1.1 1.2 1.3 *Systems A...n-2

Life-lines for Curves

Using Systems of Equations to Define Non-Linear Sequences and Functions

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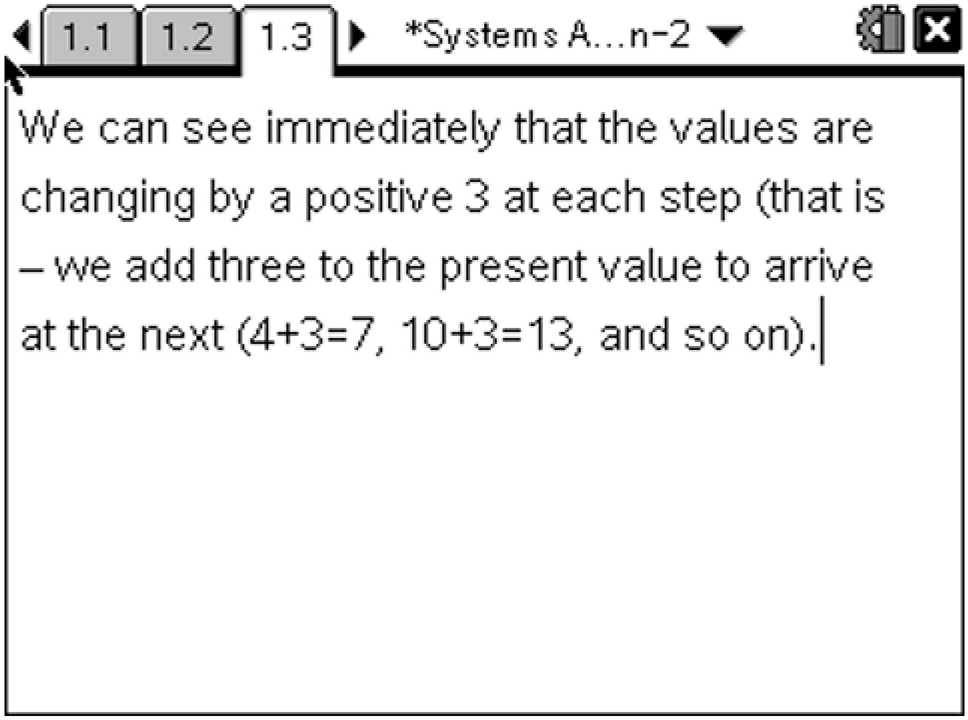
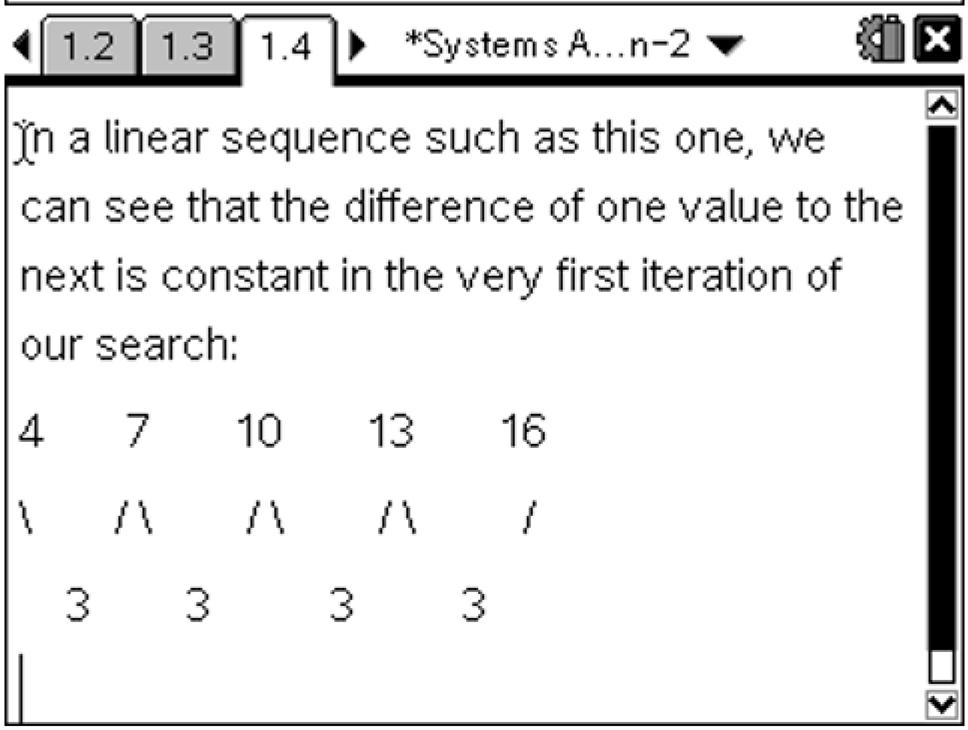
1.1 1.2 1.3 *Systems A...n-2

Systems of equations may also be applied to finding the closed recursive (or explicit) form of a function. The recursive requires that we know the previous value in a sequence to find the subsequent. For instance: Consider the sequence 4, 7, 10, 13, 16, ...



We can see immediately that the values are changing by a positive 3 at each step (that is – we add three to the present value to arrive at the next (4+3=7, 10+3=13, and so on).



In a linear sequence such as this one, we can see that the difference of one value to the next is constant in the very first iteration of our search:

```
4     7     10     13     16
\    /\     /\     /\     /
  3     3      3      3
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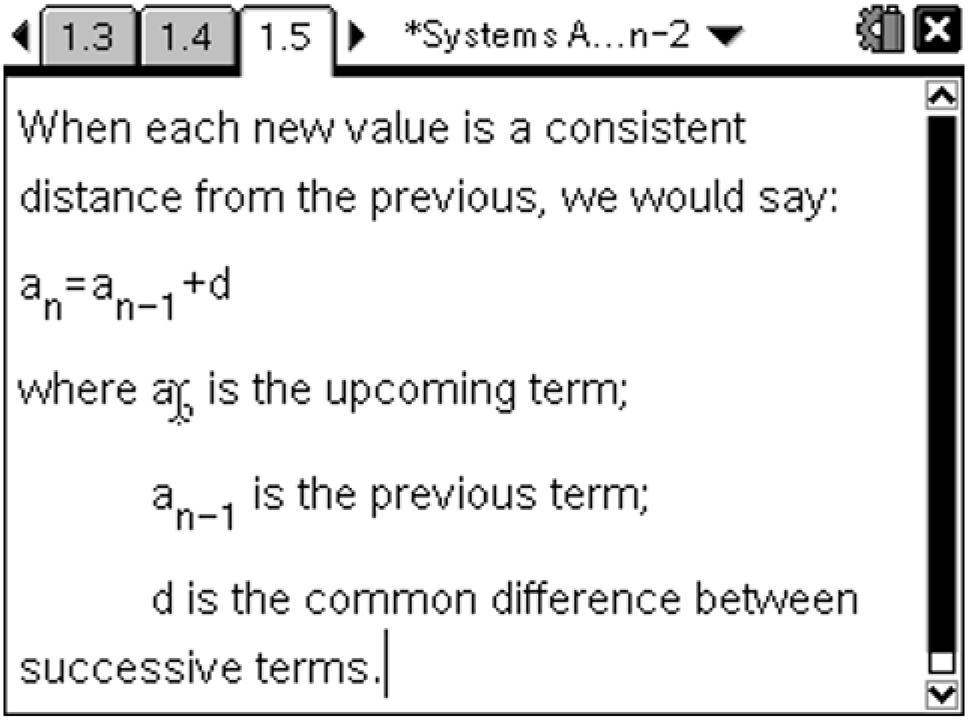
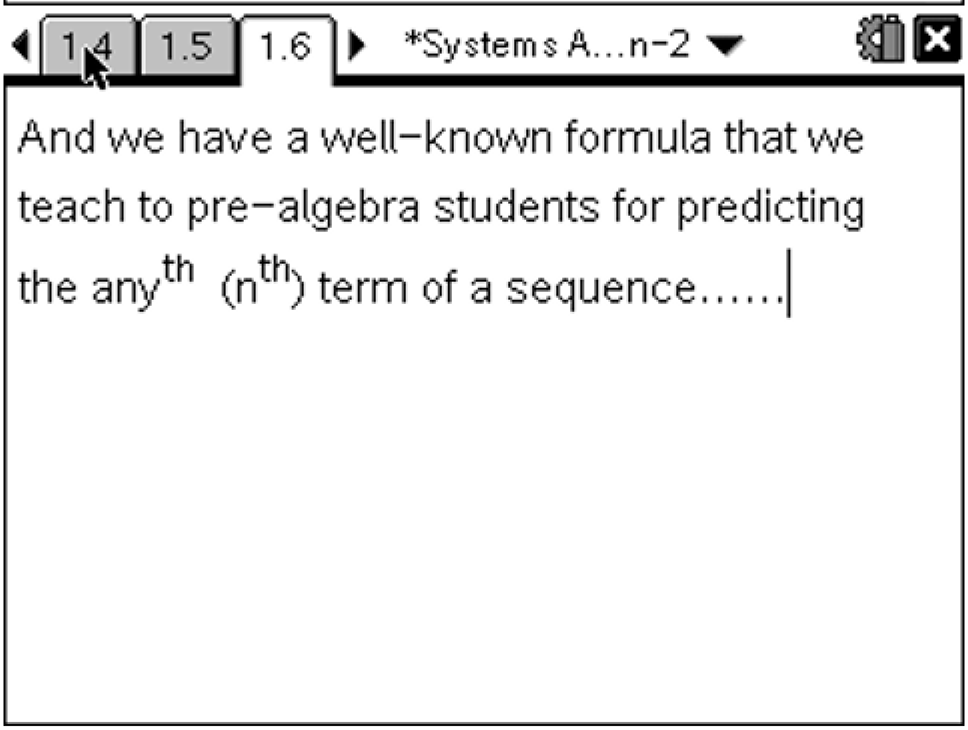
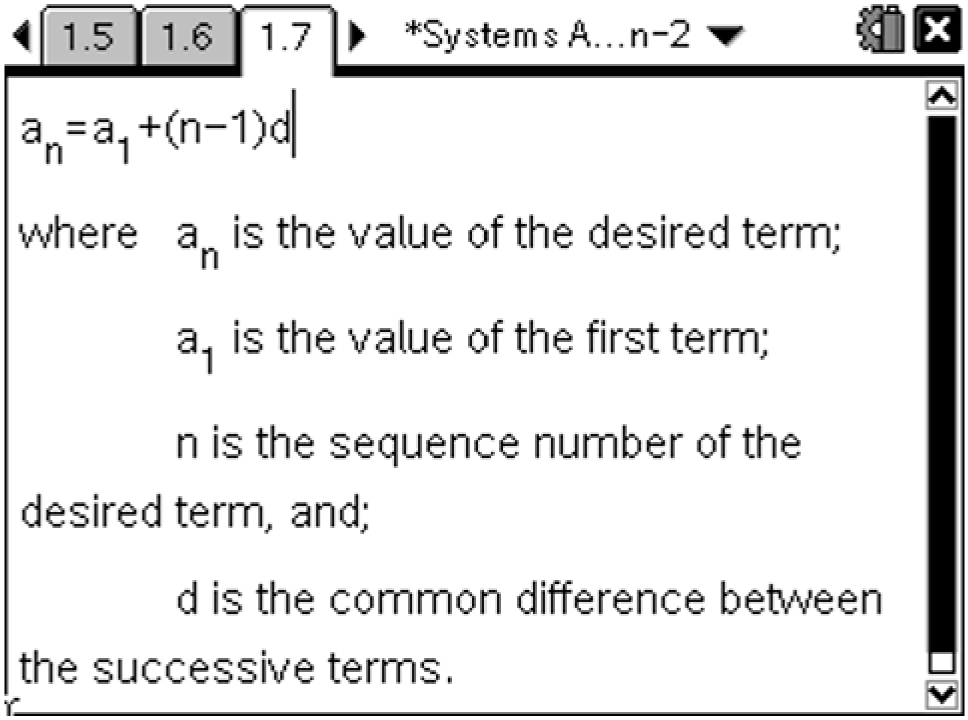
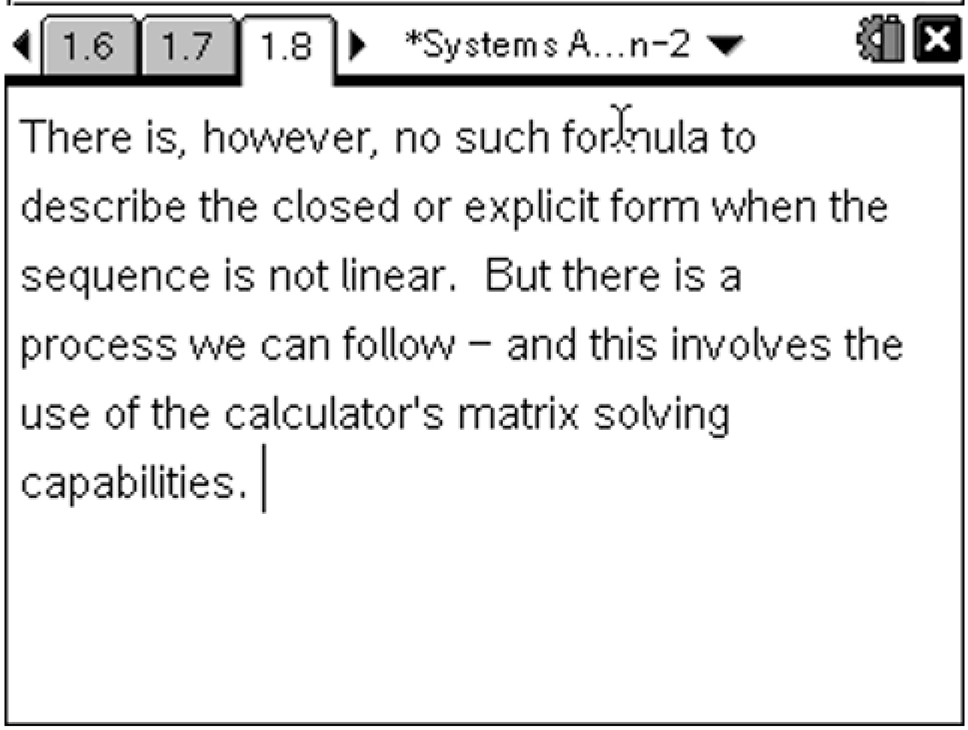
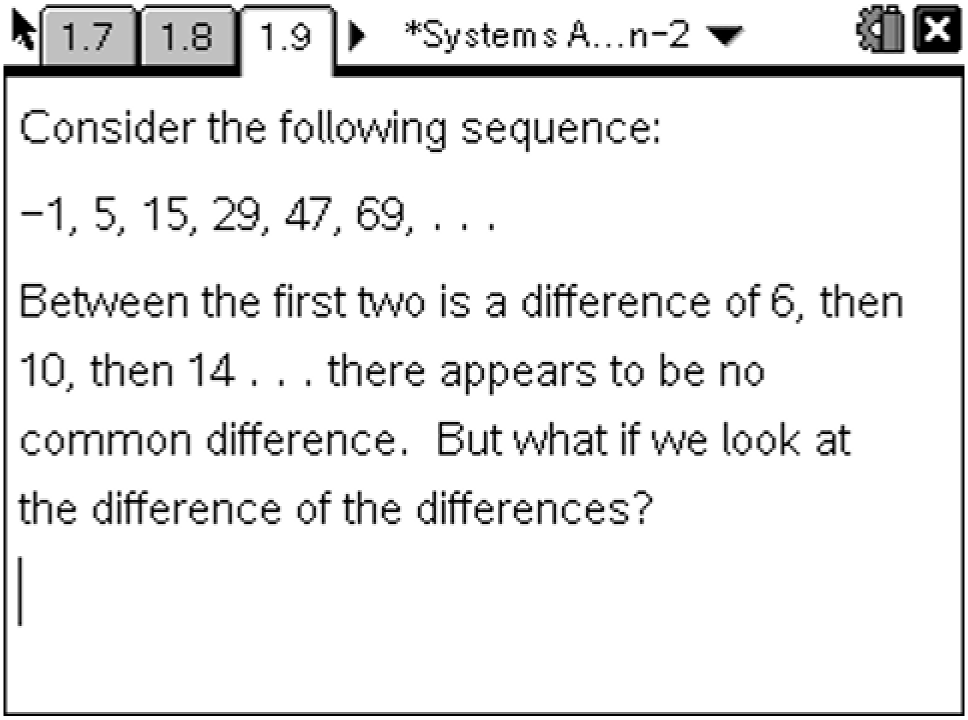
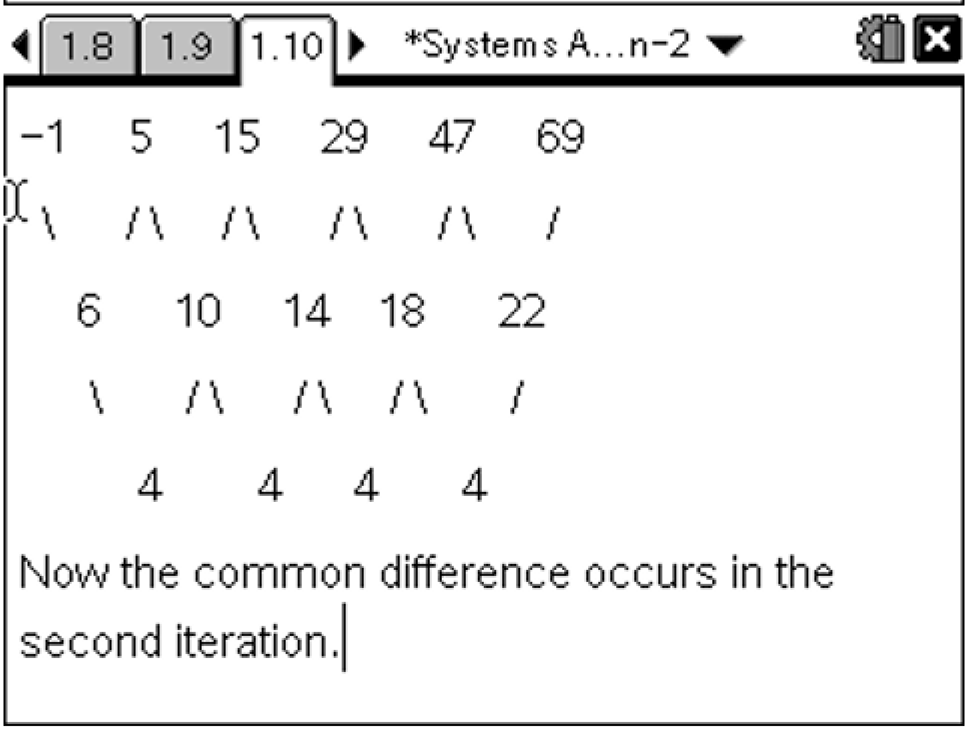

1.3
1.4
1.5
*Systems A...n-2
When each new value is a consistent distance from the previous, we would say:
$a_n=a_{n-1}+d$
where a_n is the upcoming term;
a_{n-1} is the previous term;
d is the common difference between successive terms.


1.4
1.5
1.6
*Systems A...n-2
And we have a well-known formula that we teach to pre-algebra students for predicting the anyth (n^{th}) term of a sequence......


1.5 1.6 1.7
*Systems A...n-2
$a_n=a_1+(n-1)d$
where a_n is the value of the desired term;
a_1 is the value of the first term;
n is the sequence number of the desired term, and;
d is the common difference between the successive terms.


1.6 1.7 1.8
*Systems A...n-2
There is, however, no such formula to describe the closed or explicit form when the sequence is not linear. But there is a process we can follow – and this involves the use of the calculator's matrix solving capabilities.


1.7 1.8 1.9 *Systems A...n-2
Consider the following sequence:
−1, 5, 15, 29, 47, 69, . . .
Between the first two is a difference of 6, then 10, then 14 . . . there appears to be no common difference. But what if we look at the difference of the differences?


1.8 1.9 1.10 *Systems A...n-2
−1 5 15 29 47 69
\ /\ /\ /\ /\ /
6 10 14 18 22
\ /\ /\ /\ /
4 4 4 4
Now the common difference occurs in the second iteration.

1.9 | 1.10 | 1.11 | *Systems A...n-2

If the first was linear, then the second is quadratic (the third is cubic, the fourth quartic and so on...).

Remember that the standard form of the quadratic is $Ax^2+Bx+C=y$.

1.10 | 1.11 | 1.12 | *Systems A...n-2

We know the y-values (−1, 5, 15, etc), and we know the placement in the sequence of each of the answers (1, 2, 3, etc), so we can replace x with 1 (and then 2, then 3, etc) to get the correct number of variables in a system of equations.

How many equations will we need? One for each variable, of course. So since we have a quadratic sequence we'll have to have an "A", a "B" and a "C" – three equations with three variables using the placement in the sequence for "x" and the given y–value:

$A(1)^2+B(1)+C=-1$

$A(2)^2+B(2)+C=5$

$A(3)^2+B(3)+C=15$

These give us:

$A+B+C=-1$

$4A+2B+C=5$

$9A+3B+C=15$

1.13 1.14 1.15 *Systems A...n-2

Now pull the augmented matrix –

1 1 1 −1

4 2 1 5

9 3 1 15

and run the reduced row echelon form to solve for the values of A, B, and C.

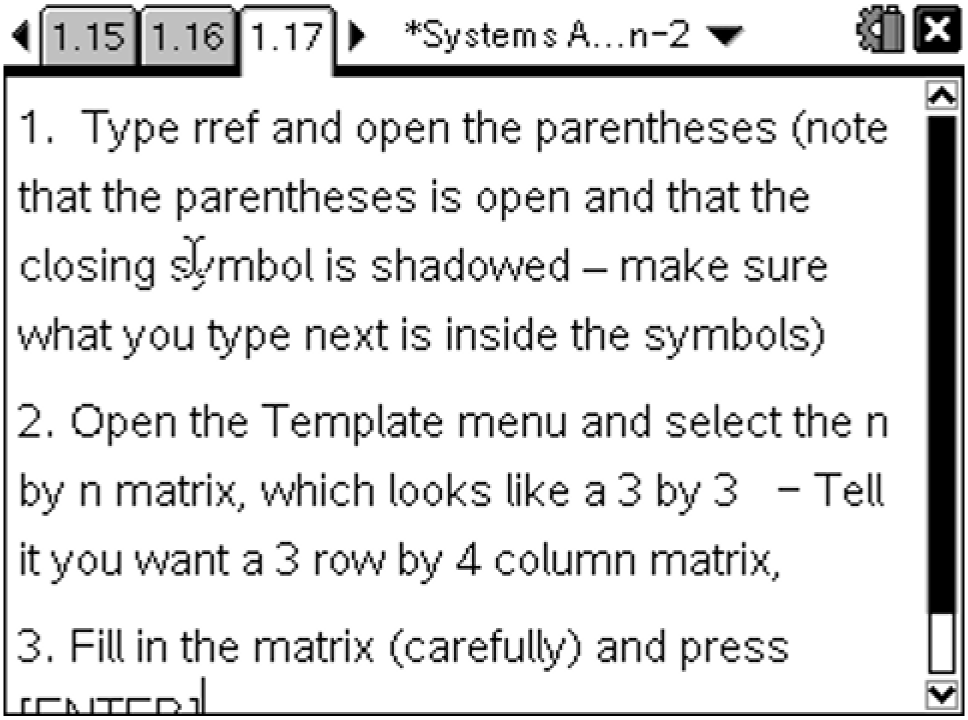
1.14 1.15 1.16 *Systems A...n-2

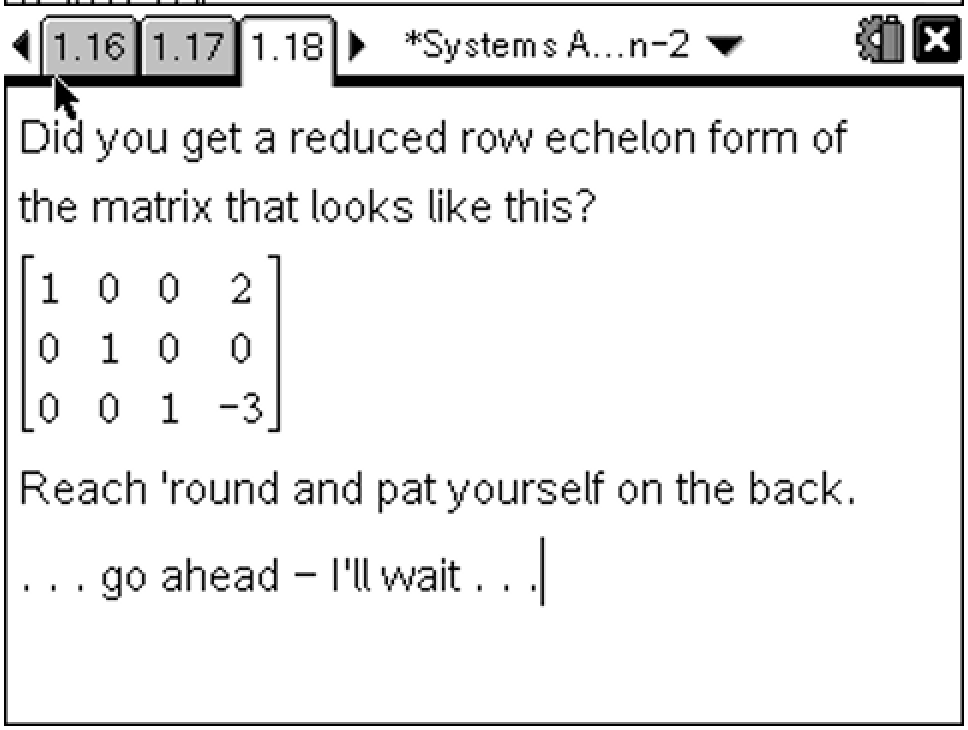
Execute the rref function by

*going to the Scratchpad (press the Scratchpad key, 2nd down from top left),

or

*by creating a new calculator page (Press HOME and choose calculator icon from bottom of screen) and:


1.15 1.16 1.17
*Systems A...n-2
1. Type rref and open the parentheses (note that the parentheses is open and that the closing symbol is shadowed – make sure what you type next is inside the symbols)
2. Open the Template menu and select the n by n matrix, which looks like a 3 by 3 – Tell it you want a 3 row by 4 column matrix,
3. Fill in the matrix (carefully) and press [ENTER]


1.16 1.17 1.18
*Systems A...n-2
Did you get a reduced row echelon form of the matrix that looks like this?
$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix}$
Reach 'round and pat yourself on the back.
. . . go ahead – I'll wait . . .

1.17 | 1.18 | 1.19 | *Systems A...n-2

But how do we read this?

It means that 1A, no Bs, and no Cs = 2, and that 1 B, no As, and no Cs = 0 and that 1C, no As and no Bs = −3. In our function, that's $2x^2+0x-3 = y$, or more properly

$2x^2-3 = y$

1.18 | 1.19 | 1.20 | *Systems A...n-2

Now you try:

Given the sequence: 2, 5, 10, 17, 26,...

Can you use TI−Nspire to find the explicit form of the function used to generate it?

1.19 | 1.20 | 1.21 | *Systems A...n-2

1. Use the difference list function to subtract each term from the next one in the sequence.

ΔList({2,5,10,17,26}) ▸ {3,5,7,9}

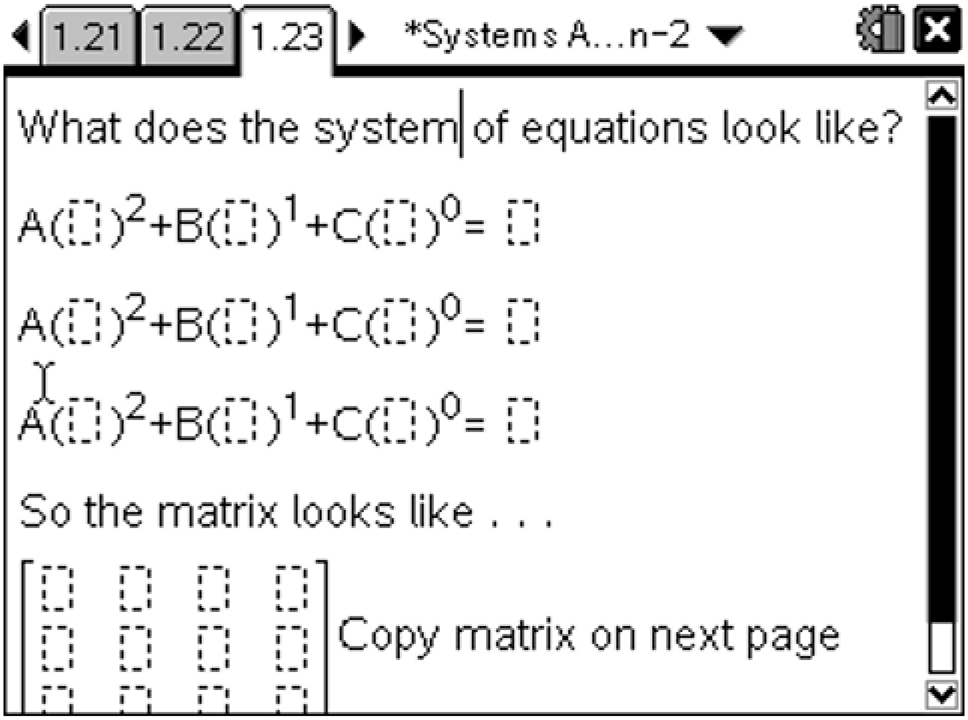
Is the difference constant? If it is we have a linear function, if not, we'll need to do the same thing for the new set of numbers. Get ΔList from the List Operations submenu in the Statistics menu. Use next page.

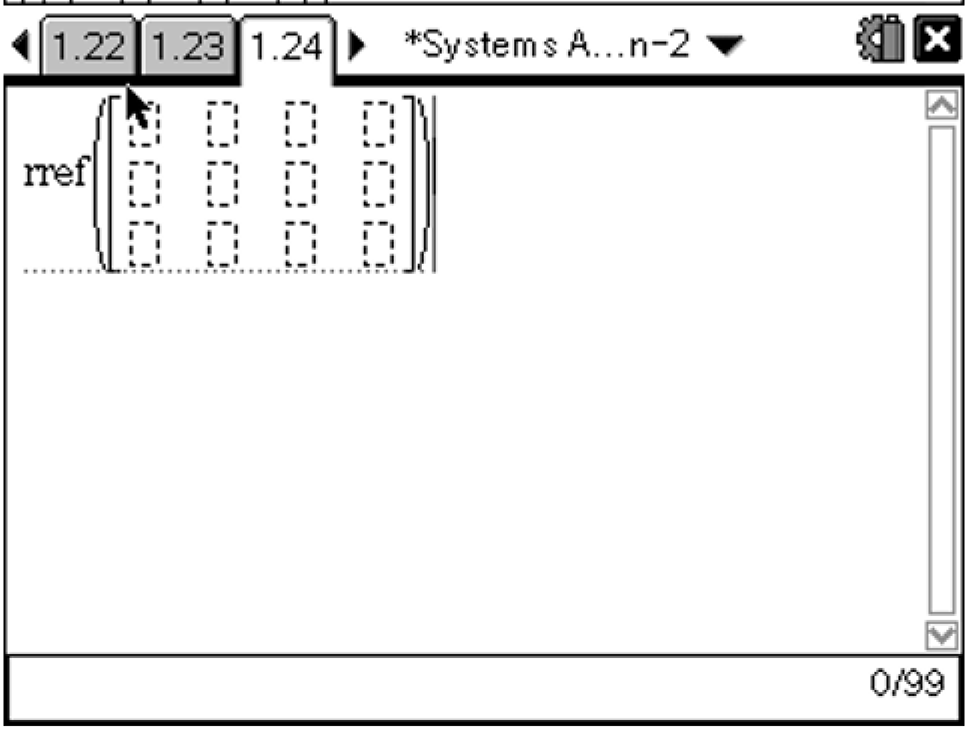
1.20 | 1.21 | 1.22 | *Systems A...n-2

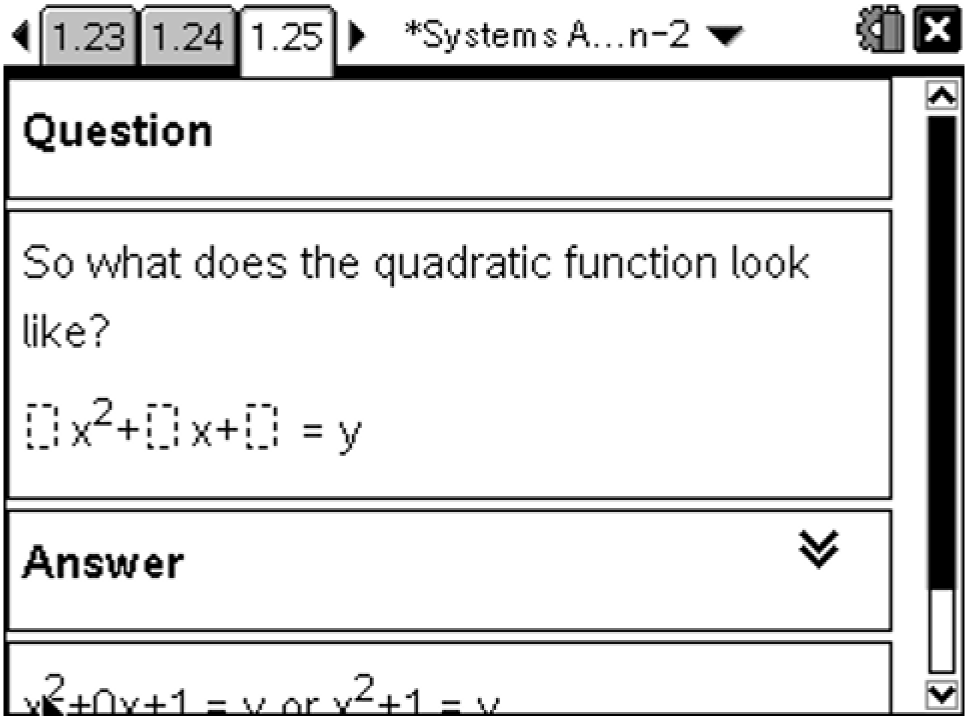
0/99

Is the difference constant?

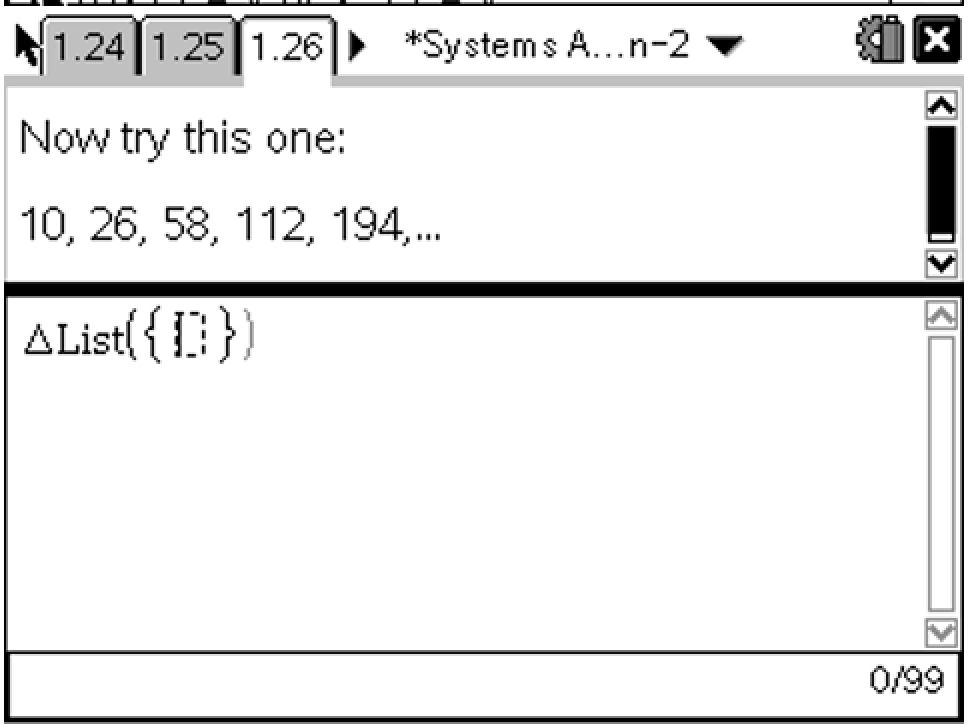
If it is, the function is quadratic, if not, we continue on. This one is, so we'll solve with matrices on the next page.


1.21
1.22
1.23
*Systems A...n-2
What does the system of equations look like?
A( )2+B( )1+C( )0=
A( )2+B( )1+C( )0=
A( )2+B( )1+C( )0=
So the matrix looks like . . .
Copy matrix on next page

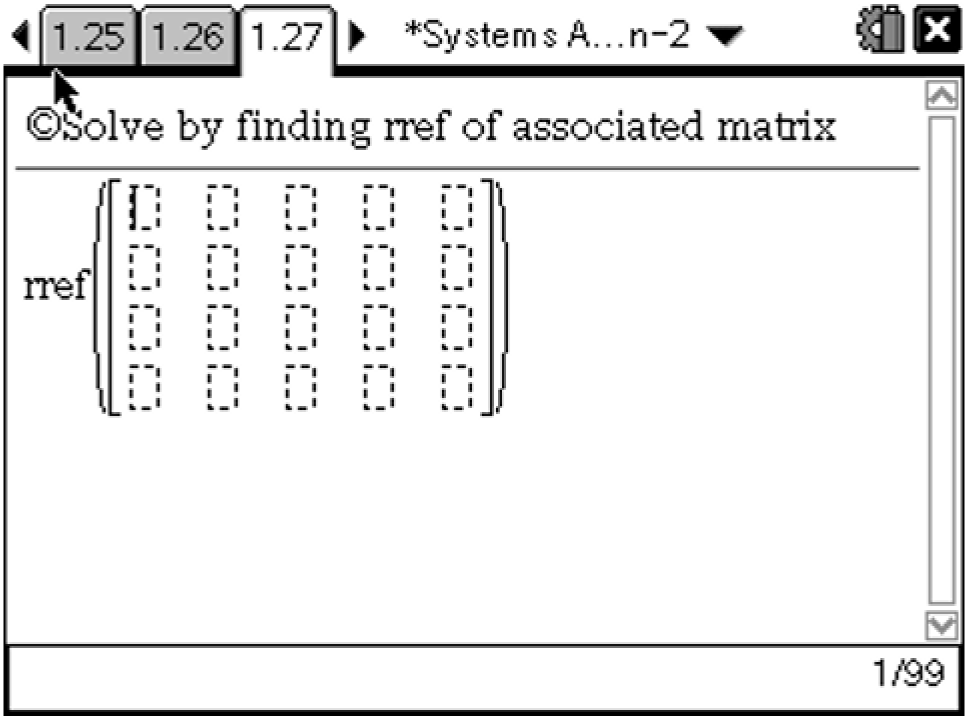

1.22
1.23
1.24
*Systems A...n-2
rref
0/99

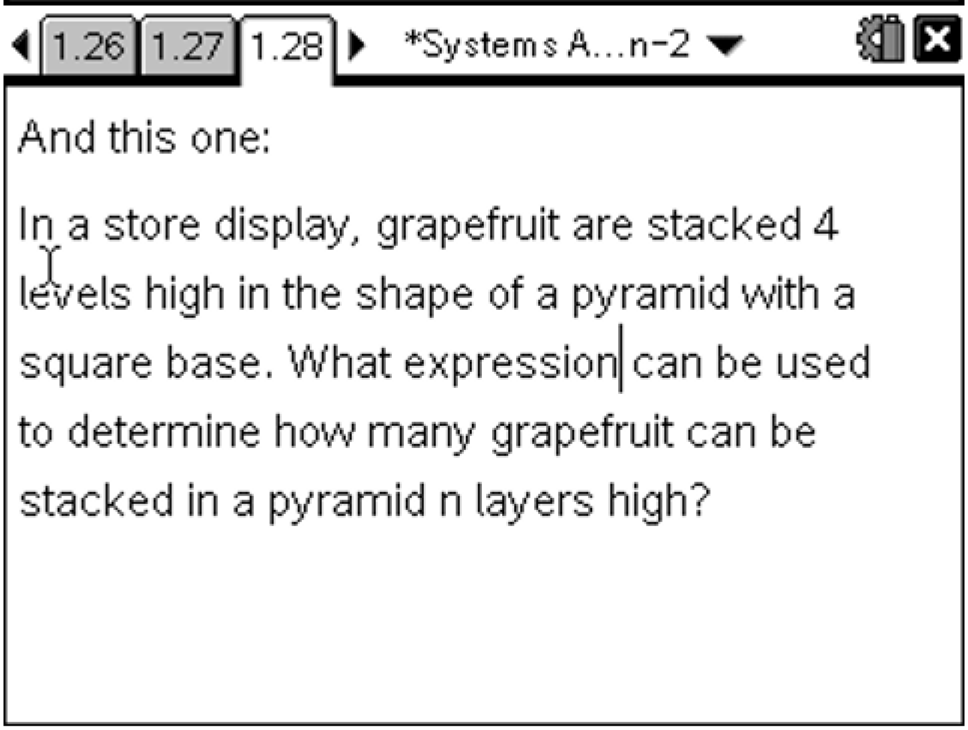


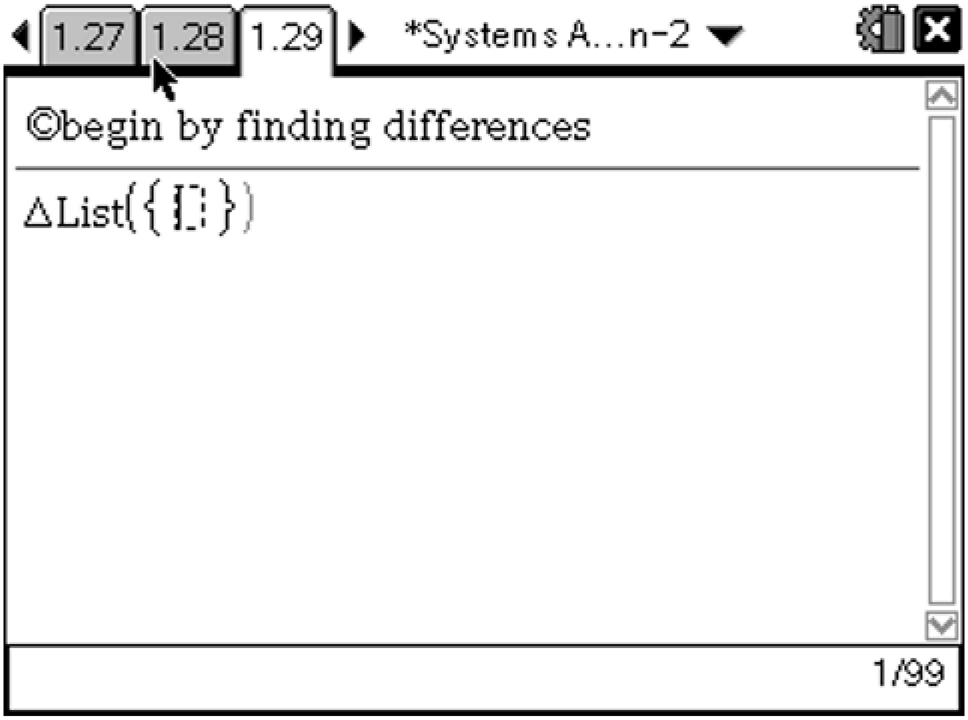
1.23 1.24 1.25
*Systems A…n-2
Question
So what does the quadratic function look like?
x2+ x+ = y
Answer

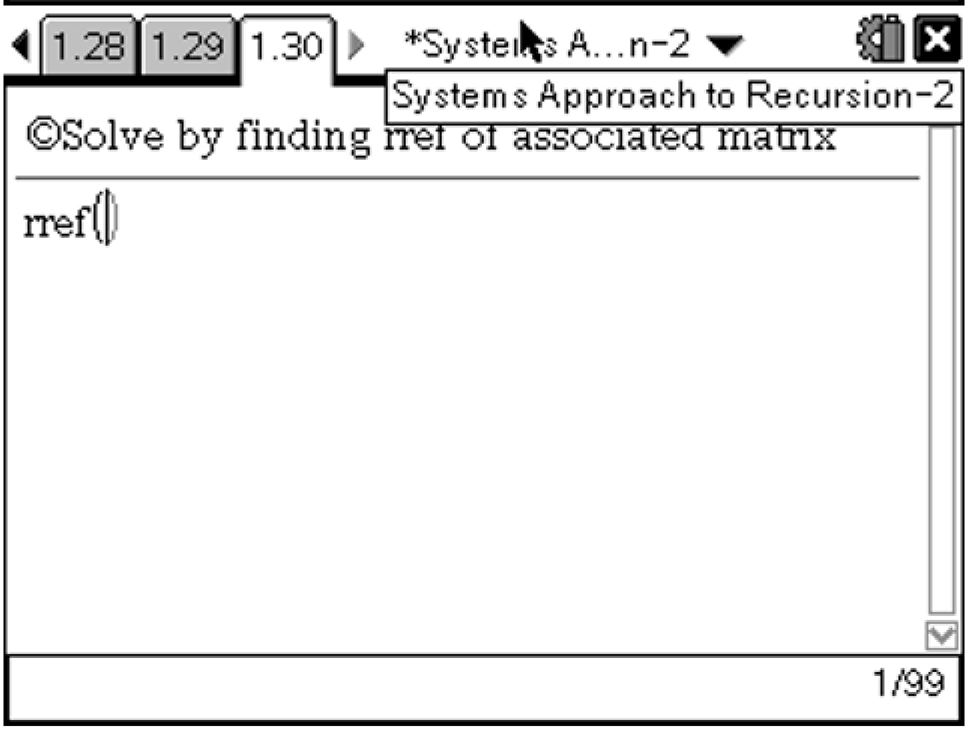


1.24 1.25 1.26
*Systems A…n-2
Now try this one:
10, 26, 58, 112, 194,...
ΔList({ })
0/99


1.25
1.26
1.27
*Systems A...n-2
©Solve by finding rref of associated matrix
rref
1/99


1.26
1.27
1.28
*Systems A...n-2
And this one:
In a store display, grapefruit are stacked 4 levels high in the shape of a pyramid with a square base. What expression can be used to determine how many grapefruit can be stacked in a pyramid n layers high?


1.27 1.28 1.29
*Systems A...n-2
©begin by finding differences
ΔList({ })
1/99


1.28 1.29 1.30
*Systems A...n-2
Systems Approach to Recursion-2
©Solve by finding rref of associated matrix
rref()
1/99