

Exploring Parametric Equations With the “Human Cannonball”

by – Lisa Blank, Math & Science Teacher, Lyme Central School, Chaumont, NY

Activity overview

Students will explore the use of parametric equations to model the motion of the “Human Cannonball” after being fired from a cannon. Key graph features will be explored, including maximum height, length of time in the air, and maximum distance traveled through the use of parametric equations and a quadratic regression equation. The quadratic equation will be determined by grabbing points from the graph of the parametric equations.

Concepts

Parametric Equations

Quadratic Regression

Interpreting Graphs of Parametric Equations

Interpreting Graphs of Quadratic Equations

Projectile Motion

Trajectories

Teacher preparation

Load the humancannons.tns file onto all student calculators. Copy the related handout for students to assist in guiding students through the activity and to provide students with directing questions.

The humancannont.tns file is for the instructor to be able to view the results the students should obtain as they work through this activity.

Classroom management tips

The instructor should direct students to open the humancannons.tns file. Once students have opened the file, the teacher should monitor students, assisting them as they work through the steps provided in this activity. Students will likely need assistance if the instructor wants the students to modify axes and/or label the graphs. Remind students that ctrl-G will remove the function window from the bottom of the graph screen to provide larger viewing windows for the graphs.

A worksheet is provided with this activity for completion by students.

TI-Nspire Applications

Parametric Graphs

Transferring Data Points from a Graph to a Spreadsheet

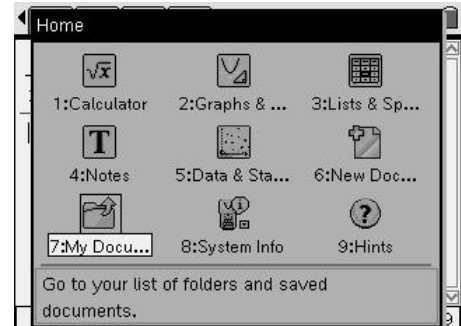
Quadratic Regression

Quadratic Graphs

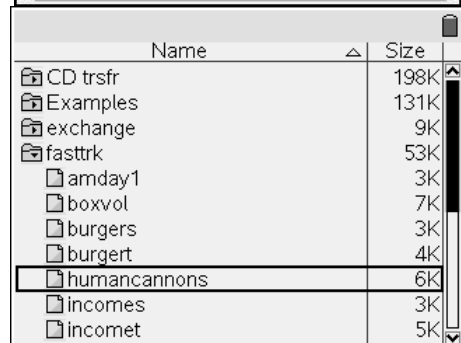
Spreadsheets

Step-by-step directions

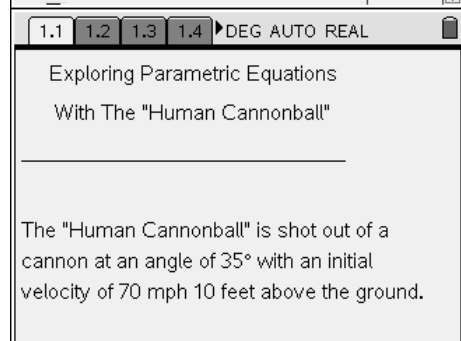
1. At the home screen, select My Documents.



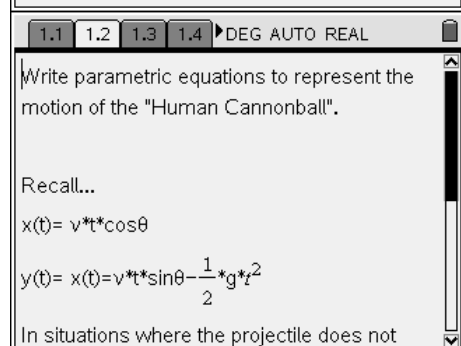
2. Browse to the folder titled *humancannons*.



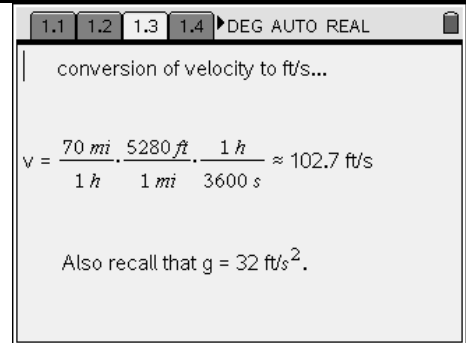
3. The first page of the document should appear as shown. Read this page to become familiar with this activity.



4. Move to the next page of the document by using ctrl \rightarrow right . On this page, you will find helpful reminders regarding parametric equations.



5. Move to the next page of the document by using $\text{ctrl} + \text{right arrow}$. On this page, you are to show your work, using dimensional analysis (factor-label), for the conversion of the velocity to ft/s.

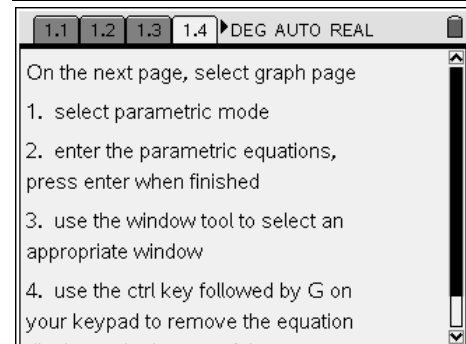


conversion of velocity to ft/s...

$$v = \frac{70 \text{ mi}}{1 \text{ h}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \approx 102.7 \text{ ft/s}$$

Also recall that $g = 32 \text{ ft/s}^2$.

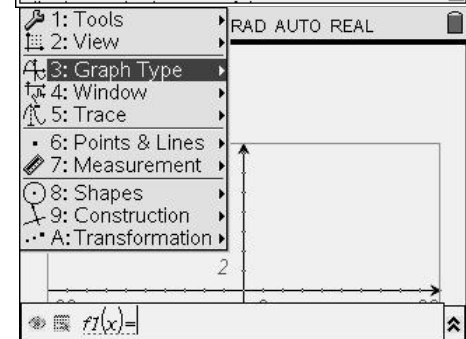
6. Move to the next page of the document by using $\text{ctrl} + \text{right arrow}$. This page provides instructions for what is to be done on the following pages.



On the next page, select graph page

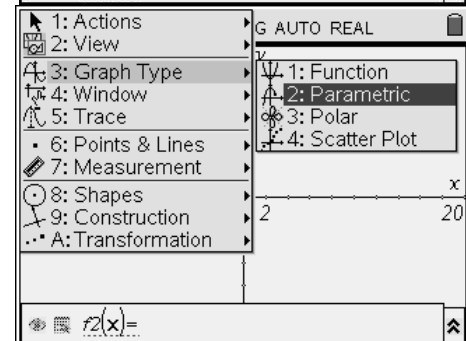
1. select parametric mode
2. enter the parametric equations, press enter when finished
3. use the window tool to select an appropriate window
4. use the ctrl key followed by G on your keypad to remove the equation

7. Move to the next page and press menu . Select Graphs & Geometry, then press menu again and select Graph Type from the pull-down menu.



1: Tools
2: View
3: Graph Type
4: Window
5: Trace
6: Points & Lines
7: Measurement
8: Shapes
9: Construction
A: Transformation

8. Next, select Parametric.



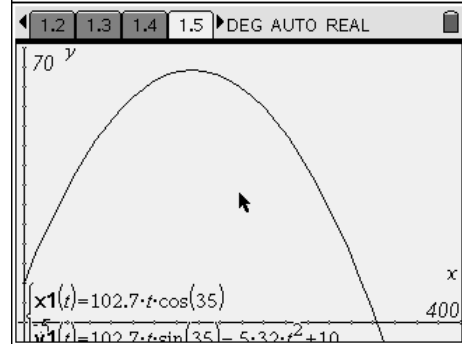
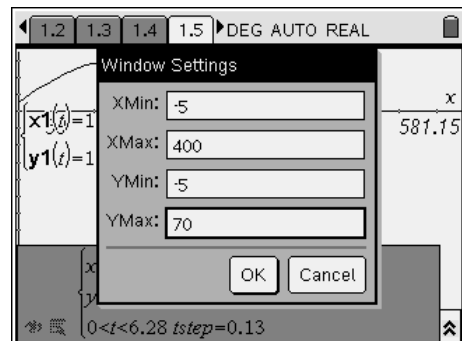
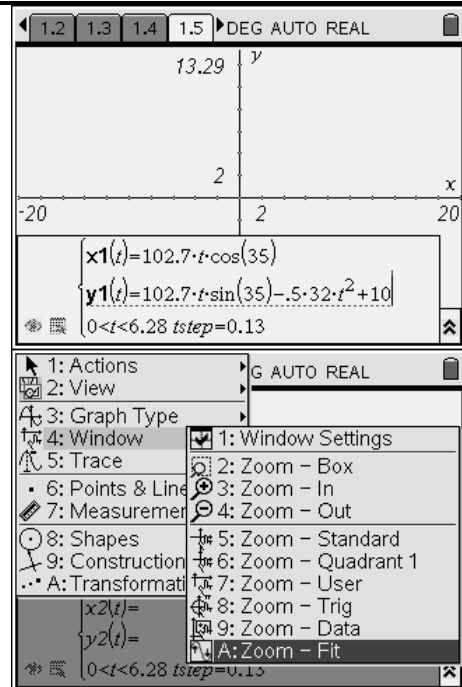
1: Actions
2: View
3: Graph Type
4: Window
5: Trace
6: Points & Lines
7: Measurement
8: Shapes
9: Construction
A: Transformation

1: Function
2: Parametric
3: Polar
4: Scatter Plot

9. The parametric graph window will appear. Move to the parametric equation box at the bottom of the screen and enter the parametric equations developed using the converted velocity and the trajectory angle provided in the problem. You can move from one entry field to another by pressing (tab).

10. Press enter following the entry of your equations and a graph will appear. To adjust the viewing window, press (menu) and select Window, followed by Zoom-Fit. If a user-friendly window does not result, settings for the window may be adjusted using choice 1: Window Settings. Window settings may also be adjusted directly on the screen by clicking and dragging axes, or editing the maximum values shown on the x- and y-axes.

11. To view more of this graph, since a portion of it is covered, fhide the entry box at the bottom of the screen by pressing (ctrl) G, which acts as a toggle to hide or reveal entry boxes for graphs.

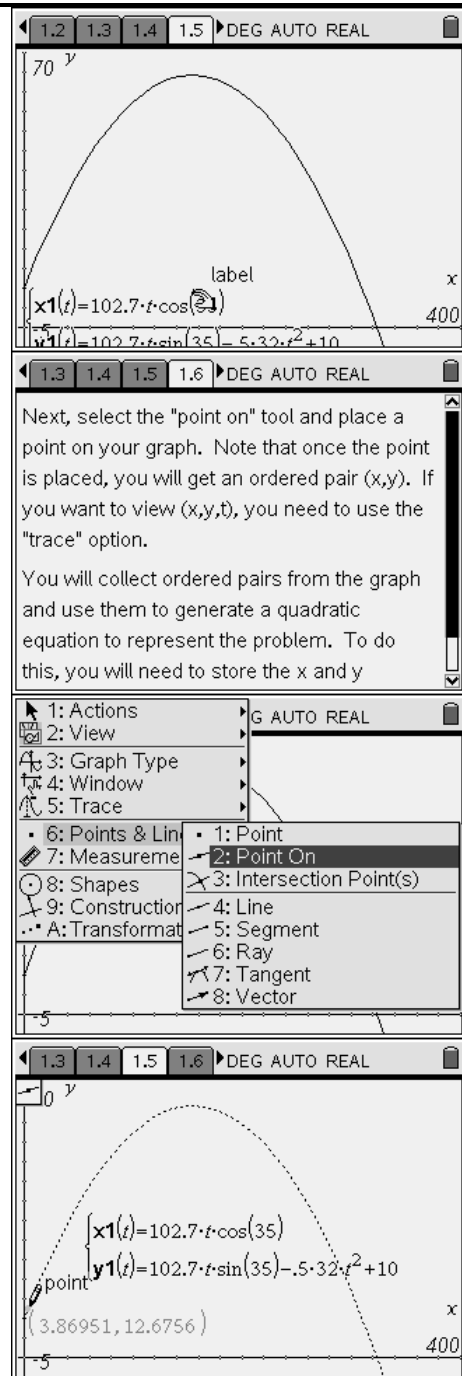


12. To make the equations visible, they can be moved by clicking and dragging to a better location. To do this, move your cursor to the equations. They should flash and a hand will appear. Press $\text{ctrl} + \text{click}$ to grab onto the equations and move them. Pressing the center of the Nav Pad will release the grip on the equations so they may be dropped in the desired location.

13. Move to the next page using $\text{ctrl} + \text{right arrow}$. Read the given instructions. Then press using $\text{ctrl} + \text{left arrow}$ to move back to the previous graph page.

14. Press menu , followed by 6: Points & Lines, followed by 2: Point On.

15. Move the cursor to the curve and a pencil tool along with the word *point* will appear. Pressing enter will drop a point label. Moving the pencil along the curve will show the point labels in grey as you trace along the curve without dropping the points.

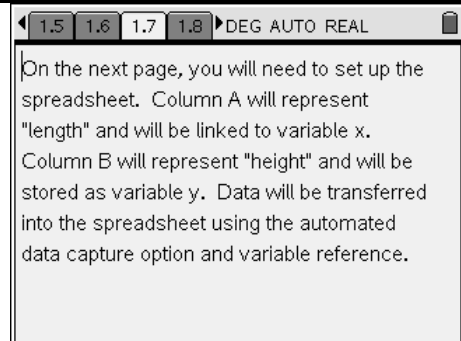


The first screenshot shows a graph of a parabola on a coordinate plane. The x-axis ranges from -5 to 400, and the y-axis ranges from -5 to 70. The parametric equations are displayed as $x1(t) = 102.7 \cdot t \cdot \cos(35)$ and $y1(t) = 102.7 \cdot t \cdot \sin(35) - .5 \cdot 32 \cdot t^2 + 10$. A label 'label' is placed near the curve.

The second screenshot shows the calculator's menu system. The '6: Points & Lines' menu is open, and '2: Point On' is selected. Other options include '1: Point', '3: Intersection Point(s)', '4: Line', '5: Segment', '6: Ray', '7: Tangent', and '8: Vector'.

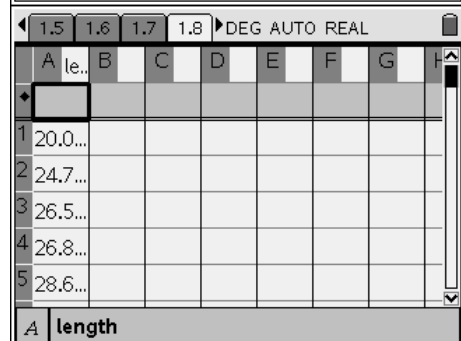
The third screenshot shows the same graph as the first, but with a point on the curve. The point is labeled with its coordinates: $(3.86951, 12.6756)$. The parametric equations are still visible on the screen.

16. Move to page 7 of the document using $\text{ctrl} \rightarrow \text{D}$. Read the given instructions



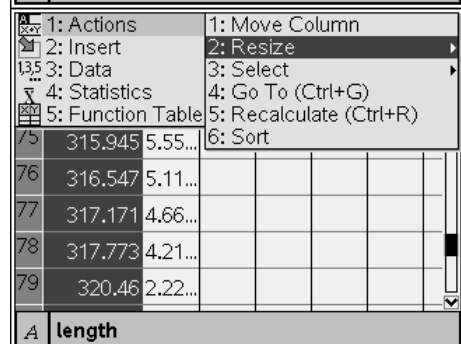
On the next page, you will need to set up the spreadsheet. Column A will represent "length" and will be linked to variable x. Column B will represent "height" and will be stored as variable y. Data will be transferred into the spreadsheet using the automated data capture option and variable reference.

17. Move to page 8 using $\text{ctrl} \rightarrow \text{D}$. Press menu . Select Lists & Spreadsheets. In the region to the right of A, type *length* and press enter . The data column will fill in with values from the tracing of the parametric graph. Similarly, label column B as *height*.



	A	B	C	D	E	F	G	H
1	20.0...							
2	24.7...							
3	26.5...							
4	26.8...							
5	28.6...							

18. To resize columns for better viewing, press menu , followed by 1: Actions, followed by 2: Resize, then 1: Resize Column Width. The selected column will then be grey. Use the left and right arrows of the keypad to resize the column width as desired. Press enter to keep the desired width.

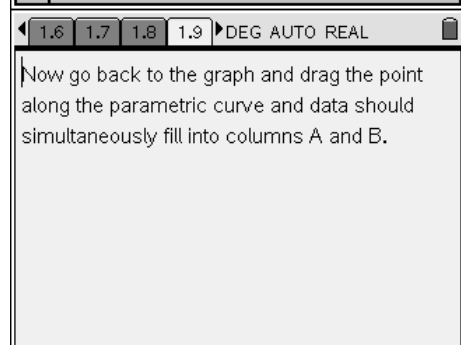


1: Actions
2: Insert
3: Data
4: Statistics
5: Function Table

1: Move Column
2: Resize
3: Select
4: Go To (Ctrl+G)
5: Recalculate (Ctrl+R)
6: Sort

	A	B	C	D	E	F	G	H
75	315.945	5.55...						
76	316.547	5.11...						
77	317.171	4.66...						
78	317.773	4.21...						
79	320.46	2.22...						

19. Move to the next page of the document by using $\text{ctrl} \rightarrow \text{D}$. Go back to page 5 and drag the point on pencil tool along the parametric curve to automatically fill additional ordered pairs into the spreadsheet.



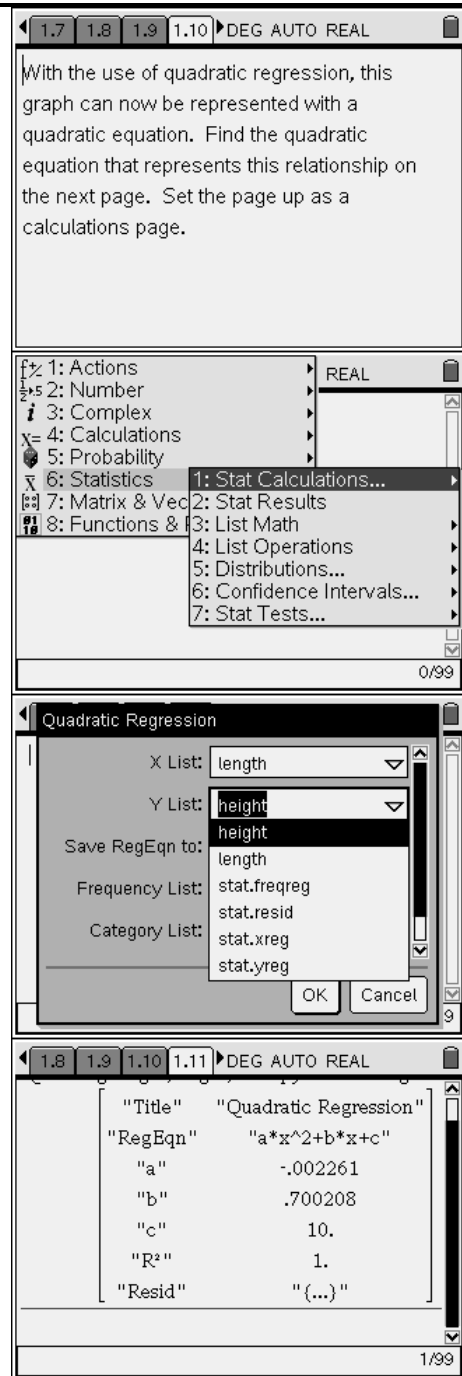
Now go back to the graph and drag the point along the parametric curve and data should simultaneously fill into columns A and B.

20. Move to the page 10 of the document by using \leftarrow (ctrl) \rightarrow .

21. Move to the next page of the document by using \leftarrow (ctrl) \rightarrow . Press $\left[\text{menu}\right]$. Select 1: Add Calculator, then press $\left[\text{menu}\right]$, followed by 6: Statistics, followed by 1: Stat Calculations, then choice 1: Quadratic Regression.

22. You must also choose the appropriate spreadsheet columns for x and y. Use $\left[\text{tab}\right]$ to move between entry fields.

23. Select "OK" at the bottom of the screen after choosing the appropriate x and y lists. The linear regression equation results will appear. These results are automatically saved for function graphing.



The image shows three screenshots of a TI-84 Plus calculator interface. The top screenshot shows a document page with text about quadratic regression. The middle screenshot shows the calculator's menu system with '1: Stat Calculations...' selected under '6: Statistics'. The bottom screenshot shows the 'Quadratic Regression' dialog box with 'length' selected for the X List and 'height' for the Y List. Below the dialog box, the results of the quadratic regression are displayed in a table.

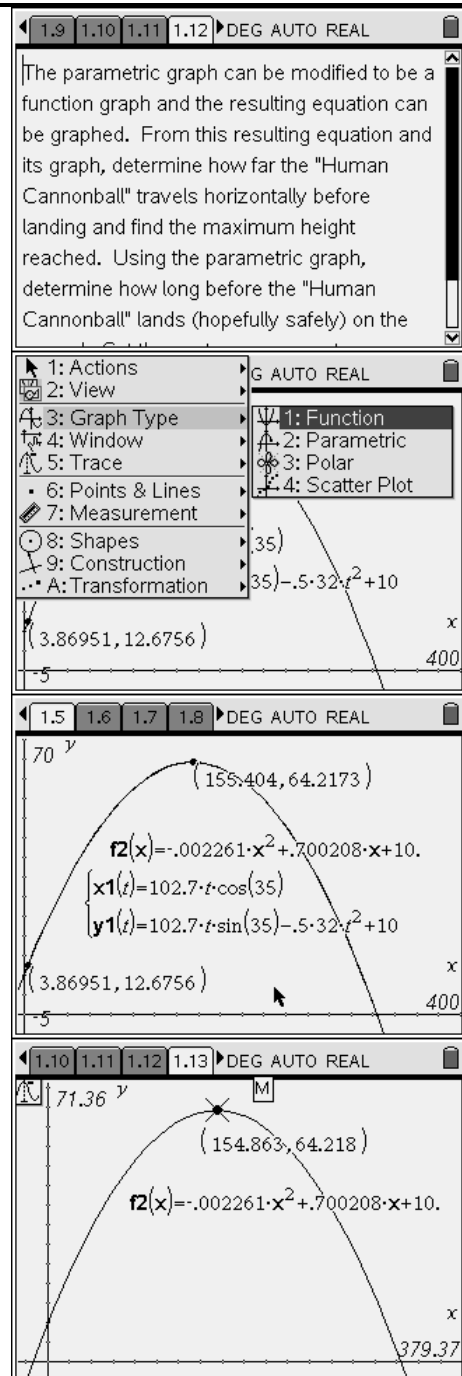
Parameter	Value
"Title"	"Quadratic Regression"
"RegEqn"	"a*x^2+b*x+c"
"a"	-.002261
"b"	.700208
"c"	10.
"R ² "	1.
"Resid"	" {... }"



24. Move to the next page of the document by using $\text{ctrl} \rightarrow \text{D}$.
Read this page for additional problem details.

25. Move back to page 5 and press menu . Select 3: Graph Type, followed by 1: Function.

26. In the function window that appears at the bottom, use the up arrow to locate the quadratic equation just obtained. Pressing $\text{ctrl} \rightarrow \text{G}$ will result in the graph being plotted on top of the existing parametric graph. Note that the quadratic function is also displayed on the graph screen. Pressing $\text{ctrl} \rightarrow \text{G}$ will remove the function box from the bottom of the screen.

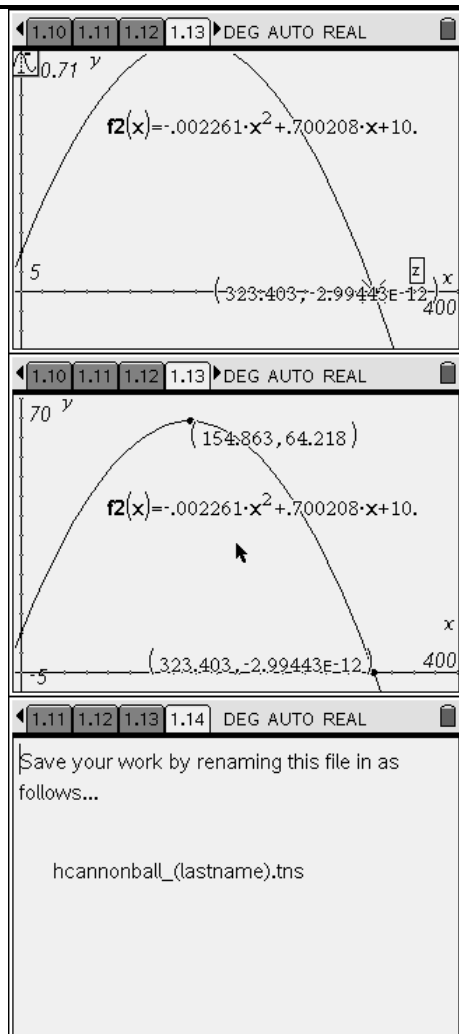
27. The graph on page 5 may be traced, or a new quadratic graph can be set up on page 13, so that quadratic equation alone can be studied. This can be easier for tracing to find key graph points such as max, min, and zeros. The maximum value is identified by the letter M and zeros are similarly identified by the letter z. Be careful to trace slowly so that these letters don't flash too quickly to be seen.



28. Once a key value has been found, pressing  will drop that point on the graph. If the point label is not fully visible on the screen, it may be grabbed and dragged using the grab hand, .

29. The quadratic graph provides the maximum height and distance for the path of the "Human Cannonball". The benefit of the parametric equation is that as you trace along the curve, you will be able to obtain time values.

30. The final page of the handheld document provides instructions to the student in saving the document created for review by the instructor.



Assessment and evaluation (*NOTE: this section can be separate or included in the step-by-step directions.*)

- Evaluate the completed student .tns files
- Collect the related student handouts and assess them for understanding.
- Follow up with problems from the Parametric Equations Problem Sampler, providing somewhat decreased instruction to check for understanding of the process and concepts involved.

Activity extensions

- Study additional parametric equation problems, such as those on the Parametric Equations Problem Sampler.
- Have students create their own parametric equations problems that relate to their personal interests. There are many great topics of interest to many students to be related to this activity!
- This activity provides an excellent opportunity for math and science teachers to work cooperatively in the study of parametric equations and projectile motion.

Student TI-Nspire Document

humancannons.tns

<p>humancannont</p> <table border="1"> <thead> <tr> <th>Name</th> <th>Size</th> </tr> </thead> <tbody> <tr><td>burgers</td><td>3K</td></tr> <tr><td>burgert</td><td>4K</td></tr> <tr><td>humancannons</td><td>6K</td></tr> <tr><td>humancannont</td><td>10K</td></tr> <tr><td>incomes</td><td>3K</td></tr> <tr><td>incomet</td><td>5K</td></tr> <tr><td>incometchr</td><td>5K</td></tr> <tr><td>inscangles</td><td>3K</td></tr> <tr><td>inscanglet</td><td>4K</td></tr> <tr><td>savings</td><td>3K</td></tr> <tr><td>savingstchr</td><td>4K</td></tr> </tbody> </table>	Name	Size	burgers	3K	burgert	4K	humancannons	6K	humancannont	10K	incomes	3K	incomet	5K	incometchr	5K	inscangles	3K	inscanglet	4K	savings	3K	savingstchr	4K	<p>1.1 1.2 1.3 1.4 DEG AUTO REAL</p> <p>Exploring Parametric Equations With The "Human Cannonball"</p> <hr/> <p>The "Human Cannonball" is shot out of a cannon at an angle of 35° with an initial velocity of 70 mph 10 feet above the ground.</p>	<p>1.1 1.2 1.3 1.4 DEG AUTO REAL</p> <p>Write parametric equations to represent the motion of the "Human Cannonball".</p> <p>Recall...</p> $x(t) = v \cdot t \cdot \cos \theta$ $y(t) = x(t) = v \cdot t \cdot \sin \theta - \frac{1}{2} \cdot g \cdot t^2$ <p>In situations where the projectile does not</p>
Name	Size																									
burgers	3K																									
burgert	4K																									
humancannons	6K																									
humancannont	10K																									
incomes	3K																									
incomet	5K																									
incometchr	5K																									
inscangles	3K																									
inscanglet	4K																									
savings	3K																									
savingstchr	4K																									
<p>1.1 1.2 1.3 1.4 DEG AUTO REAL</p> <p>Show work here for conversion of velocity to ft/s. Show units in your work.</p>	<p>1.1 1.2 1.3 1.4 DEG AUTO REAL</p> <p>On the next page, select graph page</p> <ol style="list-style-type: none"> 1. select parametric mode 2. enter the parametric equations, press enter when finished 3. use the window tool to select an appropriate window 4. use the ctrl key followed by G on your keypad to remove the equation 	<p>1.2 1.3 1.4 1.5 DEG AUTO REAL</p> <p>Press Menu</p>																								
<p>1.3 1.4 1.5 1.6 DEG AUTO REAL</p> <p>Next, select the "point on" tool and place a point on your graph. Note that once the point is placed, you will get an ordered pair (x,y). If you want to view (x,y,t), you need to use the "trace" option.</p> <p>You will collect ordered pairs from the graph and use them to generate a quadratic equation to represent the problem. To do this, you will need to store the x and y</p>	<p>1.4 1.5 1.6 1.7 DEG AUTO REAL</p> <p>On the next page, you will need to set up the spreadsheet. Column A will represent "length" and will be linked to variable x. Column B will represent "height" and will be stored as variable y. Data will be transferred into the spreadsheet using the automated data capture option and variable reference.</p>	<p>1.5 1.6 1.7 1.8 DEG AUTO REAL</p> <p>Press Menu</p>																								
<p>1.6 1.7 1.8 1.9 DEG AUTO REAL</p> <p>Now go back to the graph and drag the point along the parametric curve and data should simultaneously fill into columns A and B.</p>	<p>1.7 1.8 1.9 1.10 DEG AUTO REAL</p> <p>With the use of quadratic regression, this graph can now be represented with a quadratic equation. Find the quadratic equation that represents this relationship on the next page. Set the page up as a calculations page.</p>	<p>1.8 1.9 1.10 1.11 DEG AUTO REAL</p> <p>Press Menu</p>																								



The image shows three screenshots of the TI-Nspire software interface, arranged horizontally. Each window has a title bar with navigation buttons and the text "DEG AUTO REAL".

- Left Window:** The title bar shows buttons for 1.9, 1.10, 1.11, and 1.12. The main area contains a paragraph of text: "The parametric graph can be modified to be a function graph and the resulting equation can be graphed. From this resulting equation and its graph, determine how far the 'Human Cannonball' travels horizontally before landing and find the maximum height reached. Using the parametric graph, determine how long before the 'Human Cannonball' lands (hopefully safely) on the ground." A vertical scrollbar is visible on the right side of the text area.
- Middle Window:** The title bar shows buttons for 1.10, 1.11, 1.12, and 1.13. The main area is mostly blank with the text "Press Menu" centered.
- Right Window:** The title bar shows buttons for 1.11, 1.12, 1.13, and 1.14. The main area contains the text "Save your work by renaming this file in as follows..." followed by the filename "hcannonball_(lastname).tns".