Reflective Property of Parabolas

Materials

TI-Nspire™ or TI-Nspire CAS™

Overview

This activity involves exploring the reflective property of a parabola. It will include concepts from Algebra 2 (Conic Sections), Geometry (Parallel Lines, Perpendicular Lines, Reflections, and Proof), and Calculus (Derivatives, and Tangent Line). The mirrors in some telescopes as well as satellite dishes are parabolic in shape. This activity will look at the reason behind why this is so.

Activity

1. Open the file Paraboloids.tns.
2. To go to Page 1.3, press ctrl ▶ ctrl ▶. Observe the effects as you drag point P, and change the curve with slider **a**.
3. The mirrors in some telescopes as well as satellite dishes are parabolic in shape. Can you now explain why?

Proof (see Page 1.5)

1. Consider the parabola $y=ax^2$. Let the point P be a point on $y=ax^2$ with coordinates (x_1, y_1). Write the equation of the tangent line to $y=ax^2$ at the point $P(x_1,y_1)$.

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2. Find the x-intercept of this tangent line.

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   We will call this point G.
3. Recall that the definition of a parabola is the locus of points in a plane which are equidistant from a given point, called the focus, and a given line, called the directrix. Let f be the distance from the focus to the vertex. Let the focus be point F with coordinate (0,f). Let Q be a point on the directrix such that $\overline{PQ}$ is perpendicular to the directrix. Point Q will have coordinates______________________
4. Use the Midpoint Formula to find the midpoint of $\overline{FQ}$ and show that the midpoint of $\overline{FQ}$ is point G.

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5. Since G is the midpoint of $\overline{FQ}$, _________ $\cong$ ___________
6. By the definition of a parabola, P is equidistant from F and Q, so _________ $\cong$ ___________
7. $\overline{PG} \cong \overline{PG}$ by the ______________________Property of Congruence.
8. Δ__________ $\cong \Delta$ ___________ by the ____________ ____________ ___________ Postulate.
9. $\angle FPG \cong \angle$ __________ corresponding parts of congruent triangles are congruent.
10. Let R be a point on $\overleftrightarrow{PQ}$, and T be a point on $\overleftrightarrow{PG}$ (see Page 1.5). $\angle RPT \cong \angle$ ______________ because vertical angles are congruent.
11. From Step 9, $\angle QPG \cong \angle FPG$. Therefore $\angle$ ____________ $\cong \angle$ ___________ by the Transitive Property of Congruence.
12. Therefore any light ray that travels down a vertical line parallel to the axis of symmetry will bounce off of the parabola at point P and be reflected towards point F, the focus of the parabola.