That Area Is So Regular

ID: 11685

Time Required
45 minutes

Activity Overview

In this activity, students will investigate the formula for the area of regular polygons given the length of one side. Students will be asked inquiry questions to assist them in deriving the formula for a regular polygon with n = 3, 4, and 5 sides. They will then use the patterns they discovered to find the formula for the area of a regular polygon of n sides.

Topic: Right Triangles & Trigonometric Ratios

- *Regular Polygons*
- *Area*
- *Trigonometry*

Teacher Preparation and Notes

- *This activity was written to be explored with the TI-Nspire.*
- *This activity may be best explored as a teacher lead activity.*
- *This is an introductory activity where students will need to know how to change between pages, grab and move points, and use the Calculator.*
- *The multiple choice items are self-check and students can check them by pressing* ctrl + ▲.
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11685" in the keyword search box.***

Associated Materials

- *AreaRegular_Student.doc*
- *AreaRegular.tns*

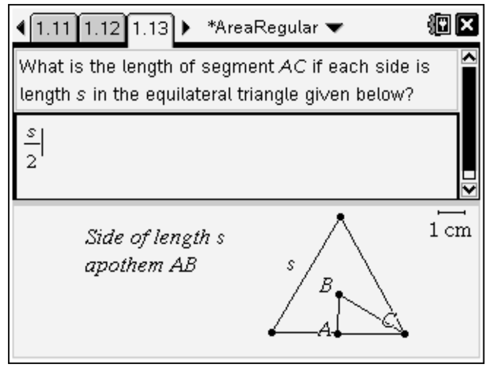
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Area of a Regular Polygons (TI-84 Plus) — 7227*
- *Inscribed Regular Polygons (TI-Nspire technology) — 9203*
- *Areas of Regular Polygons and Circles (TI-84 Plus) — 7340*

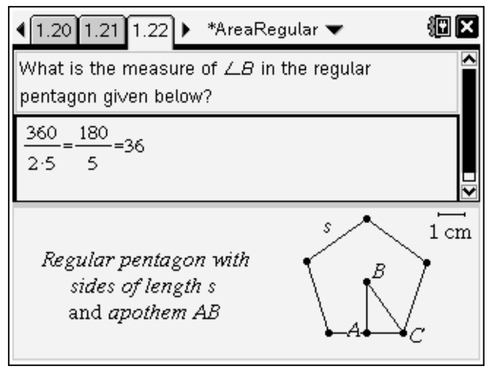
Problem 1 – Discovering the Area of a Regular Polygon Given the Length of Each Side.

In this activity, students will investigate the area of a regular polygon given the length of one side. Beginning with the formula for the area of a regular polygon (one half times the apothem times the perimeter), we will derive the formula for the area of a regular polygon in terms of the length of one side.



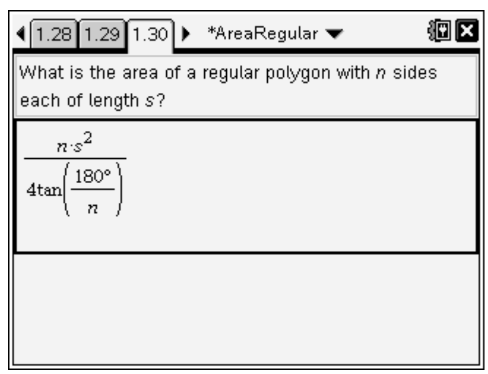
Students will first derive the formula for an equilateral triangle. Students are asked inquiry questions designed to lead them to the formula $Area = \dfrac{3 \cdot s^2}{4\tan\left(60^\circ\right)}$.

This part of the activity may be best explored as a teacher led activity.



Students will then be asked to derive the formula for a regular polygon with $n = 4$ and $n = 5$ sides. The goal is for students to develop the pattern in order to find the formula for a regular polygon of n sides. Therefore, Problem 1 is very repetitive, but the repetition is very important for pattern recognition.

The formula for $n = 4$ is $Area = \dfrac{4 \cdot s^2}{4\tan\left(45^\circ\right)}$ and the formula for $n = 5$ is $Area = \dfrac{5 \cdot s^2}{4\tan\left(36^\circ\right)}$.

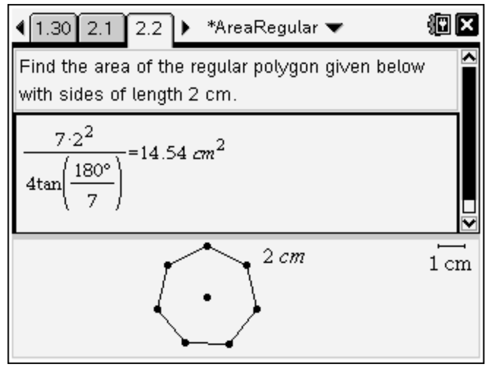


Eventually, students will be asked to derive the formula for a regular polygon with n sides each of length s through inquiry questions that are identical to the questions for $n = 3$, 4, and 5. The formula is

$$Area = \frac{n \cdot s^2}{4\tan\left(\frac{180^\circ}{n}\right)}.$$

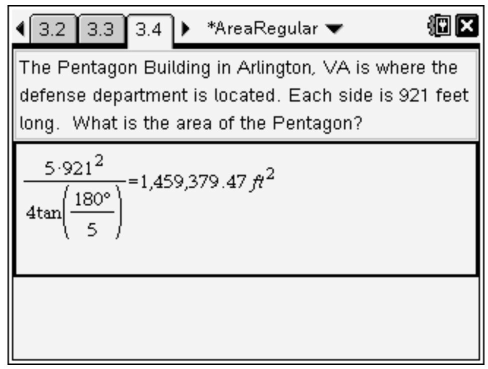
Problem 2 – Applications of the Area of a Regular Polygon

For this problem students will be asked to apply the formula they developed in Problem 1.



Problem 3 – Real-World Applications of the Area of Regular Polygons

In this problem, students are asked to solve real-world applications of the area of regular polygons. They are asked to find areas of a stop sign, a yield sign, the Pentagon Building, and the Sigil of Ameth.



Student Solutions

1. Area = $\frac{1}{2}$(apothem)(perimeter)

2. $\frac{360}{4} = 90$

3. $\frac{360}{5} = 72$

4. $\frac{360}{n}$

5. $p = ns$

6. $\frac{360^\circ}{6} = \frac{180^\circ}{3}$ = 60°; this is because the center of the polygon is formed by the angle bisectors of the vertices of the regular polygon.

7. $\frac{s}{2}$

8. $\tan(60^\circ) = \frac{s/2}{AB} \Rightarrow AB = \frac{s}{2\cdot\tan(60^\circ)}$

9. $Area = \frac{1}{2}(apothem)(perimeter)$

$$= \frac{1}{2} \cdot \frac{s}{2 \cdot \tan(60^\circ)} \cdot (3s)$$

$$= \frac{3 \cdot s^2}{4 \cdot \tan(60^\circ)}$$

10. $\frac{360}{8} = \frac{180}{4} = 45$

11. $\frac{s}{2}$

12. $\tan(45^\circ) = \frac{s/2}{AB} \Rightarrow AB = \frac{s}{2 \cdot \tan(45^\circ)}$

13. $Area = \frac{1}{2}(apothem)(perimeter)$

$$= \frac{1}{2} \cdot \frac{s}{2 \cdot \tan(45^\circ)} \cdot (4s)$$

$$= \frac{4 \cdot s^2}{4 \cdot \tan(45^\circ)}$$

14. $\frac{360}{10} = \frac{180}{5} = 36$

15. $\frac{s}{2}$

16. $\tan(36^\circ) = \frac{s/2}{AB} \Rightarrow AB = \frac{s}{2 \cdot \tan(36^\circ)}$

17. $Area = \frac{1}{2}(apothem)(perimeter)$

$$= \frac{1}{2} \cdot \frac{s}{2 \cdot \tan(36^\circ)} \cdot (5s)$$

$$= \frac{5 \cdot s^2}{4 \cdot \tan(36^\circ)}$$

18. $\frac{360}{2n} = \frac{180}{n}$

19. $\frac{s}{2}$

20. $\tan\left(\frac{180^\circ}{n}\right) = \frac{s/2}{AB} \Rightarrow AB = \frac{s}{2 \cdot \tan\left(\frac{180^\circ}{n}\right)}$

21. $$\begin{aligned} Area &= \frac{1}{2}(apothem)(perimeter) \\ &= \frac{1}{2} \cdot \frac{s}{2 \cdot \tan\left(\frac{180^o}{n}\right)} \cdot (ns) \\ &= \frac{n \cdot s^2}{4 \cdot \tan\left(\frac{180^o}{n}\right)} \end{aligned}$$

22. $Area = \frac{7 \cdot 2^2}{4 \cdot \tan\left(\frac{180^\circ}{7}\right)} = 14.54\,\text{cm}^2$

23. $Area = \frac{25 \cdot 9^2}{4 \cdot \tan\left(\frac{180^\circ}{25}\right)} = 4007.38\,\text{cm}^2$

24. $Area = \frac{12 \cdot 3^2}{4 \cdot \tan\left(\frac{180^\circ}{12}\right)} = 100.77\,\text{in.}^2$

25. $Area = \frac{8 \cdot 12.5^2}{4 \cdot \tan\left(\frac{180^\circ}{8}\right)} = 754.44\,\text{in.}^2$

26. $Area = \frac{3 \cdot 90^2}{4 \cdot \tan\left(\frac{180^\circ}{3}\right)} = 3{,}507.40\ \text{cm}^2$

27. $Area = \frac{5 \cdot 921^2}{4 \cdot \tan\left(\frac{180^\circ}{5}\right)} = 1{,}459{,}379.47\ \text{ft}^2$

28. $Area = \frac{7 \cdot 7^2}{4 \cdot \tan\left(\frac{180^o}{7}\right)} = 178.06\ \text{cm}^2$