In this activity, students will

- identify and calculate maximum and/or minimum values in a linear programming model

- apply linear programming to find optimal solutions to real-world problems

- express and interpret constraints, using inequalities

- solve problems using graphing technology

- graph equations and inequalities and analyse graphs with technology

- investigate and make and test conjectures about the solution to equations and inequalities using graphing technology

> ## **Vocabulary**
>
> Inequality   Optimal Solution
>
> Feasible Region   Constraint

Linear programming is a technique used to solve problems that are encountered in business and industry. These problems usually involve maximizing or minimizing profit or expenses. The solution will consist of graphing the region that satisfies all the inequalities. The solution will produce a feasible region and the vertices that will yield a maximum profit or a minimum cost.

Example 1: Panther Electronics is going to build computers for the students of the new high school. Two types will be produced: a laptop and a desktop. The laptop requires 1 hour putting the components together and 3 hours to test the components. The desktop requires 2 hour putting the components together and 1 hour to test the components. The maximum number of any type they can make in one week is 6 and each week the company is allotting 10 hours for putting the components together and 14 hours for testing. Write the constraints and sketch the graph that shows the feasible region of the points that represent the number of laptops and desktops that can be made. If the profit on a laptop is $40 and the profit on a desktop is $50, how many of each computer should be made each week to make a profit?

Introduction

Load the program algebra_linearprogramming.tns onto the TI-nSPIRE calculator. You should also have the student worksheet to record any answers to questions asked in the program to hand into your teacher when you are done.

Open the algebra_linearprogramming.tns on your TI-nSPIRE and follow the directions. The first few screens will give you a brief overview of linear programming and an introduction to your problem. Move to page 1.5 when you are ready to begin the problem. Remember to use the (ctrl) ▶ keys to move between pages and (ctrl) (tab) to move from one side of a screen to another.
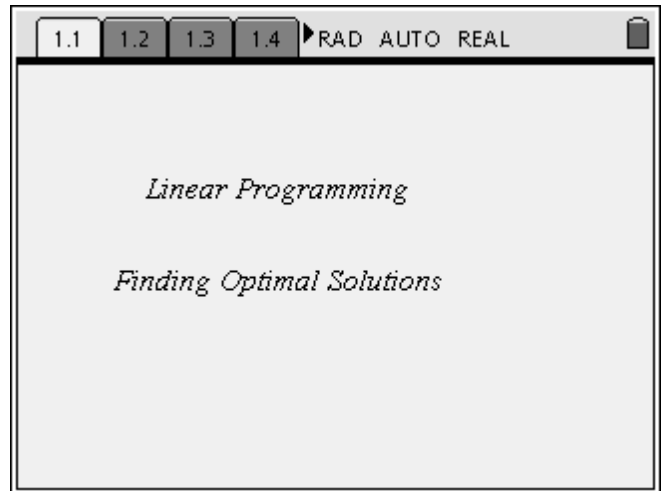

Constraints

The first step is to write your inequations. To do this, you have to consider all the constraints (restrictions) of the problem and then write inequations that will be used to write the graph. The first question on the student worksheet provides you with a table in which to enter the constraints. This will provide the data needed to write the inequations. Go to the student worksheet and fill in question 1.

Graphing

Next we need to have the inequations in a form to graph them. On the student worksheet, solve the inequations in terms of the variable y. Your new inequations will be in the form $y = mx + b$. When done, go on to page 1.7 by pressing (ctrl) ▶ to start graphing!

Answer Question 4 on your student worksheet. We need to change our window on the graph: use (menu) **1: Tools**: **4: Window**, **6: Quadrant 1.**

Now that the window is set, you now have to start graphing. Graph each inequation on pages 1.8, 1.9, and 1.10 adding one inequation at a time. This allows you to see your feasible region develop.

Graph the first
inequation:
y <= −0.5x + 5

6.77

2

−1.0 2 8.18
1.35

✷ ▧ f3(x)=
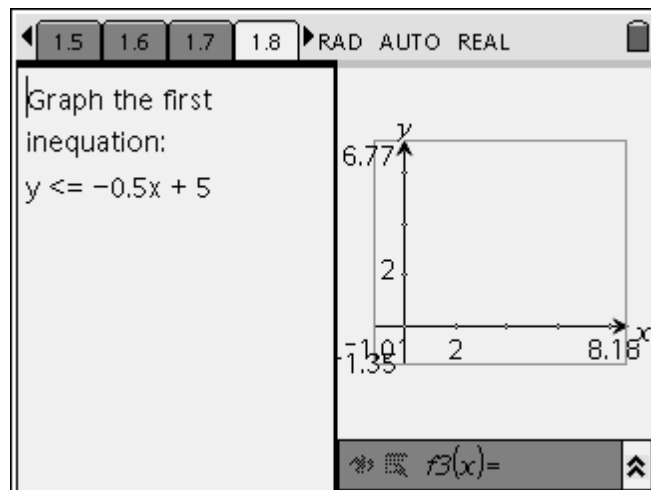
*Finding the Coordinates of the Feasible Region*

After graphing the inequations it is necessary to find the coordinates of the feasible region. There are three different tasks we have to do.

1. *Along the y-axis*. Add the 2 points by choosing ⟨menu⟩: **1: Tools, 6: Points and Lines, 1: Point.** Move the cursor over the *y*-intercept point of the feasible region. Press Enter. You should now see the point. Repeat for the point where the *y*- and *x*- axes cross. Next choose ⟨menu⟩: **1: Tools, 6: Coordinates and Equations**. Move the pointer over the two points you just made and you

Add a point (menu,
6:, 1) at the
y−intercept of the
feasible region.
Choose the tools
menu, then 6, run
the cursor over the
points along the
feasible region on
the y axis. Press

15

(0,5)

−1 4 2 10.84
(0,0)
y≤−x+6
y≤−.5·x+5

✷ ▧ f1(x)=

should see the points appear in gray. If you hit enter, these points darken. Record the two *y*-intercept points in question 5 of the student worksheet.
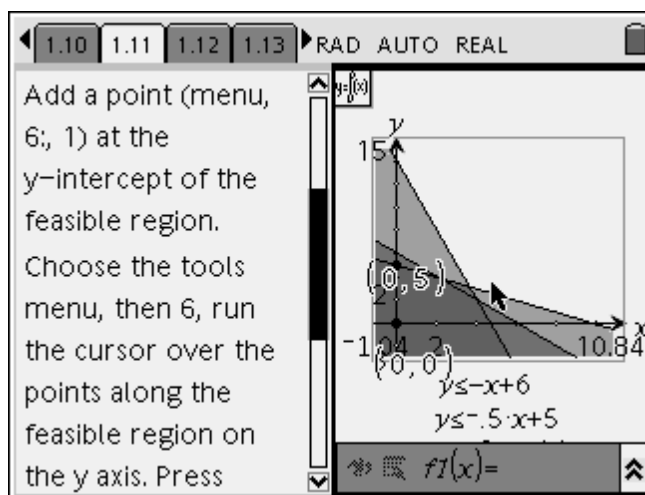
2. The next two points of the feasible region are found by using the intersection points command. On page 1.13, choose ⟨menu⟩: **6: Points and Lines, 3: Intersection Points**. Move the cursor over the two lines you want to find the intersection of, you will see both the

point and the coordinates of that point appear. Press Enter or click to darken the coordinates of this intersection point. Repeat this procedure for the second intersection point in the feasible region. Finally, record these two points in the student worksheet.
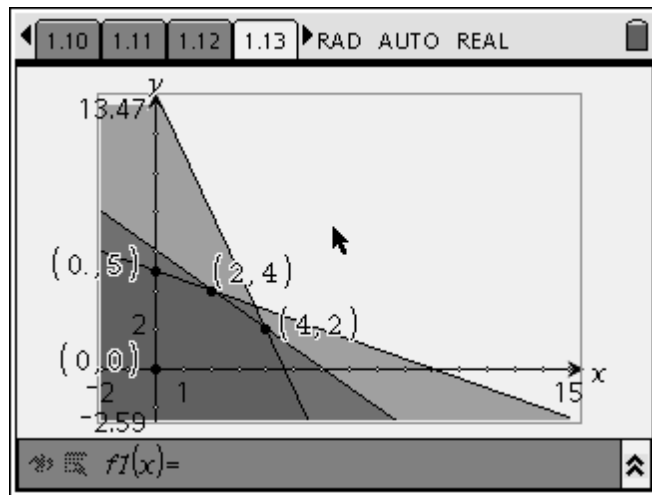

You can also find these points using (menu): **1: Tools, 6: Points and Lines, 1: Point**, as you used in #1.

3. *Zero Point*: The last point in the feasible region is the point on the *x*-axis. In order to find this point we will use the Trace feature. Choose (menu): **1: Tools, 5: Trace**. Move the point down a little on the line $y \le -3x+14$. Click to position the point. Press (esc). Move the hand on the point and move it down to the x-axis. You know when you are to stop because you see a z appear on the screen
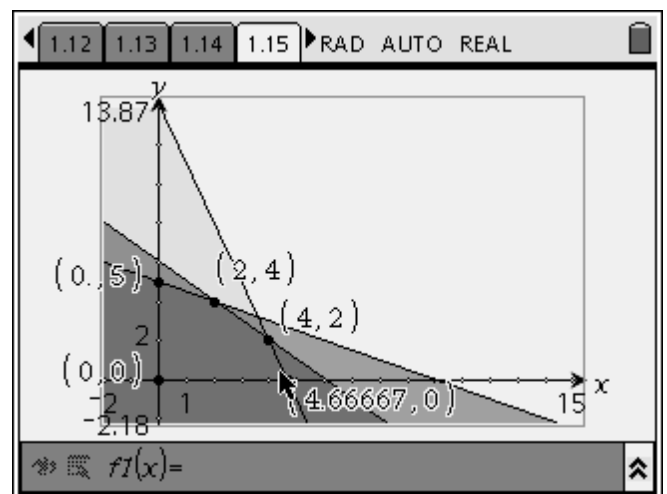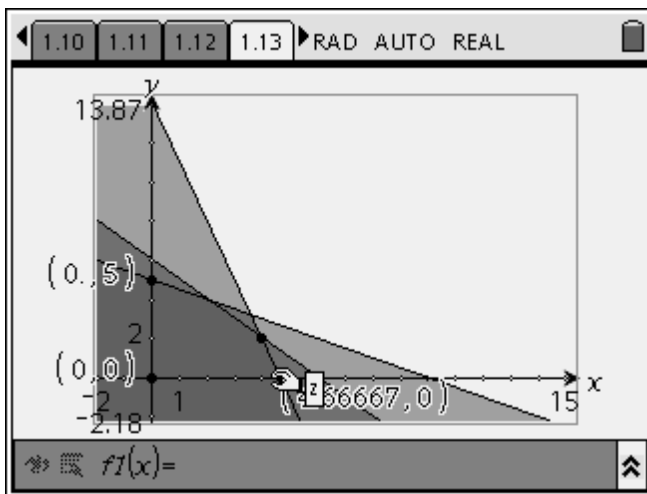
You can use Attributes to improve the precision of the zero point. Go under ⓜ**: 1: Tools, 3: Attributes**. Click on the point, choose ⓜ**: 1: Tools, 3: Attributes**. You should see a window pop up that shows you that the number 4.66667 has six significant figures. You can increase or decrease this level by clicking on the top box. Decrease the number of significant figures so that there are zero decimal places. *Why would we do this?*
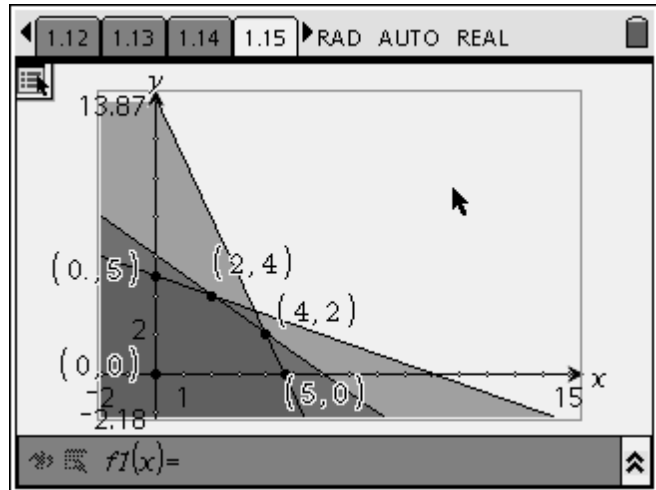
Final Product

After finding all of the vertices of the feasible region, you have to place these into the profit equation from the original problem. This will help determine the maximum profit but also how many laptops and desktops should be made each week.

It is also important to think what effect technology has had in the problem-solving process for graphing linear inequations. Before handing in your assignment, take a moment to write or produce a graphical representation of how you think this technology has affected your learning of graphing and analyzing linear inequations.
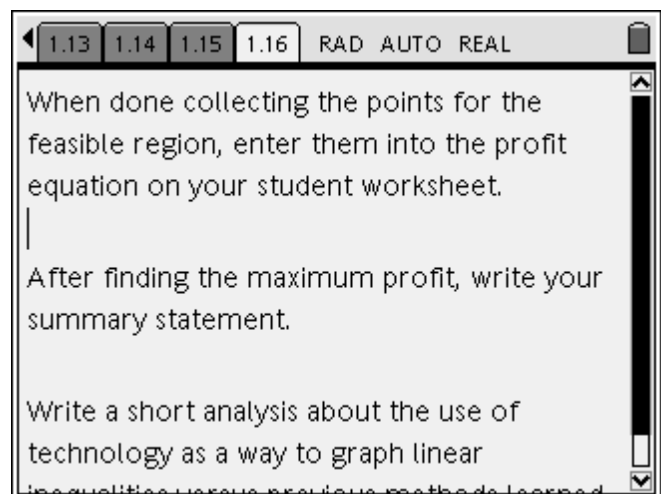
Inequalities

$x \geq 0$

$y \geq 0$

$x + 2y \leq 10$ }   1 hour to put the components together in laptops and 2 hours

to put the components together in desktops. Company only allotted

10 hours maximum per week for this job.

$3x + y \leq 14$ }   3 hour to test the components in laptops and 2 hours

to test the components in desktops. Company only allotted

14 hours maximum per week for this job.

$x + y \leq 6$ Company can only make a maximum of 6 computers in one week.

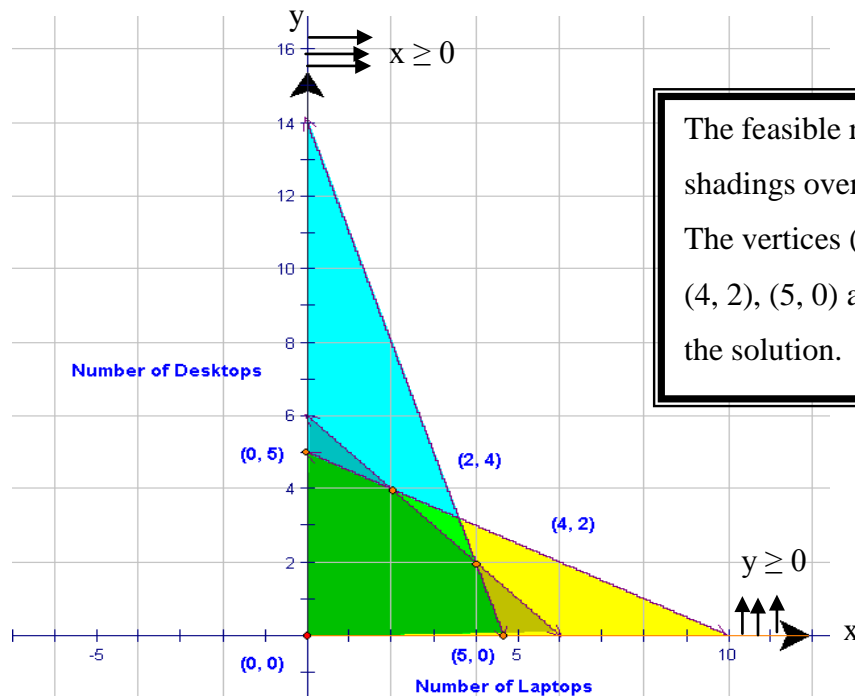|		
$x + 2y \leq 10$	$3x + y \leq 14$	$x + y \leq 6$
$2y \leq -x + 10$	$y \leq -3x + 14$	$y \leq -x + 6$

$\dfrac{2y}{2} \leq \dfrac{-x}{2} + \dfrac{10}{2}$

$y \leq \dfrac{-1}{2}x + 5$

The feasible region where the shadings overlap (green area). The vertices (0, 0), (0, 5), (2, 4), (4, 2), (5, 0) are also included in the solution.

Test (0, 0)

$$x + 2y \leq 10$$

$$0 \leq -\tfrac{1}{2}(0) + 10$$

$0 \leq 10$ {True} Therefore shade the region that contains (0, 0)

Test (0, 0)

$$y \leq -3x + 14$$

$$0 \leq -3(0) + 14$$

$0 \leq 14$ {True} Therefore shade the region that contains the point (0, 0).

Test (0, 0)

$$y \leq -x + 6$$

$$0 \leq -(0) + 6$$

$0 \leq 6$ {True} Therefore shade the region that contains the point (0, 0).

Profit equation

$P(x, y) = 40x + 50y$

$P(0, 5) = 40(0) + 50(5)$

$P(0, 5) = \$250$

$P(2, 4) = 40(2) + 50(4)$

$P(2, 4) = \$280$

$P(4, 2) = 40(4) + 50(2)$

$P(4, 2) = \$260$

$P(5, 0) = 40(5) + 50(0)$

$P(5, 0) = \$200$

Therefore the largest profit is made when the company makes 2 desktops and 4 laptops a week.