *G* coincides with the *y*-intercept of the locus? With the *x*-intercept?

   $\overline{BC}$ is zero when *G* coincides with the *y*-intercept of the locus.

   $\overline{AD}$ is zero when *G* coincides with the *x*-intercept of the locus.

5. As point *G* moves from left to right, what happens to its *y*-coordinate?

   It decreases.

6. What does this mean in terms of $\overline{BC}$?

   $\overline{BC}$ **increases.**

7. What is the relationship between congruent chords of a circle and their respective distances from the center of the circle?

   They are always equal.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are correctly manipulating the movable point *B*.

Note 2
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are correctly hiding the perpendicular bisector and adding the segment $\overline{AD}$. This is also a good place to make sure students are measuring the segments and affixing labels appropriately.

Note 3
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are correctly constructing the intersection point, *G*, as well as the using the **Locus** tool properly.