Perpendicular Slopes – ID: 8973

Time required
45 minutes

Topic: Linear Functions

- *Graph lines whose slopes are negative reciprocals and measure the angles to verify they are perpendicular.*

Activity Overview

In this activity, students investigate the “negative reciprocal” relationship between the slopes of perpendicular lines. The final phase of the activity is appropriate for more advanced students as they are led through an algebraic proof of the relationship. Optional geometric activities (Problems 5 and 6) use the result to verify that (1) the radius of a circle and its tangent line are perpendicular and (2) a triangle inscribed in a circle with the diameter as one side is a right triangle.

Teacher Preparation

This activity is appropriate for students in Algebra 1. It is assumed that students have recently been introduced to the notion of slope and perhaps the fact that two lines are parallel if and only if they have the same slope.

- ***To download the student files PERP1-6, go to education.ti.com/exchange and enter “8973” in the quick search box.***

Classroom Management

- *This activity is designed to have students explore* ***individually and in pairs****. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on the slope of perpendicular lines.*
- *This worksheet is intended to guide students through the main ideas of the activity. You may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *Information for an optional extension is provided at the end of this activity; information for students is provided in the CabriJr Files* ***PERP5*** *and* ***PERP6****.*

TI–84 Plus Applications

CarbriJr

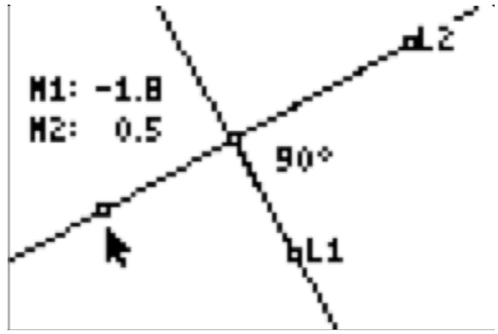
Perpendicular Slopes

ID: 8973

In this activity, you will explore:

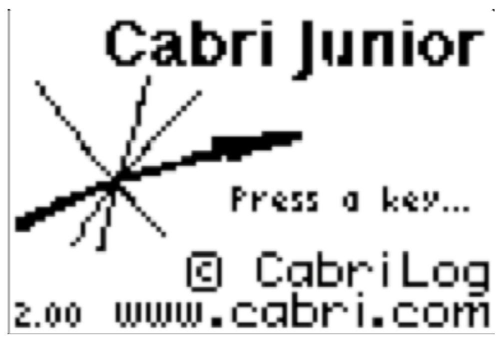
- *an algebraic relationship between the slopes of perpendicular lines*
- *a geometric proof relating these slopes*

Use this document as a reference and record your answers on a separate sheet of paper.



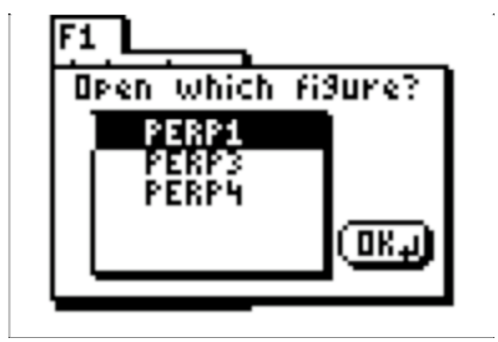
Problem 1 – An initial investigation

Open the **Cabir Jr.** app by pressing APPS and choosing it from the menu. Press ENTER. Press any key to begin.



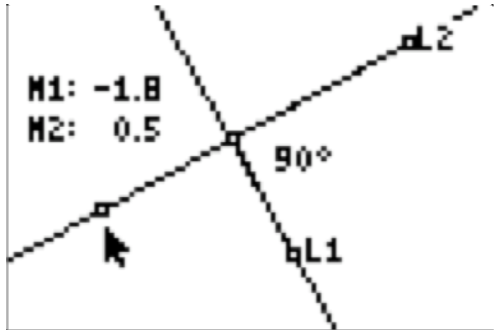
The calculator displays the Cabri Jr. window. Press Y= to open the **F1: File** menu. Arrow down to the **Open…** selection and press ENTER.

Choose figure **PERP1** and press ENTER.

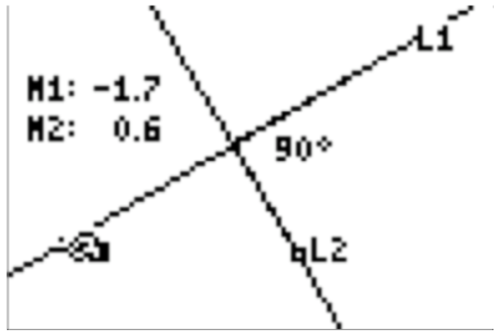


Two lines are displayed:
line *L1* with a slope of *m1* and line *L2* with a slope of *m2*.

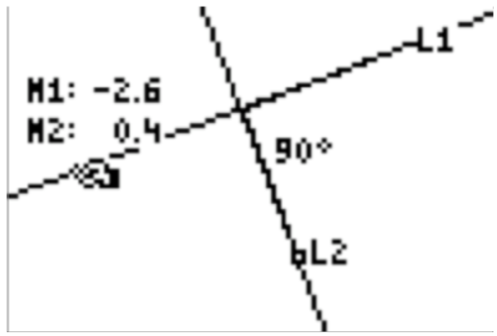
Notice that the angle formed by the intersection of the lines measures 90°; that is, the two lines are perpendicular.



Grab line *L1* by moving the cursor over the point pressing ALPHA. The cursor turns into a hand to show that you have grabbed the point.



Rotate *L1* by dragging the point using the arrow keys. Observe that as the slopes of the lines change, the two lines remain perpendicular. Explore the relationship between the slopes by answering the questions below.



1. Can you rotate *L1* in such a way that *m1* and *m2* are both positive? Both negative?

2. Can you rotate *L1* so that *m1* or *m2* equals 0? If so, what is the other slope?

3. Can you rotate *L1* so that *m1* or *m2* equals 1? If so, what is the other slope?

4. Rotate *L1* so that *m1* is a negative number close to zero. What can be said *m2*?

5. Rotate *L1* so that *m1* is a positive number close to zero. What can be said about *m2*?

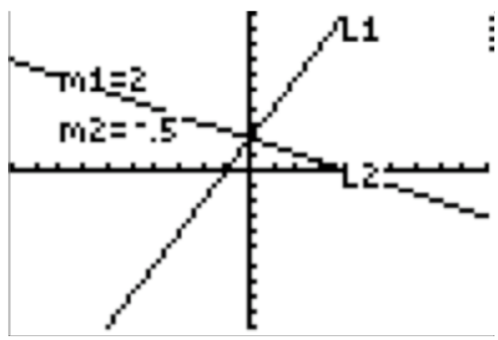
Problem 2 – A closer examination

Now that you have observed some of the general relationships between the slopes of two perpendicular lines, it is time to make a closer examination. Press [2nd] + [MODE] to exit Cabri Jr.

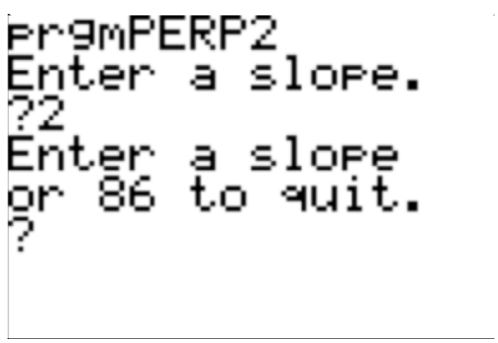
Press [PRGM] to open the program menu. Choose **PERP2** from the list and press [ENTER] twice to execute it.

Enter a slope of 2 and press [ENTER].

The program graphs a line *L1* with the slope you entered and a line *L2* that is perpendicular to *L1*. *m1* is the slope of *L1* and *m2* is the slope of *L2*.

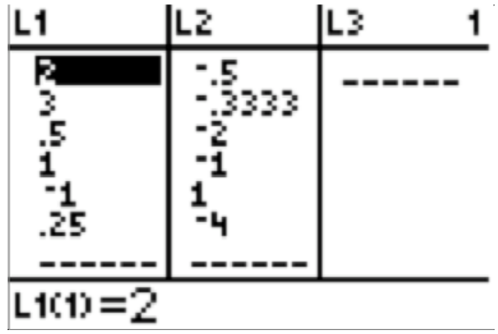


Press [ENTER] and the calculator prompts you for another slope. Use the graph to complete the following.



1. Enter **0** to make the slope of *L1* equal to 0. What is the slope of *L2*?
2. What is the slope of *L2* when the slope of *L1* is 1?
3. What is the slope of *L2* when the slope of *L1* is –1?

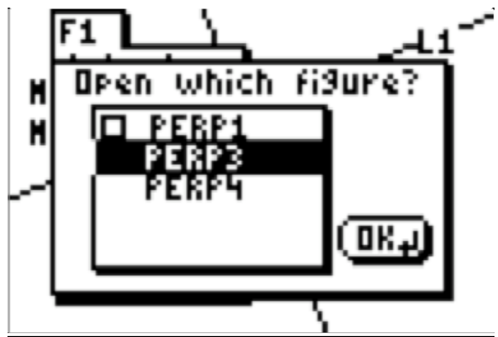
Enter other values for the slope of *L1* and examine the corresponding slope of *L2*. For each slope that you enter, *m1* and its corresponding value of *m2* are recorded in the lists $\mathbf{L_1}$ and $\mathbf{L_2}$. To see a history of your "captured" values, enter a slope of **86** to exit the program. Then press [STAT] and [ENTER] to enter the **List Editor**. The values of *m1* are recorded in $\mathbf{L_1}$ and the values of *m2* are recorded in $\mathbf{L_2}$.



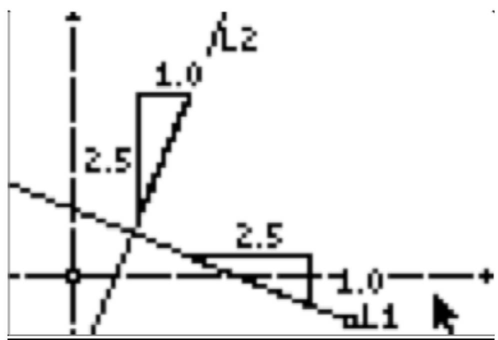
4. Conjecture a formula that relates the slope of two perpendicular lines. Enter your formula in the top of $\mathbf{L_3}$ (with variable $\mathbf{L_1}$) to test your conjecture.

Problem 3 – A geometric look

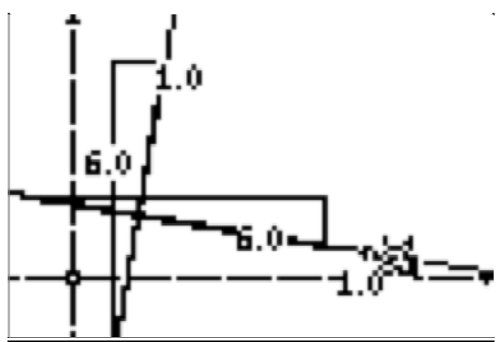
Start the Cabri Jr. app and open the file **PERP3**.



This figure shows another way to examine the slopes of perpendicular lines, geometrically.



Grab line *L1*, rotate it, and compare the rise/run triangles.



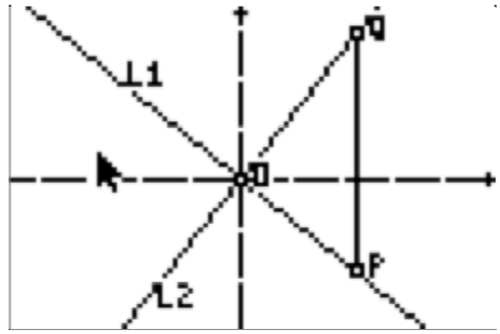
1. What do you notice about the two triangles?

Problem 4 – The analytic proof

We now will analytically verify that two lines with slopes *m1* and *m2* are perpendicular if and only if $m1 \cdot m2 = -1$.

(All of the following assumes *m1* ≠ 0. What can be said about the case when *m1* = 0?)

Open the CabriJr file **PERP4**. This graph shows two perpendicular lines *L1* and *L2* with slopes *m1* and *m2* respectively, translated such that their point of intersection is at the origin. Refer to the diagram to answer the questions below.

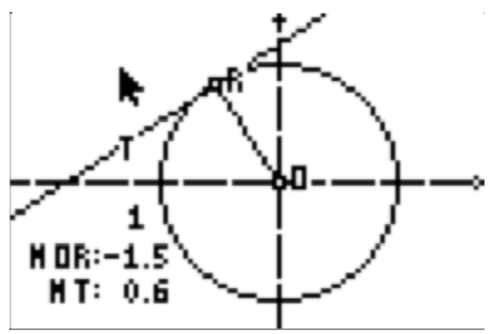


1. What are the equations of these translated lines as shown in the diagram?
2. Let *P* be the point of intersection of line *L1* and the vertical line *x* = 1 and let *Q* be the point of intersection of line *L2* and the line *x* = 1. What are the coordinates of points *P* and *Q*?
3. Use the distance formula to compute the lengths of $\overline{OP}$, $\overline{OQ}$, and $\overline{PQ}$.(Your answers should again be in terms of *m1* and *m2*.)
4. Apply the Pythagorean Theorem to triangle *POQ* and simplify. Does this match your conjecture from Problem 2?

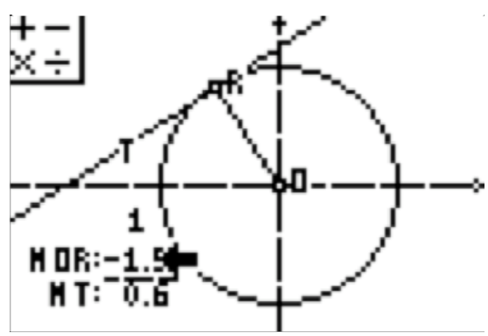
Problem 5 – Extension activity #1

The CabriJr file **PERP5** shows a circle with center *O* and radius *OR*. Line *T* is tangent to the circle at point *R*.

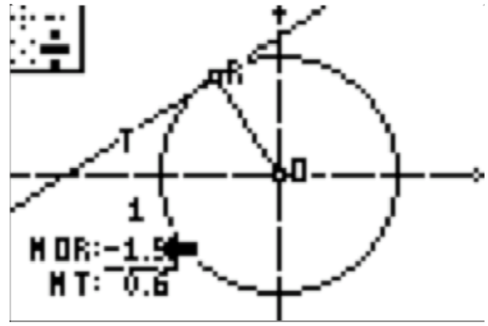
The slopes of line *T* and segment *OR* are shown (*mT* and *mOR*, respectively.)



Your first task is to calculate $\frac{1}{mOR}$. Activate the **Calculate** tool, found in the **F5: Appearance** menu. Move the cursor over 1 and press ENTER. Repeat to select *mOR*, the slope of the segment *OR*.



Press [÷] to divide the two numbers. Drag the quotient to a place on the screen where you can see it clearly and press [ENTER] again to place it.

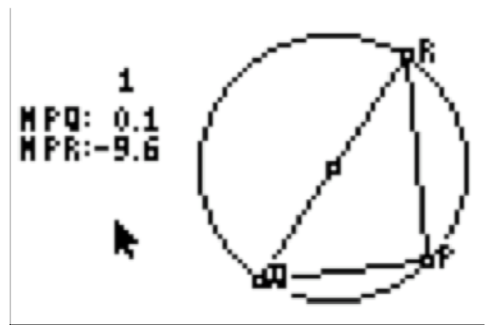


1. Grab point *R* and drag it around the circle. Observe the changing values of *mT*, *mOR*, and $\frac{1}{mOR}$. What can you conjecture about the relationship between a tangent line to a circle and its corresponding radius?

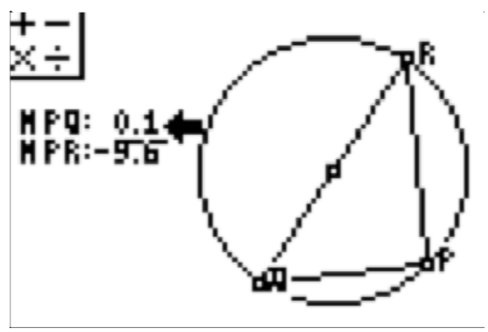
Problem 6 – Extension activity #2

The CabriJr file **PERP6** shows a circle with an inscribed triangle *QPR*. The segment *QR* is a diameter of the circle.

The slopes of segments *PR* and *PQ* are shown (*mPR* and *mPQ*, respectively.)



Compute $\frac{1}{mPQ}$ using the **Calculate** tool.



1. Grab point *P*, drag it around the circle, and examine the changing values. What can you conjecture about a triangle inscribed in a circle such that one side is a diameter?