More Power to Ya!

ID: 11308

Time Required
10–15 minutes

Activity Overview

In this activity, students will use a script to explore the Power Rule. Students will graphically and algebraically discover the derivative of x^n. They will examine "true" statements about derivatives of x^n where n is an integer. They will observe patterns, and use these patterns to create a rule for finding the derivative of x^n with respect to x. They will then use their rule to create examples of their own.

Topic: Power Rule

- *Derivative of x^n with respect to* x
- *Definition of a derivative*

Teacher Preparation and Notes

- *This activity uses the $\frac{d}{dx}f(x)$ notation to denote the derivative. Some students may attempt to type the letter d in the numerator of a fraction and dx in the denominator. This will not work. The script models the notation to use with* [2nd] *8. d(function, variable).*
- *With TI-Connect, send power2ya.89t to your TI-89. From VAR-LINK (*[2nd] [−]*), press* [F3] *and send the file to students 89. Their TI-89s must say Receive first.*
- ***To download the student TI-89 script file (.89t) and student worksheet, go to education.ti.com/exchange and enter "11308" in the keyword search box.***

Associated Materials

- *MorePowerToYa_Student.doc*
- *Calculus.Power2ya.89t*

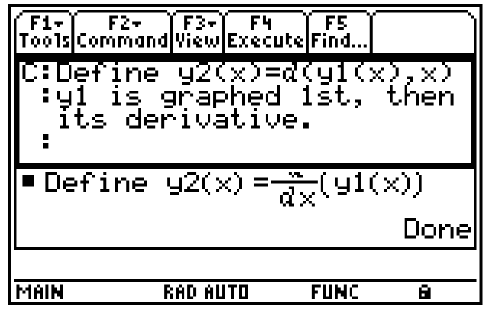
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

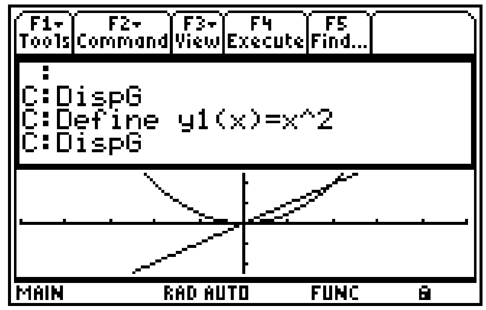
- *The Derivative of a Polynomial (TI-Nspire technology) — 9858*

Problem 1 – Graphical Exploration

To run the script, press [APPS], select **Text Editor**, and open *power2ya*. Press [F3] to select "Script view." Press [F4] to execute each command line. Students should read each line as they execute the commands, paying close attention to those that do not begin with **C:**.



By observing the graph of the function and its derivative, students should begin to predict the relationship between y(x) = x^n and its derivative.

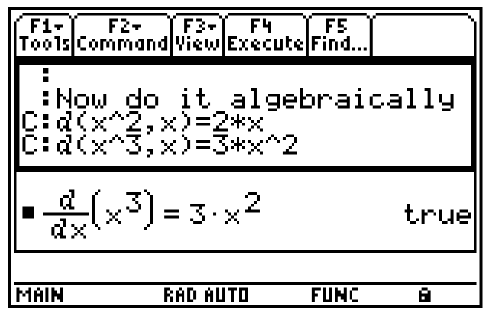


Student Solutions

- The degree of the derivative is one less than that of the original function.

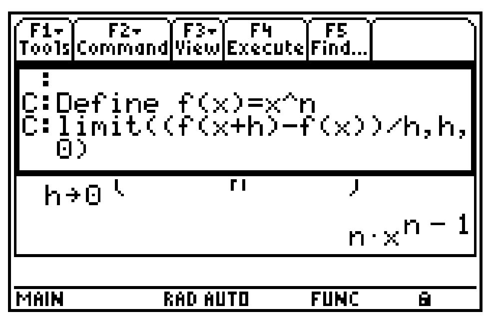
Problem 2 – Defining the Derivative of x^n

After the graphical exploration, students continue the script to examine four true statements of the derivative of x^n with respect to x where n is an integer. They are then asked to observe any patterns. After coming up with a pattern, students will test several examples of their own on bottom half of the screen. To toggle between applications on a split screen, press [2nd] + [APPS].



Students should create a rule for taking the derivative of x^n that fits the given examples and the examples that they created.

When students finish the script they find the derivative of x^n with respect to x, using the definition of a derivative. They are to compare this result with the rule they just developed. Students should be aware that this result is called the Power Rule.



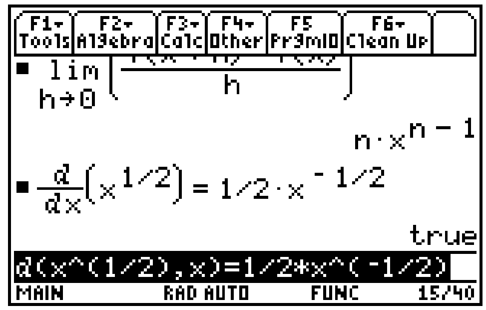
Student Solutions

- The exponent becomes the coefficient of x and the degree is one less.

- Sample answers: $\frac{d}{dx}x^6 = 6 \cdot x^5$, $\frac{d}{dx}x^7 = 7 \cdot x^6$, $\frac{d}{dx}x^{-2} = -2 \cdot x^{-3}$, $\frac{d}{dx}x^{-3} = -3 \cdot x^{-4}$
- Sample answer: $\frac{d}{dx}x^n = n \cdot x^{n-1}$
- Sample answer: The result is the same as the rule I found.

Extension

On the worksheet students are asked if the Power Rule applies when *n* is a non-integer, rational number. They should use the HOME screen to test their conjecture.



Student Solution

- Yes, the Power Rule also applies when *n* is a non-integer, rational number.

  For example, $\frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$

Students are asked to expand the binomial $(x + h)^n$ on a separate piece of paper. They will use this to evaluate the limit previously evaluated to prove the Power Rule by hand.

Student Solution

- $$\lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$
$$= \lim_{h\to 0}\frac{\left({}_nC_0x^n + {}_nC_1x^{n-1}h + {}_nC_2x^{n-2}h^2 + \ldots + {}_nC_{n-1}xh^{n-1} + {}_nC_nh^n\right) - x^n}{h}$$
$$= \lim_{h\to 0}\frac{\left({}_nC_1x^{n-1}h + {}_nC_2x^{n-2}h^2 + \ldots + {}_nC_{n-1}xh^{n-1} + {}_nC_nh^n\right)}{h}$$
$$= \lim_{h\to 0}\frac{h\left({}_nC_1x^{n-1} + {}_nC_2x^{n-2}h + \ldots + {}_nC_{n-1}xh^{n-2} + {}_nC_nh^{n-1}\right)}{h}$$
$$= \lim_{h\to 0}\left({}_nC_1x^{n-1} + {}_nC_2x^{n-2}h + \ldots + {}_nC_{n-1}xh^{n-2} + {}_nC_nh^{n-1}\right)$$
$$= {}_nC_1x^{n-1}$$
$$= n \times x^{n-1}$$
where ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$

