

## The Lunes of Hippocrates

ID: 10589

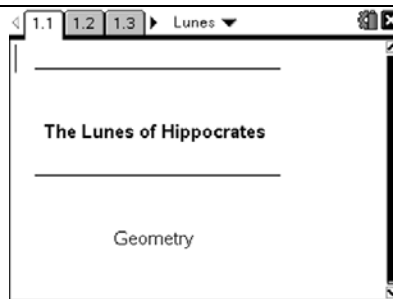
Name \_\_\_\_\_

Class \_\_\_\_\_

*In this activity, you will explore a figure involving circles and a right triangle.*

*A Lune is the area enclosed between arcs of intersecting circles*

Use this document to record your answers.



### Problem 1 – Constructing the Lunes

Open the file *Lunes*.

1. On page 1.3, construct a segment  $\overline{AC}$  and a perpendicular line through point C. Then create point B on the perpendicular line, and use the **Triangle** tool to draw  $\triangle ABC$ .
2. Hide the perpendicular line, and construct the midpoints of the sides of  $\triangle ABC$ .
3. Use the midpoints as centers of 3 circles. Each circle will have a side of  $\triangle ABC$  as its diameter.
4. Measure the areas of the three circles and the triangle.
5. Use the expression  $\frac{\text{circle}}{2}$  to calculate the areas of the 3 semicircles.

### Problem 2 – Sum of the Areas of the Lunes

Use your figure on page 1.3 or the pre-made figure on page 2.2 to continue.

6. Record the areas of the semicircles below. Drag a vertex of  $\triangle ABC$  to get different values.

semiAC	semiBC	semiAB

7. Do you notice a relationship between the areas of the 3 semicircles? Make a conjecture.
8. What well-known theorem justifies this result?

9. On page 2.3, the lunes are defined as the regions *inside* the two small semicircles (with diameters  $\overline{AC}$  and  $\overline{BC}$ , but *outside* the large semicircle (with diameter  $\overline{AB}$ ).

What areas would you add and subtract to find the sum of the areas of the lunes? Write a formula below:

Sum of the Lunes = \_\_\_\_\_

10. Record the data you collected in the spreadsheet on page 2.6.

triangle	semiAC	semiBC	semiAB	lunes

11. What do you notice about the area of the lunes and another area already found?

12. Complete the conjecture:

The sum of the areas of the lunes = \_\_\_\_\_

13. On page 2.8, drag point A, B or C to change the triangle's shape and observe if your conjecture is true.

### Problem 3 – Algebraic Approach (Extension)

14. Let  $a$  = length of side  $\overline{BC}$ ,  $b$  = length of side  $\overline{AC}$ , and  $c$  = length of side  $\overline{AB}$ .

Write expressions for the areas of the triangle and semicircles in terms of  $a$ ,  $b$ , and  $c$ .

Area triangle = \_\_\_\_\_      Area semiAC = \_\_\_\_\_

Area semiBC = \_\_\_\_\_      Area semiAB = \_\_\_\_\_

15. Substitute your expressions into your formula from #9 above and simplify.