Properties of Parabolas

ID: 8852

Time required
35 minutes

Activity Overview

This investigation offers an approach to show students the basic definition of a parabola as the locus of all points equidistant from a fixed point (focus) and a fixed line (directrix). Students will also interpret the equation for a parabola in vertex form and gain a visual understanding of a parabola's focal width.

Topic: Quadratic Functions and Equations

- *Observe the changes in the equation of a quadratic function under a translation and/or a stretch.*
- *Approximate the real zeros, vertex and extrema of a quadratic function graphically.*

Teacher Preparation and Notes

- *This activity was designed for use in an Algebra 2 classroom as an introduction to the parabola as a conic section. It may also be used in Precalculus as a review of properties of parabolas.*
- *Students should be familiar with quadratics as transformations and should have some experience using tools on the handheld or students. Encourage students to ask for help if they are unsure how to complete a certain task.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8852" in the keyword search box.***

Associated Materials

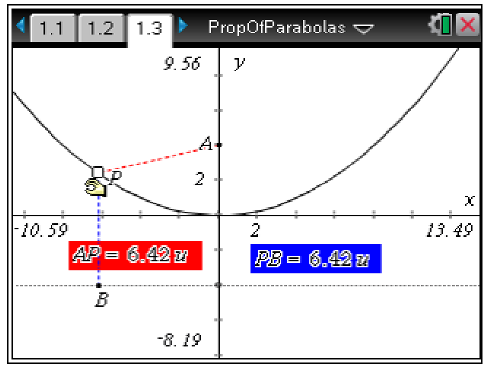
- *PropOfParabolas_Student.doc*
- *PropOfParabolas.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Around the Vertex in 80 Days (TI-Nspire technology) — 11683*
- *An Application of Parabolas (TI-Nspire technology) — 13364*

Problem 1 – Focus and directrix

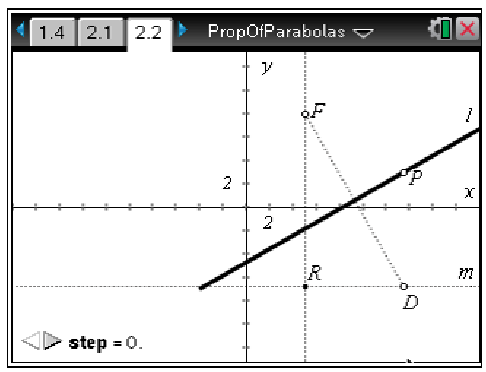


On page 1.3, students drag point *P* along the parabola and observe the lengths of $\overline{AP}$ and $\overline{BP}$. It should not take long for them to conclude that the lengths are equal for all points *P* on the parabola.

Define a parabola as the locus of all points that are equidistant from a fixed point, called the ***focus***, and a fixed line, called the ***directrix***. Also, explain that the vertex of the parabola (with which students should have studied when graphing quadratics) is included in this locus of points, and thus it, too, is equidistant from the focus and directrix. Moreover, the vertex is the point that is *closest* to the focus and directrix.

Students should conclude (by calculating, not measuring) that the distance from the focus to the vertex of the parabola is 4 units.

Problem 2 – Constructing a parabola and writing its equation



On page 2.2, students find a diagram like the one shown to the right. Direct students' attention to each geometric object as you describe it:

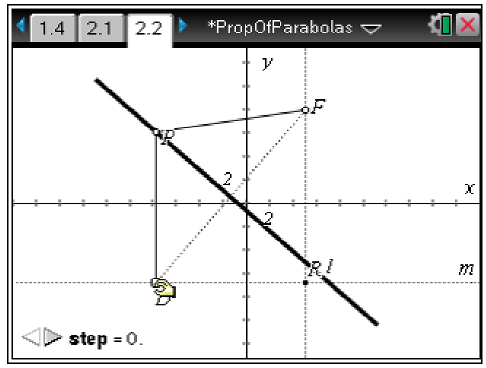
- Point *D* lies on line *m*, which is parallel to the *x*-axis (later found to be the directrix);
- Point *F* is a free point, through which a perpendicular line intersects line *m* at point *R*;
- Line *l* is the perpendicular bisector of $\overline{FD}$; and
- Point *P* lies on line *l* such that $\overline{PD}$ (not shown) is perpendicular to line *m*.

Students should drag point *D* and observe the movement of line *l* and point *P*. Encourage them to notice that the line and point appear to trace out a parabola. Discuss with students that, for this parabola, point *F* is the *focus* and point *D* is moving along line *m*, the *directrix*.

TI-Nspire Navigator Opportunity: *Live Presenter*

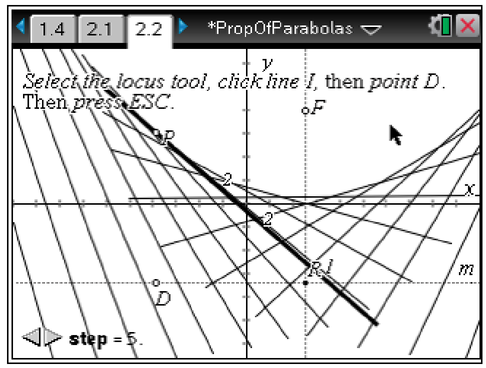
See Note 1 at the end of this lesson.

Students should now verify that point *P* is equidistant from the focus and directrix. Have them use the **Segment** tool from the Points & Lines menu to draw segments *FP* and *PD*. Next, they should select the **Length** tool from the Measurement menu to measure and display the length of each segment.

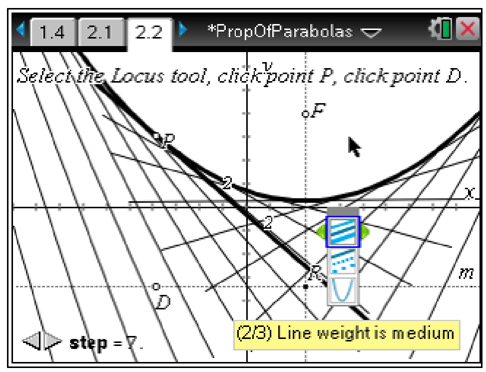


Dragging point *D* along the directrix—as well as point *F*, the focus—confirms that the segments have the same length. Before proceeding, students should use the **Hide/Show** tool to hide the segments and their lengths.

To show the shape of the parabola, students can use the **Locus** tool from the Construction menu. After selecting the **Locus** tool, they should click on line *l,* click on point *D*, and press esc. Have them drag point D again and observe. Discuss with students the shape of this locus of lines.

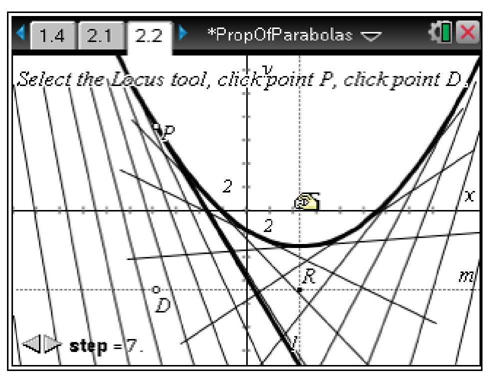


Selecting the **Locus** tool a second time, have students click on point *P*, followed by point *D*. The locus of points—the parabola—is constructed. Students may use the **Attributes** tool to change the appearance of the parabola, increasing the line weight. Dragging point *D* once more reveals that point *P* does, in fact, trace out the parabola.

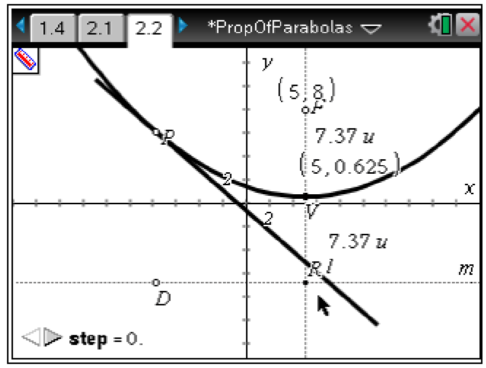


Next, students will move point *F* vertically to observe what effect changing the distance between vertex and directrix has upon the parabola. They should conclude that the greater the distance between focus and directrix, the "wider" the parabola.

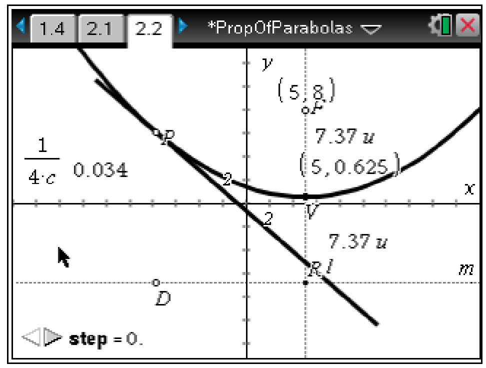
Before advancing to the next part of the problem, tell students hide the locus of lines and use the **Coordinates and Equations** tool to display and "reset" the coordinates of point *F* to (5, 8).



The students' next task is to write an equation for this parabola. The equation, in vertex form, is described on page 2.3. The first step will be to identify the vertex. Return to the graph on page 2.2 and hide the loci. Select the **Midpoint** tool to construct the midpoint of $\overline{FR}$, and label it *V* (this is the vertex). Using the **Coordinates and Equations** tool once more, the vertex (*h*, *k*) may be identified as (5, 0.625). Using the **Length** tool, students can measure each segment to verify *V* is the midpoint.



Since this parabola opens up, the value of *c* (and *a*) is positive. Students need only to substitute 7.375 (the length of $\overline{FV}$) for *c* and evaluate to obtain $a \approx 0.033898$. This may be accomplished by using the **Text** tool to display the expression $\frac{1}{4 \cdot c}$, and the **Calculate** tool to evaluate the expression, as shown to the right.



The correct equation for the parabola is $y = 0.033898(x-5)^2 + 0.625$.

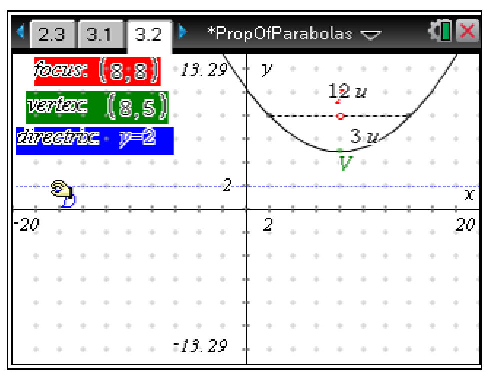
TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Have students verify it by pressing ctrl G to show the Entry Line, defining **f1**(*x*) as needed, and pressing enter. If the parabolas coincide, their equation is correct.

Problem 3 – Focal width and the value of |*a*|

On page 3.2, students will determine the relationship between |*a*| and the focal width of a parabola (the length of the dashed segment, shown at right). They should find the focal width of the parabola using the **Length** tool. Encourage them to realize that displaying the length of $\overline{PV}$ (not shown) would be very helpful—its length is *c*, and *a* is dependent upon the value of *c*. Dragging points *D* and *F* after both measurements are displayed, the relationships should be obvious:



The focal width is 4 times the distance from the vertex to the focus, and thus |a| is the reciprocal of the focal width.

The parabolas explored in this activity have had positive values of *a*—opening up. The model on page 3.2 allows for the directrix to be placed *above* the focus, so that they may experiment with negative values of *a*, with the parabola opening down. Be sure students take care in dealing with the absolute values.

TI-Nspire Navigator Opportunities

Note 1
Problem 2, *Live Presenter*
For each step, have one student be the *Live Presenter* and complete the directions on the screen while the other students follow along.

Note 2
Problem 2, *Quick Poll*
Send a Quick Poll asking students to submit the equation of the parabola they found. Have a class discussion if the equations vary to determine possible errors.