Math Objectives

- Students will be able to identify the graphical connections between a function and its accumulation function.
- For a given function, students will recognize the accumulation function as an antiderivative of the original function.
- Students will be able to apply and explain the second Fundamental Theorem of Calculus.
- Construct viable arguments and critique the reasoning of others. (CCSS Mathematical Practice)
- Look for and make use of structure. (CCSS Mathematical Practice)

Vocabulary

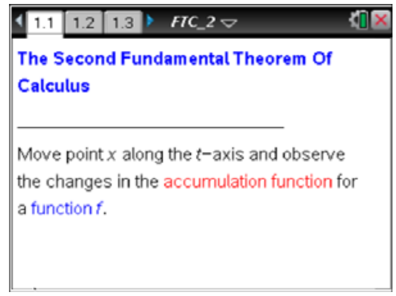
- accumulation function
- definite integral
- antiderivative

About the Lesson

- The intent of this lesson is to help students make visual connections between a function and its definite integral.
- Students will use the accumulation function with a fixed starting point to find definite integrals of a function over different intervals.
- Students will observe that the accumulation function is an antiderivative of the original function.
- Students will apply the antiderivative property of the accumulation function in combination with their use of the accumulation function to determine a definite integral.
- The lesson concludes with students stating and applying an informal statement of the second Fundamental Theorem of Calculus.
- This lesson is designed to follow the lesson *The First Fundamental Theorem of Calculus*.

TI-Nspire™ Navigator™ System

- Use Screen Capture to demonstrate that students can grab and drag the point *x* properly.
- Use Document Collection to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- Press ctrl G to either hide the function line or access the function line in a *Graphs & Geometry* page.

Lesson Materials:
Student Activity

- FTC_2_Student.pdf
- FTC_2_Student.doc

TI-Nspire document

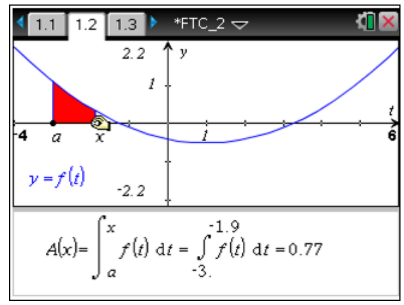
- FTC_2.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Then press ctrl to grab the point and close the hand ( ).

Move to page 1.2.



1. The graph shown is of the function $y = f(x)$. The **accumulation function** of $f(t)$ from a to x is given by $A(x) = \int_a^x f(x)\,dx$. The accumulation function gives the value of the definite integral of $f(t)$ between a and x. Set $a = -3$ and find the following:

   a. $A(3) = \int_{-3}^{3} f(x)\,dx$ = ______________

   <u>Answer:</u> –0.6

   b. $A(0) = \int_{-3}^{0} f(x)\,dx$ = ______________

   <u>Answer:</u> 0.6

2. Without changing the value of a, how could you use the values of the accumulation function in question 1 to find $\int_0^3 f(t)\,dt$? Explain your thinking.

   <u>Answer:</u> Looking at the graph, you see that the integral from 0 to 3 is the same as the integral from –3 to 0 subtracted from the integral from –3 to 3. So you can use the accumulation function, taking $A(3) - A(0) = -0.6 - 0.6 = -1.2$.

3. Without changing the value of a, use the accumulation function and your thinking from question 2 to find the following. For each, be sure to explain your thinking.

   a. $\int_1^4 f(t)\,dt$ = ___________________________

   <u>Answer:</u> –0.6.

   $\int_1^4 f(t)\,dt = A(4) - A(1) = -0.467 - 0.133 = -0.6$

b. $\int_{-2}^{2} f(t)\,dt$ = ______________________

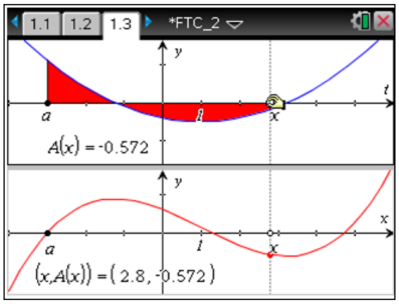
Answer: –1.066
$\int_{-2}^{2} f(t)\,dt = A(2) - A(-2) = -0.333 - 0.733 = -1.066$

c. $\int_{0}^{-1} f(t)\,dt$ = ______________________

Answer: 0.267
$\int_{0}^{-1} f(t)\,dt = A(-1) - A(0) = 0.867 - 0.6 = 0.267$

TI-Nspire Navigator Opportunity: *Embedded Assessment*
See Note 1 at the end of the lesson.

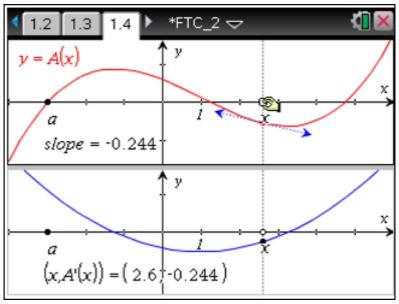
Move to page 1.3.



4. The top graph shows the original function, *f*, and the measurement of an accumulation function as the point *x* is dragged along the *t*-axis. The bottom graph shows the accumulation function as a function of *x*. What relationship, if any, do you notice between the original function and the accumulation function? Explain.

Answer: The original function appears to be the derivative of the accumulation function. The interval over which *A* is decreasing is the same as that over which *f* is negative, while the intervals over which the graph of *A* is increasing are the same as those over which *f* is positive. When the graph of *A* is concave up, *f* is increasing, while when the graph of *A* is concave down, *f* is decreasing.

CCSS Mathematical Practice: Construct viable arguments and critique the reasoning of others.
Mathematically proficient students will make a conjecture that the original function appears to be the derivative of the accumulation function, building on the logical progression of ideas noted above regarding when A is increasing, decreasing, concave up, and concave down.



Move to page 1.4.

5. The top graph on page 1.4 is the graph of the accumulation function, $y = A(x)$, for the function f from the previous pages, and the bottom graph shows the graph of its derivative, $y = A'(x)$.

   a. Choose several values of x and find the corresponding values of $A'(x)$. For each of these, how do they compare to the value of $f(x)$ for that x? What do you observe? Does this make sense? Explain.

<u>Sample Answers:</u> $A'(1) = -0.5$, $A'(-2) = 0.4$, $A'(5) = 1.1$. Looking back at the graph of f, these appear to be the same as the values of $f(x)$ at the same x-values. Also, the graph of A' looks like the graph of f. In question 4, I said that f appears to be the derivative of A, so this is consistent.

   b. Given your response to a, complete the following:
   $f(x)$ is ______________________ of $A(x)$.

<u>Answer:</u> the derivative

   $A(x)$ is ______________________ of $f(x)$.

<u>Answer:</u> an antiderivative

6. Drag point a on the top graph on page 1.4.

   a. What are you changing in the accumulation function when you change a? What are you changing in the graph of the accumulation function? Explain.

<u>Answer:</u> In the accumulation function, you are changing the lower bound of the definite integral by which the accumulation function is defined. It appears in the graph that this results in a shift in the graph. This would make sense, because, for example, if you change a from –3 to –2, you are now looking at integrals from –2 to x instead of from –3 to x. But this gives $\int_{-2}^{x} f(t)\,dt = \int_{-3}^{x} f(t)\,dt - \int_{-3}^{-2} f(t)\,dt$, and $\int_{-3}^{-2} f(x)\,dx$ is simply a constant value. Adding or subtracting a constant from a function results in a vertical shift of the graph of the function.

b. Using what you know about the accumulation function, why do you think the bottom graph doesn't change when you change the value of *a*? Explain.

Answer: The accumulation function is an antiderivative of the original function. When you change to a new value of the lower limit *a*, you get the previous accumulation function (using the original value of the lower limit *a*) plus or minus a constant. (This constant represents the definite integral between your original lower limit and your new lower limit.) Of course, an antiderivative plus or minus a constant is still an antiderivative, so its slope (or derivative) will be the same as any other antiderivative. This yields the same graph of the derivative function below.

> **CCSS Mathematical Practice:** Look for and make use of structure.
> Mathematically proficient students will step back for an overview to connect why the bottom graph doesn't change to the zero value of the derivative of the constant that has been added or subtracted from the accumulation function by changing the value of *a*.

7. Suppose you are given that an accumulation function for a continuous function $f(x)$ can be expressed as $A(x) = x^2 + 3$. Explain how you can use this to find $\int_2^4 f(x)\,dx$.

Answer: $\int_2^4 f(x)\,dx = A(4) - A(2) = [(4)^2 + 3] - [(2)^2 + 3] = 19 - 7 = 12$

8. Based on your answers to questions 5 and 6, how do you think you would find a formula for an accumulation function of a continuous function without using the integral? Explain.

Answer: From question 5, you know that the accumulation function is an antiderivative of the original function, and from question 6, you know that any antiderivative will work, since constants added to the accumulation function do not change the result. Thus, you should be able to find an antiderivative of the original function, and that would be an accumulation function.

9. Using your response to question 8, describe how you would find the value of a definite integral, for a continuous function *f*.

Answer: To find the definite integral from *a* to *b* of a continuous function *f*, find an antiderivative *F* such that $F'(x) = f(x)$, and calculate $F(b) - F(a)$.

10. Use your response to question 8 to find $\int_0^3 2x\,dx$. Explain your solution. How can you check your work?

Answer: An antiderivative of $2x$ is x^2, so $\int_0^3 2x\,dx = (3)^2 - (0)^2 = 9 - 0 = 9$. One way to check is to use geometry to find the area under $2x$ between 0 and 3.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The second Fundamental Theorem of Calculus.

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Note 1

Question 3, *Embedded Assessment*

- After completing questions 1–3 on the student worksheet, students respond to the question added to the .tns file. Upon completion of the activity, use the TI-Nspire Navigator™ Class Analysis feature to show the slide show of student responses and discuss the results.

  Answers:
  question 1: First choice
  question 2: Third choice
  question 3: Second choice
  question 4: Third choice