Pendulum Explorations – ID: 12227

By Irina Lyublinskaya

Time required
90 minutes

Topic: Circular and Simple Harmonic Motion

- *Explore what factors affect the period of pendulum oscillations.*
- *Measure the period of oscillation for a pendulum in simple harmonic motion.*
- *Determine the equation relating a pendulum's period to its length.*

Activity Overview

In this activity, students experimentally explore the factors that might affect the period of a pendulum. They collect distance–time data for various masses, initial angles, and pendulum lengths. They use these data to determine the formula for the period of a pendulum using regression and dimensional analysis.

Materials

To complete this activity, each student or student group will require the following:

- *TI-Nspire™ technology*
- *Vernier CBR2™ or Go!™Motion sensor*
- *string*
- *hooked weight set*
- *ring stand and pendulum clamp*
- *protractor*
- *measuring tape or meter stick*
- *scissors*
- *copy of student worksheet*
- *pen or pencil*

TI-Nspire Applications

Graphs & Geometry, Data & Statistics, Lists & Spreadsheet, Notes, Calculator

Teacher Preparation

Students should be familiar with the concepts of amplitude, frequency, angular frequency, and period of oscillations, and they should know how to determine the amplitude and the period of a sine or cosine function from its graph. Students should know how to use dimensional analysis.

- *The screenshots on pages 2–8 demonstrate expected student results. Refer to the screenshots on pages 9 and 10 for a preview of the student TI-Nspire document (.tns file). Pages 11–13 show the student worksheet.*
- ***To download the .tns file, go to education.ti.com/exchange and enter "12227" in the search box.***

Classroom Management

- *This activity is designed to be* ***student-centered****, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students may record their answers to the questions on blank paper or answer in the .tns file using the Notes application.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration during this activity:

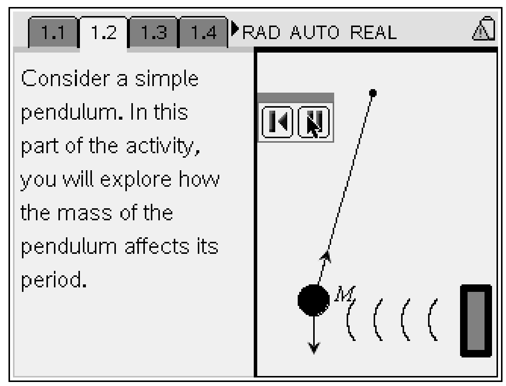
- What factors affect the period of pendulum oscillations?
- What is the equation for the period of pendulum oscillations?

The goals of this activity for students are a) to explore how the mass of the pendulum bob, the initial angle of displacement, and the length of the pendulum affect the period of pendulum oscillations; b) to develop a mathematical model for the period of oscillations using experimental data; and c) to use dimensional analysis to verify the mathematical model and derive a final equation for the period of a pendulum.

Students use a motion sensor to collect data on distance as a function of time for a swinging pendulum. They use these data to determine the period of oscillations for various masses, initial angles, and pendulum lengths. After comparing these data, they derive an equation for the period of pendulum oscillations. Finally, they solve several problems involving the equation for the period of pendulum oscillations.

Problem 1 – Effect of mass on the period of pendulum oscillations

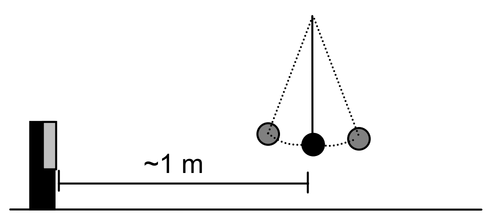
Step 1: Students should open the file **PhyAct_12227_Pendulum.tns**, move to page 1.2, start the simulation of the pendulum motion, and answer question 1. Students can press ctrl tab to move between applications on the screen.



Q1. What factors do you think affect the period of the pendulum's oscillations?

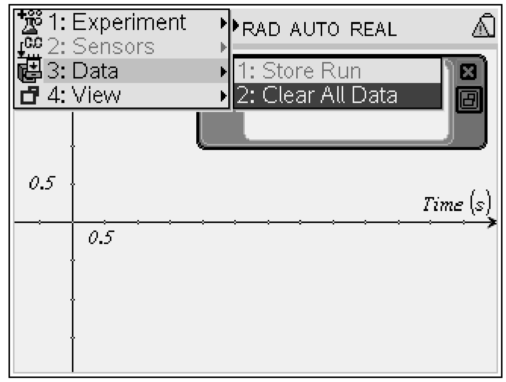
A. *Students' answers will vary. Students will probably suggest one or more of the following variables: the mass of the pendulum, the initial angle (amplitude), and the length of the pendulum, measured from the support point to the center of the mass. Encourage students to summarize how they think each factor will affect the pendulum's period.*

Step 2: Next, students should set up the experiment, as shown to the right. Students should use the ring stand with the pendulum clamp to hang the 50 g mass from the string. They should measure and record the pendulum length, which should be at least 50 cm. They should place the CBR2 (if they are using TI-Nspire handhelds) or Go!Motion sensor (if they are using TI-Nspire computer software) on a flat surface so the emitter screen is level with the pendulum bob. At rest, the pendulum bob should be about 1 m away from the motion sensor. The bob should be far enough from the motion sensor that it is never closer to the detector than 0.4 m while it is swinging.



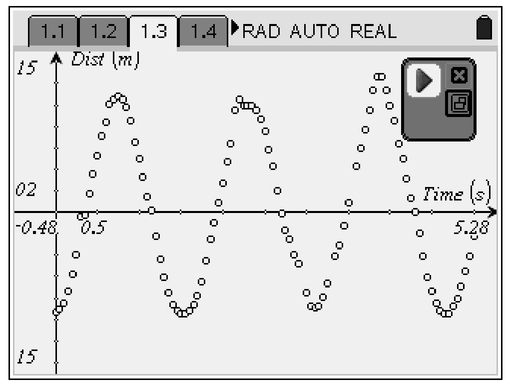
Step 3: Next, students should locate the switch under the head of the motion detector and set it to the Normal position, which is indicated by a picture of a person and a basketball.

Step 4: Next, students should move to page 1.3 and connect the motion sensor to their handheld or computer. A distance data collection display should appear.

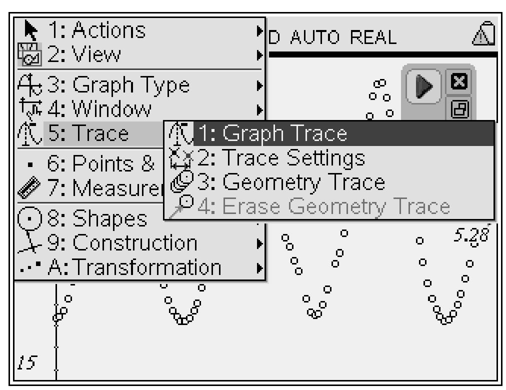


Step 5: Students should clear any stored data (**Menu > Data > Clear All Data**). Then, with the pendulum bob in the equilibrium position and stationary, they should zero the motion sensor (**Menu > Sensors > Zero**).

Step 6: Next, students should initiate pendulum oscillations and start data collection. They should be sure to measure and record the initial angle to which they displace the pendulum. (They should use an angle less than about 10°.) The Play button (►) should be highlighted in the data collection box. If this is the case, students need only click (press) to begin data collection. If the Play button is not highlighted, students should press tab until it is selected. When data collection has ended, a scatter plot will be displayed in the *Graphs & Geometry* application on page 1.3. If the collected data do not have a sinusoidal shape (as shown to the right), students should repeat the experiment. To avoid storing too much data on the device, students should discard any previous data before collecting a new data set.

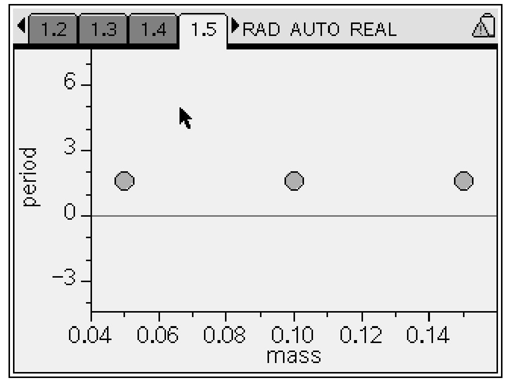


Step 7: Next, students should analyze the scatter plot and determine the period of oscillations from the graph. Students should minimize the display by selecting the button just below the × button. They should then switch to the *Graphs & Geometry* application by pressing ctrl tab. They should trace the data points (**Menu > Trace > Graph Trace**) to identify the *x*-coordinates of several maxima or minima on the graph. The difference in *x*-coordinates of two successive maxima or minima is the period of the curve. Students should move to page 1.4 and record the average period of the 50 g mass.



Step 8: Next, students should move back to page 1.3 and repeat steps 6 and 7 two more times, increasing the mass by 50 g each time. They should make sure to keep the string length and initial angle the same each time.

Step 9: Next, students will produce a visual representation of the relationship between mass and period. They should move to page 1.5, which contains an empty *Data & Statistics* application. They should use this application to create a plot of period vs. mass. The scatter plot of period vs. mass should be a horizontal line. Students should then answer questions 2 and 3.



Q2. Based on your data and observations, what can you conclude about the effect of the pendulum mass on its period?

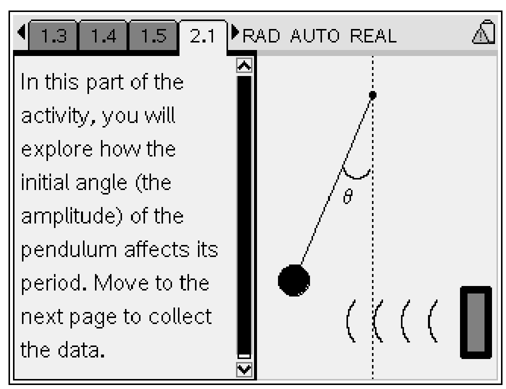
A. *The pendulum period does not depend on mass.*

Q3. Do these results agree with your prediction in question 1? If not, describe any errors in your reasoning.

A. *Students' answers will vary. Encourage metacognitive thinking to help students recognize their errors in reasoning.*

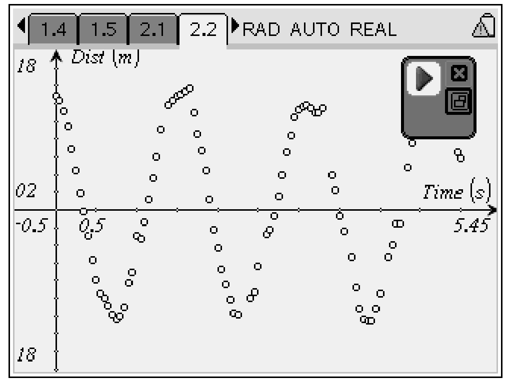
Problem 2 – Effect of initial angle (amplitude) on the period of pendulum oscillations

Step 1: Next, students should move to page 2.1 and read the text there. They should again set up the pendulum with a 50 g bob. They should then move to page 2.2, where they will carry out data collection.

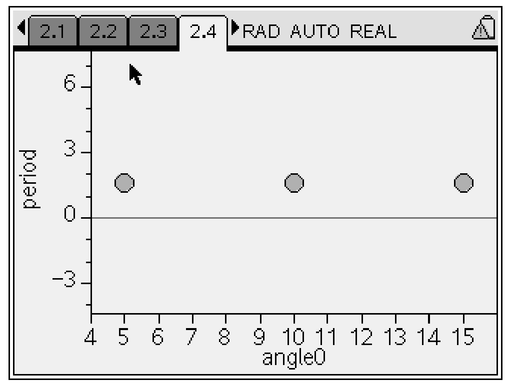


Step 2: Students should insert a data collection box by pressing ctrl D. A distance data collection box should be inserted automatically. Students should repeat steps 5–7 from problem 1. They should initially displace the pendulum by 5°.

Step 3: Students should repeat the data collection for initial angles of 10° and 15°. They should keep the mass of the pendulum bob and the length of the string the same for all three trials. They should record the period of the pendulum in each case in the *Lists & Spreadsheet* application on page 2.3.



Step 4: Students should move to page 2.4 and create a scatter plot of period vs. angle. The data should lie along a horizontal line. Students should examine the graph and answer questions 4 and 5.



Q4. Based on your data and observations, what can you conclude about the effect of the initial angle on the pendulum period?

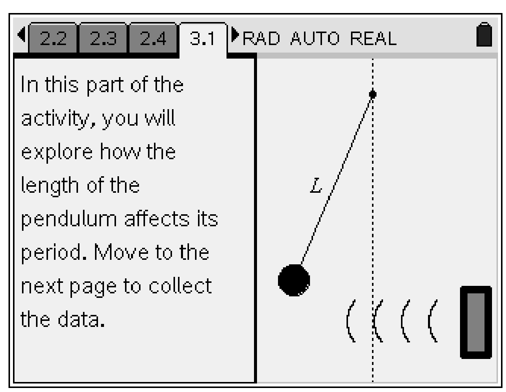
A. *The pendulum period does not depend on the initial angle. Note: The oscillations have to be small for this relationship to hold. Mathematically, this condition means that* $\frac{\sin\theta}{\theta} \approx 1$ *(where θ is in radians), or* $\theta \ll 1$ *radian.*

Q5. Do these results agree with your prediction in question 1? If not, describe any errors in your reasoning.

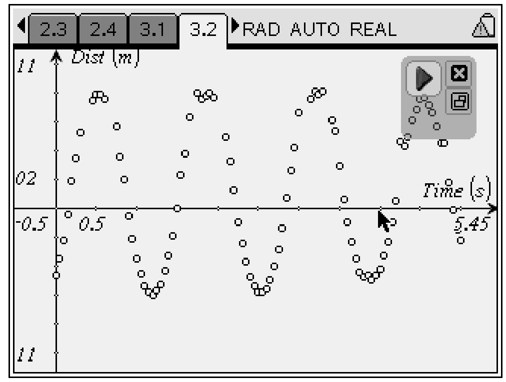
A. *Students' answers will vary. Encourage metacognitive thinking to help students recognize their errors in reasoning.*

Problem 3 – Effect of pendulum length on the period of pendulum oscillations

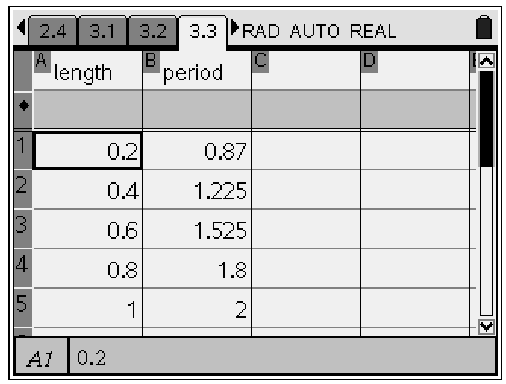
Step 1: Next, students should move to page 3.1 and read the text there. Then, they should again set up the pendulum experiment. They should again use the 50 g bob, and they should use the measuring tape or meter stick to make the length of the string 1 m. They should then move to page 3.2, where they will carry out the data collection.



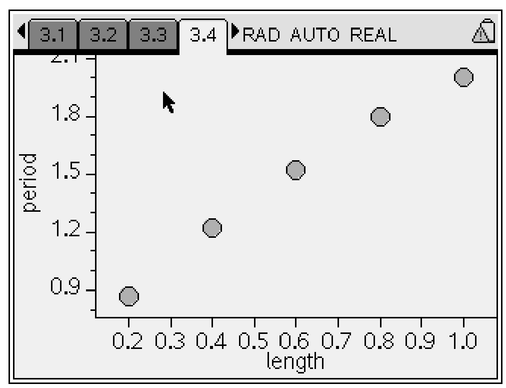
Step 2: Students should insert a data collection box by pressing ctrl D. A distance data collection box should be inserted automatically. Students should repeat steps 5–7 from problem 1 using the 1 m pendulum.



Step 3: Students should repeat the data collection four more times, increasing the length of the pendulum in each trial. They should keep the mass of the pendulum bob and the initial angle the same for all three trials. They should record the period of the pendulum in each case in the *Lists & Spreadsheet* application on page 3.3. (Suggested pendulum lengths, in meters, are included on page 3.3, but students can use other lengths if they wish.)



Step 4: Students should move to page 3.4 and create a scatter plot of period vs. length. Students should examine the graph and then answer questions 6 and 7.

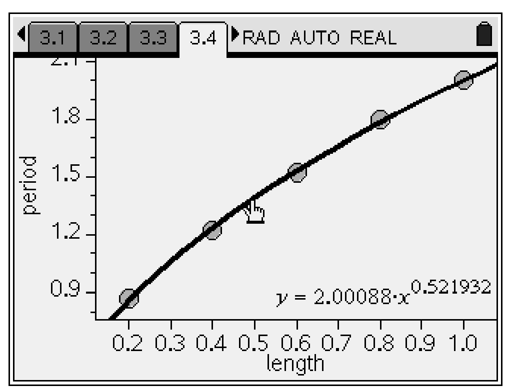


Q6. Based on your data and observations, what can you conclude about the effect of the length of the pendulum on its period?

A. *The pendulum period increases as the length increases.*

Q7. Develop a mathematical model for your experimental data.

A. *Students should use the* ***Power Regression*** *tool (**Menu > Analyze > Regression > Show Power**) to determine the equation that best fits the data. Students' equations should be of the form $T = aL^{0.5}$, where* T *is period,* a *is a constant, and* L *is length.*



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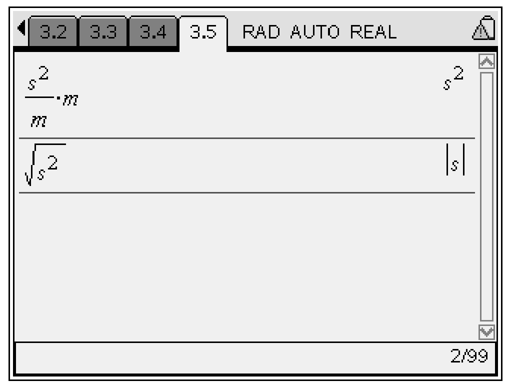
Step 5: Next, students should answer questions 8–11. Students may use the provided *Calculator* application to check their work.

Q8. Use dimensional analysis to confirm your model.

A. *You may wish to discuss with students the implications of the models they identified in question 7. Remind them that the period of oscillations depends on length, and the oscillations are caused by gravity, so period should depend on acceleration due to gravity. The units of length,* L*, are meters, and the units of gravitational acceleration,* g*, are meters per second squared. The units of period,* T*, are seconds. Therefore, students should manipulate the units of length and gravitational acceleration to yield seconds, as shown below:*



$$\frac{1}{g} \rightarrow \frac{s^2}{m}$$

$$\frac{1}{g}L \rightarrow \frac{s^2}{m}m = s^2$$

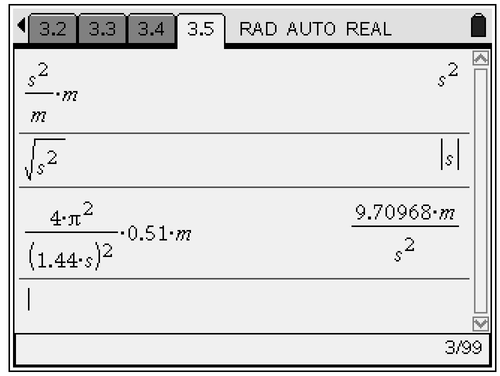
$$\sqrt{\frac{L}{g}} \rightarrow \sqrt{s^2} = s$$

Thus, according to the dimensional analysis, $T = k\sqrt{\frac{L}{g}}$*. By comparing the experimental formula identified in question 7 and the expression derived from dimensional analysis, students can determine the value of the constant* k*. Students' values of* k *should approximate 2π. Therefore, the formula for the period of a pendulum is* $T = 2\pi\sqrt{\frac{L}{g}}$*. Note that the dimensional analysis can be confirmed using the* Calculator *application only if students are using TI-Nspire CAS technology.*

Q9. Using the value of the constant determined in question 7 and g = 9.81 m/s^2, calculate the value of the constant k.

A. *The regression in question 7 yields an equation of the form $T = aL^{0.5}$. Therefore, the constant* a *in that equation is equal to $\frac{k}{\sqrt{g}}$. Therefore,*

$$k = a\sqrt{g} = (2.00)\left(\sqrt{9.81}\right) = 6.26 \approx 2\pi .$$



Q10. A simple pendulum is used to obtain an experimental value for gravitational acceleration, g. A student displaces a 0.510 m pendulum 10° from the equilibrium position and releases it. The period of the pendulum is 1.44 s. Determine the experimental value of gravitational acceleration.

A. *Students should solve the formula for the period of the pendulum for* g *to yield $g = \frac{4\pi^2}{T^2}L$.*

Substituting the given values yields

$$g = \frac{4\pi^2}{T^2}L = \frac{4\pi^2}{(1.44\ s)^2}(0.510\ \ m) = 9.71\ m/s^2 .$$

Q11. A pendulum has a length of 0.79 m and a mass of 0.24 kg. The pendulum is pulled away from its equilibrium position by an angle of 8.50° and released. Assume that friction can be neglected and that the resulting oscillatory motion is simple harmonic motion. What is the angular frequency, ω, of the motion? Remember that $\omega = 2\pi f$, where f is the frequency of the pendulum.

A. *Because $T = \frac{1}{f}$, angular frequency is given by*

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

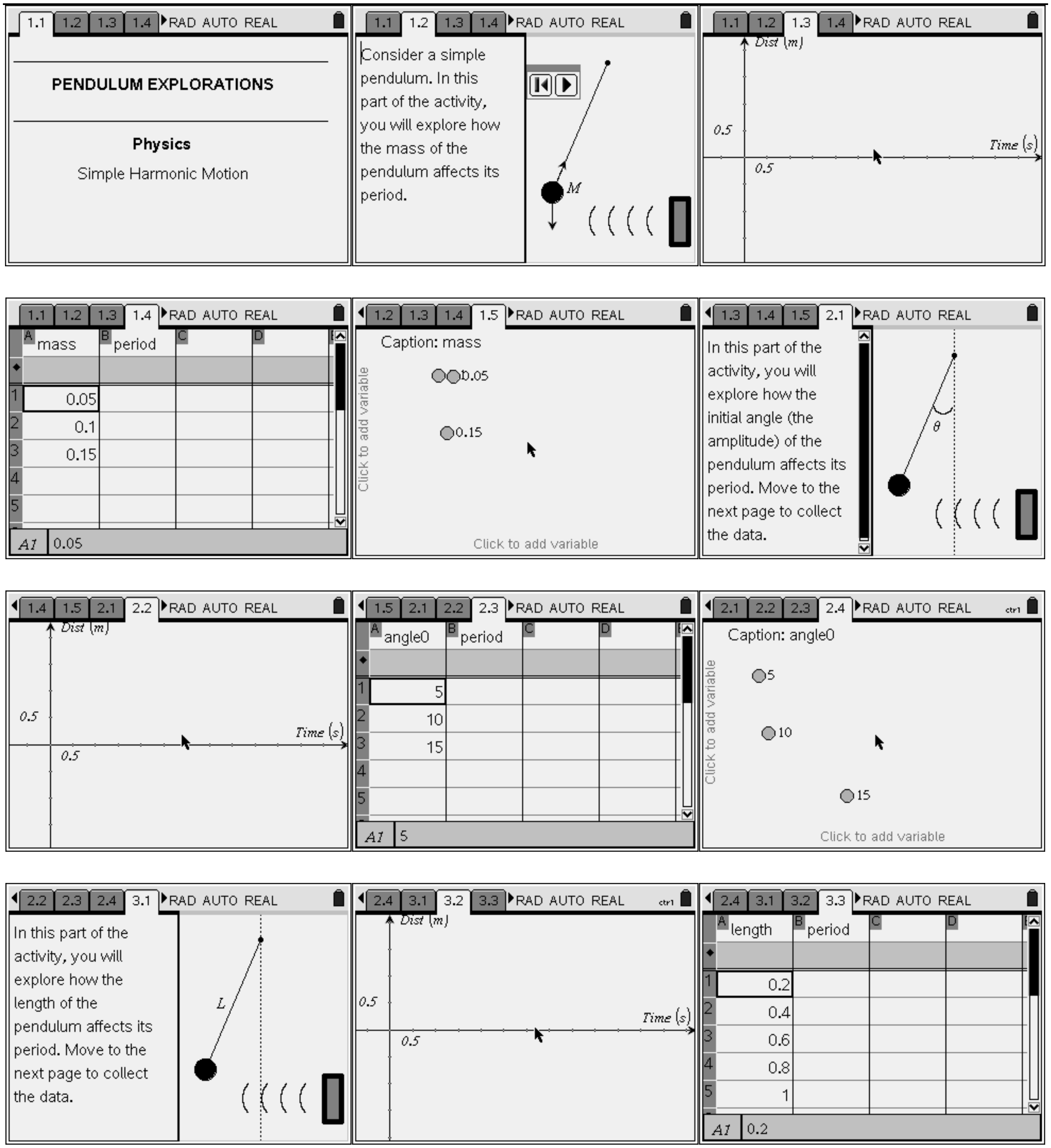
. Substituting the given values yields

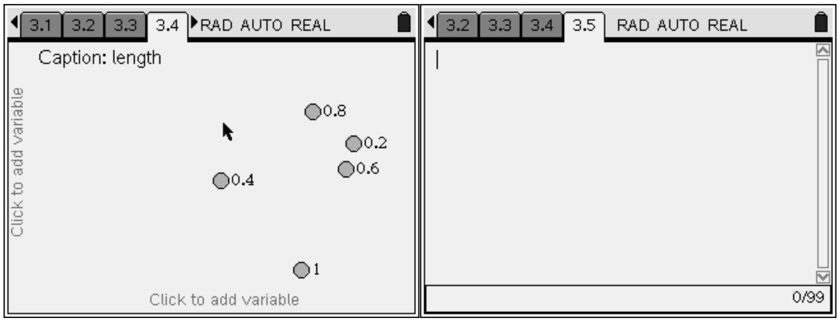
$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ \ m/s^2}{0.79\ \ m}} = 3.52\ \ rad/s$$

. Period (and, therefore, frequency) depend only on length; all other given information is irrelevant.

Pendulum Explorations – ID: 12227

(Student)TI-Nspire File: *PhyAct_12227_Pendulum.tns*




3.1 3.2 3.3 3.4 RAD AUTO REAL
Caption: length
Click to add variable
0.8
0.2
0.6
0.4
1
Click to add variable
3.2 3.3 3.4 3.5 RAD AUTO REAL
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Pendulum Explorations

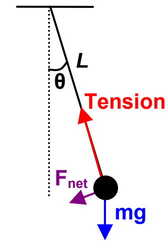
ID: 12227

Name ______________________

Class ______________________

In this activity, you will:

- *explore how various factors affect the period of pendulum oscillations*
- *measure the period of pendulum oscillations using a motion sensor*
- *develop a formula for the period of oscillations based on experimental data and dimensional analysis*



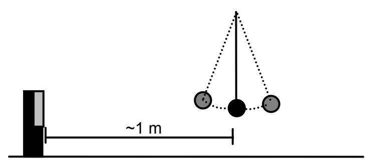
A simple pendulum is one which can be considered to be a point mass suspended from a string or rod of negligible mass. There are two forces acting on the pendulum mass: the force of tension and the force of gravity (weight). When the pendulum is displaced from its equilibrium point, the restoring force brings it back to the center. For small amplitudes, the motion of a simple pendulum is a simple harmonic motion.

In this activity, you will experimentally explore the factors that might affect the period of pendulum oscillations. You will use a motion sensor to collect distance–time data for various masses, initial angles, and pendulum lengths. You will use these data to determine the period of oscillations in each case. You will then determine the formula for the period of the pendulum by finding the best fit line and using dimensional analysis.

Problem 1 – Effect of mass on the period of pendulum oscillations

Step 1: Open the file **PhyAct_12227_Pendulum.tns**, move to page 1.2, start the simulation of pendulum motion, and answer question 1.

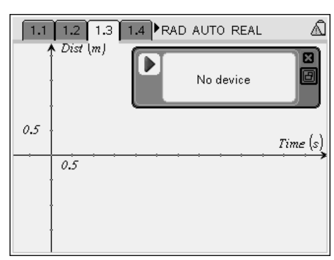
Q1. What factors do you think affect the period of the pendulum's oscillations?



Step 2: Set up the experiment, as shown to the right. Use the ring stand with the pendulum clamp to hang the 50 g mass from the string. Measure and record the length of the pendulum (it should be at least 50 cm long). Place the CBR2 (if you are using TI-Nspire handhelds) or Go!Motion sensor (if you are using TI-Nspire computer software) on a flat surface so the emitter screen is level with the pendulum bob. At rest, the pendulum bob should be about 1 m away from the motion sensor. The bob should be far enough from the motion sensor that it is never closer to the detector than 0.4 m while it is swinging.

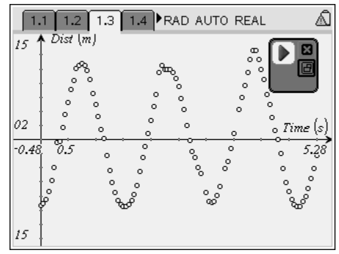
Step 3: Locate the switch under the head of the motion detector and set it to the Normal position, which is indicated by a picture of a person and a basketball.

Step 4: Move to page 1.3 and connect the motion detector to your handheld or computer. A distance data collection display should appear.

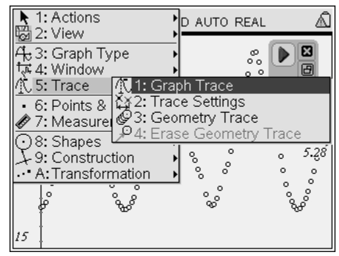


Step 5: Clear any stored data (**Menu > Data > Clear All Data**). Then, with the pendulum bob in the equilibrium position and stationary, zero the motion sensor (**Menu > Sensors > Zero**).

Step 6: Next, initiate pendulum oscillations and start data collection. Be sure to measure and record the initial angle to which you displace the pendulum. (Use an angle less than about 10°.) The Play button (►) should be highlighted in the data collection box. If this is the case, you need only click (press) to begin data collection. If the Play button is not highlighted, press tab until it is selected. When data collection has ended, a scatter plot will be displayed in the *Graphs & Geometry* application on page 1.3. If the collected data do not have a sinusoidal shape (as shown to the right), repeat the experiment. To avoid storing too much data on the device, discard any previous data before collecting a new data set.

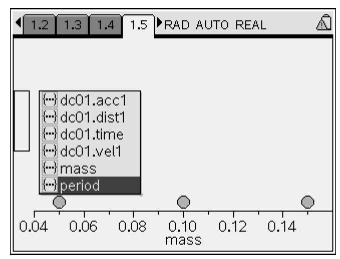


Step 7: Next, analyze the scatter plot and determine the period of oscillations from the graph. Minimize the display by selecting the button just below the × button. Then, switch to the *Graphs & Geometry* application by pressing ctrl tab. Trace the data points (**Menu > Trace > Graph Trace**) to identify the *x*-coordinates of several maxima or minima on the graph. The difference in *x*-coordinates of two successive maxima or minima is the period of the curve. Move to page 1.4 and record the average period of the 50 g mass.



Step 8: Move back to page 1.3 and repeat steps 6 and 7 two more times, increasing the mass by 50 g each time. Make sure to keep the string length and initial angle the same each time.

Step 9: Move to page 1.5, which contains an empty *Data & Statistics* application. Use this application to create a scatter plot of period vs. mass. Then, answer questions 2 and 3.

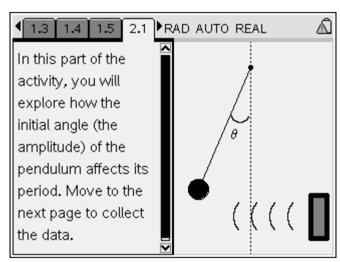


Q2. Based on your data and observations, what can you conclude about the effect of the pendulum mass on its period?

Q3. Do these results agree with your prediction in question 1? If not, describe any errors in your reasoning.

Problem 2 – Effect of initial angle (amplitude) on the period of pendulum oscillations

Step 1: Move to page 2.1 and read the text there. Again, set up the pendulum with a 50 g bob. Then, move to page 2.2, where you will carry out data collection.



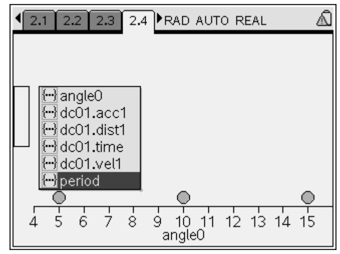
Step 2: Insert a data collection box by pressing ctrl D. A distance data collection box should be inserted automatically. Repeat steps 5–7 from problem 1. For the first trial, displace the pendulum by 5°.

Step 3: Repeat the data collection for initial angles of 10° and 15°. Keep the mass of the pendulum bob and the length of the string the same for all three trials. Record the period of the pendulum in each case in the *Lists & Spreadsheet* application on page 2.3.

Step 4: Move to page 2.4 and create a scatter plot of period vs. angle. Examine the graph and then answer questions 4 and 5.

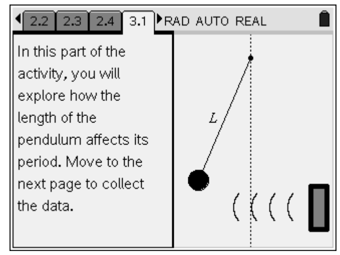
Q4. Based on your data and observations, what can you conclude about the effect of the initial angle on the pendulum period?

Q5. Do these results agree with your prediction in question 1? If not, describe any errors in your reasoning.



Problem 3 – Effect of pendulum length on the period of pendulum oscillations

Step 1: Next, move to page 3.1 and read the text there. Then, again set up the pendulum experiment. Use the 50 g bob, and use the measuring tape or meter stick to make the length of the string 1 m. Then, move to page 3.2, where you will carry out the data collection.



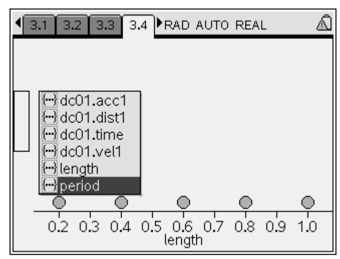
Step 2: Insert a data collection box by pressing ctrl D. A distance data collection box should be inserted automatically. Repeat steps 5–7 from problem 1 using the 1 m pendulum.

Step 3: Repeat the data collection four more times, increasing the length of the pendulum in each trial. Keep the mass of the pendulum bob and the initial angle the same for all three trials. Record the period of the pendulum in each case in the *Lists & Spreadsheet* application on page 3.3. (Suggested pendulum lengths are included on page 3.3, but you can use other lengths if you wish.)

Step 4: Move to page 3.4 and create a scatter plot of period vs. length. Examine the graph and then answer questions 6 and 7.

Q6. Based on your data and observations, what can you conclude about the effect of the length of the pendulum on its period?

Q7. Develop a mathematical model for your experimental data.



Step 5: Next, answer questions 8–11. You may use the provided *Calculator* application to check your work.

Q8. Use dimensional analysis to confirm your model.

Q9. Using the value of the constant determined in question 7 and g = 9.81 m/s^2, calculate the value of the constant k.

Q10. A simple pendulum is used to obtain an experimental value for gravitational acceleration, g. A student displaces a 0.510 m pendulum 10° from the equilibrium position and releases it. The period of the pendulum is 1.44 s. Determine the experimental value of gravitational acceleration.

Q11. A pendulum has a length of 0.79 m and a mass of 0.24 kg. The pendulum is pulled away from its equilibrium position by an angle of 8.50° and released. Assume that friction can be neglected and that the resulting oscillatory motion is simple harmonic motion. What is the angular frequency, ω, of the motion? Remember that $\omega = 2\pi f$, where f is the frequency of the pendulum.

