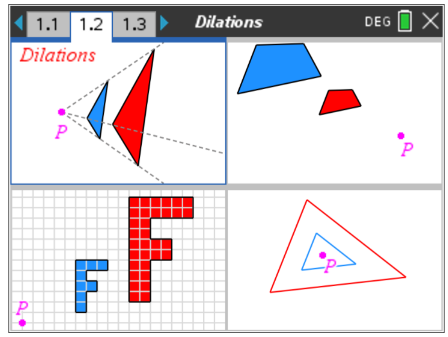
Transformational Geometry Dilations

TEACHER NOTES



Transformational Geometry is a way to study geometry by focusing on geometric "movements" or "transformations" and observing/studying properties about these figures.

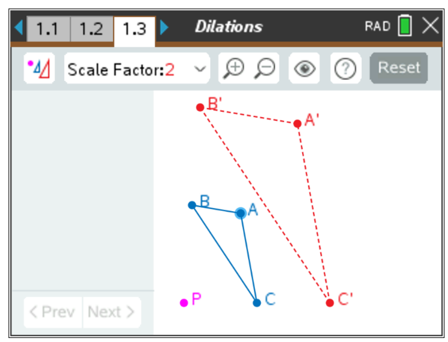
There are four geometric transformations:
< Reflections < Translations < Rotations **< Dilations**

Play - Investigate - Explore - Discover
P I E D

Download and install the red TI-Nspire student software and the Dilations TNS file from the website where you obtained this document.
Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the [videos.]

1. What do the 4 parts of the screen have in common in the screen shot above?
Make at least two conjectures. (a **conjecture** is an opinion or conclusion based upon what is observed.)

< Each part has a pink point P.
< Each part has a red and a blue figure in it.
< Each part has a smaller figure and a larger figure, both have the same shape, different size (similar)



In the figure to the right, ΔABC is dilated about point P with a scale factor of 2.
ΔABC is called the pre-image while $\Delta A'B'C'$ is called the image (of dilation).
$\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."
Point P is called the point of dilation.

2. Using the TI-Nspire technology, dilate ΔABC about point P using a Scale Factor of 2.
Make a conjecture about what seems to be true about ΔABC and its image, $\Delta A'B'C'$.

When a triangle is dilated about a point with Scale Factor 2, the pre-image triangle and the image triangle seem to have the same shape, but a different size.
Corresponding angles appear to have the same measure.

3. a. Dilate ΔABC about point P using a Scale Factor of 3. Zoom (out) as needed.
P I E D

b. Dilate ΔABC about point P using a Scale Factor of $\frac{1}{2}$ (1/2). P I E D

4. Dilate ΔABC about point P using a Scale Factor of 1.5. Zoom (out) as needed.

a. Grab and pull point P to coincide with vertex A. What two segments appear to be parallel?
$\overline{BC} \parallel \overline{B'C'}$

b. Grab and pull point P to coincide with vertex B. What two segments appear to be parallel?
$\overline{AC} \parallel \overline{A'C'}$

c. Before you grab and pull point P to coincide with vertex C, what two segments would you expect to appear to be parallel?
$\overline{AB} \parallel \overline{A'B'}$

d. Now grab and pull point P to coincide with vertex C. What two segments appear to be parallel?
$\overline{AB} \parallel \overline{A'B'}$

5. Dilate ΔABC about point P using a Scale Factor of – 1.
Play – Investigate – Explore – Discover.
How would you describe the relationship between the pre-image and the image?

The pre-image and image triangles are the same size and same shape, congruent. That occurs because of the '1'. It appears that the pre-image is rotated 180 degrees about point P to obtain the image. This occurs because of the ' – '.

Angles and Sides
Note: the measurements given using TI-Nspire technology are rounded to the nearest hundredth.

6. Dilate ΔABC about point P using a Scale Factor of 1.5. Zoom (out) as needed.
P I E D.
a. What appears to be true about the angles of the two triangles?
The corresponding angles appear to have the same measures.

b. Do the corresponding sides have the same lengths?
No!

7. Tools > Templates > 1. Angles & Sides
Dilate ΔABC about point P using a Scale Factor of 1.5. Zoom (out) as needed.
a. What is now shown on the screen?
Angle measures rounded to the nearest hundredth.

P I E D.
b. Make a conjecture about what you observe?
The corresponding angles appear to have the same measures.

c. Let's make sure our conjecture holds true for other Scale Factors.

Repeat parts a and b above but with a Scale Factor of $\frac{1}{2}$. P I E D.

Does your conjecture still seem to be true?
Yes.

8. Tools > Templates > 1. Angles & Sides
Dilate ΔABC about point P using a Scale Factor of 2. Zoom (out) as needed.

a. Click on Next >

Look at the lengths of corresponding sides of the two triangles.
P I E D.

Note: we suggest that you try to make one or two of the side lengths a whole number.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.
It appears that the lengths of corresponding sides of the image triangle are twice the lengths of the pre-image triangle.

b. Without pressing Reset, change the Scale Factor to 3.

Look at the lengths of corresponding sides of the two triangles.
P I E D.
Note: we suggest that you try to make one or two of the side lengths a whole number, if possible.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.
It appears that the lengths of corresponding sides of the image triangle are three times the lengths of the pre-image triangle.

c. Click on Next >

What is shown on this screen?
P I E D.
The ratios of the lengths of corresponding sides, image to pre-image, is 3.

Does this screen validate your conjecture?
This does not "prove" it for all cases, but it illustrates that it is probably true.

d. Without pressing Reset, change the Scale Factor back to 2.

Look at the lengths of corresponding sides of the two triangles.
P I E D.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.
It appears that the lengths of the corresponding sides of the image triangle are twice the lengths of the pre-image triangle.

e. What would you expect to observe if the Scale Factor were to be $\frac{1}{2}$?

The ratios of the lengths of corresponding sides, image to pre-image, is $\frac{1}{2}$.

Use the technology to see if you are correct.

Perimeters and Areas

Note: the measurements given using TI-Nspire technology are rounded to the nearest hundredth.

9. Tools > Templates > Perimeters and Areas
Dilate ΔABC about point P using a Scale Factor of **2**. Zoom (out) as needed.
Grab and move one of the vertices of ΔABC until the **perimeters are whole numbers, if easily possible. If not, just use the values from the video.**

a. Record these values into the Perimeter Table below, a. Scale Factor = **2** column.
Notice that the image perimeter is listed first in the column, the pre-image perimeter below it.
Look for patterns as you grab and move vertices of the pre-image triangle.

Perimeter Table

In units	**a. Scale Factor 2**	**b. Scale Factor 3**	**c. Scale Factor $\frac{1}{2}$**
Perimeter of image $\Delta A'B'C'$	**72**	**210**	**13**
Perimeter of pre-image ΔABC	**36**	**70**	**26**
$\frac{Perim(\Delta A'B'C')}{Perim(\Delta ABC)}$	**2**	**3**	$\frac{1}{2}$

b. Reset. Dilate ΔABC about point P using a Scale Factor of **3**. Zoom (out) as needed.
Grab and move one of the vertices of ΔABC until the **perimeters are whole numbers, if possible**.
If not easily possible, just use the values from the video.
Place these values into the Perimeter Table above, b. Scale Factor = **3** column.

c. Reset. Dilate ΔABC about point P using a Scale Factor of $\frac{1}{2}$. Zoom as needed.

Grab and move one of the vertices of ΔABC until the **perimeters are whole numbers, if possible**.

Place these values into the Perimeter Table above, c. Scale Factor = $\frac{1}{2}$ column.

d. Look at each pair of corresponding perimeters in the columns. What pattern do you see?
The ratio of the perimeter of the image triangle to the perimeter of the pre-image triangle is the same as the Scale Factor.

10. Let's continue to use TI-Nspire technology to discover or confirm the pattern.
a. Reset. Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.

b. Click on Next > twice to see **ratios of the perimeters of the image to the pre-image triangle**.
Grab and move the vertices of ΔABC and notice the perimeters change. But what does **NOT** change?
The ratio stays the same.
c. Notice that the 4th (last) row of the Perimeter Table is empty.

Into the first column of the 4th row, write the following:

$$\frac{Perim(\Delta A'B'C')}{Perim(\Delta ABC)}$$

d. Into the last row of the column marked "**a.** Scale Factor **2**", write the ratio of the perimeters, image to pre-image.

e. Change the scale factor to **3**. Zoom as needed. Grab and move vertices and observe. Into the last row of the column marked "**b.** Scale Factor **3**", write the ratio of the perimeters, image to pre-image.

f. Change the scale factor to $\frac{1}{2}$. Zoom as needed. Grab and move vertices and observe.

Into the last row of the column marked "**c.** Scale Factor $\frac{1}{2}$", write the ratio of the perimeters, image to pre-image.

11. a. Reset. Change the Scale Factor to be **3**.

Dilate ΔABC about point P using a Scale Factor of **3**. Zoom (out) as needed.

Click on Next > to see the **areas** displayed.

Grab and move vertices until the area of the pre-image ΔABC is a multiple of ten.

Record these values into the Area Table on the top of the next page, **a**. Scale Factor = **3** column.

Notice that the image area is listed first in the column, the pre-image area below it.

Look for patterns as you grab and move vertices of the pre-image triangle.

Area Table

In square units	**a. Scale Factor 3**	**b. Scale Factor 2**	**c. Scale Factor** $\frac{1}{2}$
Area of image $\Delta A'B'C'$	**450**	**120**	**20**
Area of pre-image ΔABC	**50**	**30**	**80**
$\frac{Area(\Delta A'B'C')}{Area(\Delta ABC)}$	**9**	**4**	$\frac{1}{4}$

b. Reset. Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.

Grab and move one of the vertices of ΔABC until the **areas are multiples of ten**.
Place these values into the Area Table above, **b**. Scale Factor = **2** column.

c. Reset. Dilate ΔABC about point P using a Scale Factor of $\frac{1}{2}$. Zoom as needed.

Grab and move one of the vertices of ΔABC until the **areas are multiples of ten**.

Place these values into the Perimeter Table above, **c**. Scale Factor = $\frac{1}{2}$ column.

d. Look at each pair of areas in the columns. What pattern do you see?
The ratio of the image triangle area to the pre-image area triangle is the Scale Factor squared.

12. Let's continue to use TI-Nspire technology to discover or confirm the pattern.
a. Reset. Dilate ΔABC about point P using a Scale Factor of **3**. Zoom as needed.

Click on Next > three times to see the **ratios of the areas of the image to the pre-image triangle**.
Grab and move the vertices of ΔABC and notice the areas change. But what does **NOT** change?

b. Notice that the 4th (last) row of the Area Table is empty.
Into the first column of the 4th row, write the following:

$$\frac{Area(\Delta A'B'C')}{Area(\Delta ABC)}$$

c. Into the last row of the column marked "**a.** Scale Factor **3**", write the ratio of the areas, image to pre-image.

d. Change the scale factor to **2**. Zoom as needed. Grab and move vertices and observe.
Into the last row of the column marked "**b.** Scale Factor **2**", write the ratio of the areas, image to pre-image.

e. Change the scale factor to $\frac{1}{2}$. Zoom as needed. Grab and move vertices and observe.

Into the last row of the column marked "**c.** Scale Factor $\frac{1}{2}$", write the ratio of the areas,

image to pre-image.

13. Let's a) summarize with a specific number and then b) generalize with a variable.

a. Suppose ΔABC is dilated about a point with a Scale Factor of 4.
Based upon what we have observed, complete the following:

$$\frac{Perimeter(\Delta A'B'C')}{Perimeter(\Delta ABC)} = \underline{\quad 4 \quad} \qquad \frac{Area(\Delta A'B'C')}{Area(\Delta ABC)} = \underline{\quad 16 \quad}$$

b. Suppose ΔABC is dilated about a point with a Scale Factor of some number, R.
Based upon what we have observed, complete the following:

$\dfrac{Perimeter(\Delta A'B'C')}{Perimeter(\Delta ABC)} = $ ____ R ____	$\dfrac{Area(\Delta A'B'C')}{Area(\Delta ABC)} = $ ____ R^2 ____

Grid and Coordinates

14. Tools > Templates > Grid & Coordinates
Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.

a. Observe the coordinates of the vertices of the pre-image triangle and the image triangle and look for patterns. Record the coordinates into the Table on the next page in the 'Figure 1' row.

b. Grab and move points A, B, C. Record the coordinates into the 'Figure 2' row. Pattern?

c. Repeat: Grab and move points A, B, C. Record the coordinates into the 'Figure 3' row. Pattern?
d. Now grab and move point P and record the coordinates into the 'Figure 4' row. Pattern?

Scale Factor = **2**	A	B	C	A'	B'	C'
Figure 1	**(3, 4)**	**(- 2, 7)**	**(6, - 5)**	**(6, 8)**	**(- 4, 14)**	**(12, - 10)**
Figure 2	**(2, 7)**	**(- 3, 4)**	**(4, - 7)**	**(4, 14)**	**(- 6, 8)**	**(8, - 14)**
Figure 3	**(0, 11)**	**(- 6, - 3)**	**(7, 1)**	**(0, 22)**	**(- 12, - 6)**	**(14, 2)**
Figure 4 (move P) **(2, 1)**	**(0, 11)**	**(- 6, - 3)**	**(7, 1)**	**(- 2, 21)**	**(-14, - 7)**	**(12, 1)**

e. Look at the Figure 1, 2, and 3 rows. Make a conjecture about the coordinates of the vertices of a triangle and its image under a dilation about the origin.
If a triangle is dilated about the origin with a Scale Factor of 2, the coordinates of the image triangle are twice the coordinates of the pre-image triangle.

f. Does your conjecture still hold true if the dilation point P is not at the origin (Figure 4)? Explain.
No! See the data.

15. Let's repeat the investigation we did in number 14 above, but with a different scale factor, $\frac{1}{2}$.

Dilate ΔABC about point P using a Scale Factor of $\frac{1}{2}$. Zoom as needed.

a. Observe the coordinates of the vertices of the pre-image triangle and the image triangle and look for patterns. Record the coordinates into the Table on the next page in the 'Figure 1' row.

b. Grab and move points A, B, C. Record the coordinates into the 'Figure 2' row. Pattern?

c. Repeat: Grab and move points A, B, C. Record the coordinates into the 'Figure 3' row. Pattern?

d. Now grab and move point P and record the coordinates into the 'Figure 4' row. Pattern?

Scale Factor = $\frac{1}{2}$	A	B	C	A'	B'	C'
Figure 1	(3, 4)	(- 2, 7)	(6, - 5)	(1.5, 2)	(- 1, 3.5)	(3, - 2.5)
Figure 2	(- 1, 8)	(- 4, 0)	(- 3, - 8)	(- 0.5, 4)	(- 2, 0)	(- 1.5, - 4)
Figure 3	(4 , 5)	(- 4, 2)	(1, - 6)	(2, 2.5)	(- 2, 1)	(0.5, - 3)
Figure 4 (move P) **(- 1, - 2)**	(4 , 5)	(- 4, 2)	(1, - 6)	(1.5, 1.5)	(- 2.5, 0)	(0, - 4)

e. Look at the Figure 1, 2, and 3 rows. Make a conjecture about the coordinates of the vertices of a triangle and its image under a dilation about the origin.

If a triangle is dilated about the origin with a Scale Factor of $\frac{1}{2}$, the coordinates of the image triangle are half the coordinates of the pre-image triangle.

f. Does your conjecture still hold true if the dilation point P is not at the origin (Figure 4)? Explain.
No! See the data.

16. Summary and generalization.
a. Suppose ΔDEF is dilated about the origin using a Scale Factor of **3**. The coordinates of the pre-image triangle are listed below. Based upon what you have observed, list what you expect the coordinates of the image triangle to be.

Scale Factor = **3**	$D: (1, 4)$	$E: (-2,\ 5)$	$F: (-2,\ -6)$
	D': **(3, 12)**	E': **(- 6, 15)**	F': **(- 6, - 18)**

b. Suppose ΔXYZ is dilated about the origin using a Scale Factor of **R**.
The coordinates of point X are (a, b). What do you expect the coordinates of X' to be?

$X': \left(R\cdot a. \ R\cdot b \right)$

Distance from P to the Vertices

17. Tools > Templates > Dist P to Vertices

a. Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.

Investigate the distances from P to each of the 6 vertices by grabbing and moving each of the vertices of ΔABC. Try to make as many distances as possible to be whole numbers. Look for patterns.

b. Record the 6 lengths below.

$Scale \ \ Factor = 2$ $\quad PA = 5$ $\quad PB = 13$ $\quad PC = 11$

$PA' = 10$ $\quad PB' = 26$ $\quad PC' = 22$

c. Make a conjecture based on what you have observed.

If a triangle is dilated about a point with a Scale Factor of 2, the distance from the point of dilation to a point on the image is twice the distance from the point of dilation to the corresponding point on the pre-image.

d. Click on Next > . Observe.

Grab and move each of the vertices of ΔABC and point P.

Does this seem to validate your conjecture?

Yes.

e. Change the Scale Factor to 3 and observe what happens as you grab and move the vertices and P.

f. Change the Scale Factor to $\frac{1}{2}$ and observe what happens as you grab and move the vertices and P.

g. **Summary.**

Given: ΔDEF is dilated about point P with a Scale Factor of 4. Complete the following:

If $PD = 3$, then $PD' =$ **12** If $PE = 6.2$, then $PE' =$ **24.8** If $PF = 8.7$, then $PF' =$ **34.8**

18. Tools > Templates > Slopes

Click on the menu and click on Grid.

a. Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.
Observe the slopes of corresponding sides.
Grab and move the 3 vertices and continue to observe the slopes of corresponding sides.
Make a conjecture.
When a triangle is dilated about a point with a Scale Factor of 2, the slopes of the corresponding sides are equal.

And what does that mean about the corresponding sides?
That implies that the corresponding sides are parallel.

b. Change the Scale Factor to 3. Zoom as needed.
Observe the slopes of corresponding sides.
Grab and move the 3 vertices and continue to observe the slopes of corresponding sides.
Make a conjecture.
When a triangle is dilated about a point with a Scale Factor of 3, the slopes of the corresponding sides are equal.

And what does that mean about the corresponding sides?
That implies that the corresponding sides are parallel.

c. Change the Scale Factor to $\frac{1}{3}$.
Do a similar investigation as you did in parts a and b.

d. **Summary.**
Given: ΔDEF is dilated about point P with a Scale Factor of 4.
State 3 pairs of segments that are parallel.

$\overline{DE} \parallel \overline{D'E'}$ $\overline{EF} \parallel \overline{E'F'}$ $\overline{DF} \parallel \overline{D'F'}$

19**. Dilate ΔABC about point P with a Scale Factor of 2 by hand.**
We suggest that you do this along with the video and pause as needed.
You will need a compass and straightedge.

If you don't have a compass, do the best that you can without one.
If you have TI-Nspire technology, you can use the Dilations TNS file, pages 2.1 – 2.3.
It will take you through the steps at your own pace.